PROLOGUE

The main part of this book, **Chapters 3-8**, consists of dialogues between two characters M, a mathematician, and RME, a researcher in mathematics education. These dialogues focus on a range of issues regarding the learning and teaching of mathematics at university level. Each dialogue starts from a discussion of a sample of data (students' writing or speaking) that exemplify these issues. Both the samples that M and RME discuss as well as their discussions are grounded in data collected in the course of several studies that I have been involved in since 1992. In **Chapter 1** I outline the studies that formed the raw material for the book as well as the wider area of mathematics education research these studies are embedded in and the book aims to contribute to. In **Chapter 2** I outline the processing that the data collected in these previous studies has gone through in order to reach the dialogic form in which it is presented in Chapters 3-8. **Chapter 3** focuses on students' mathematical reasoning and in particular their conceptualization of the necessity for Proof and their enactment of various proving techniques. **Chapter 4** shifts the focus towards the students' mathematical expression and their attempts to mediate mathematical meaning through words, symbols and diagrams. **Chapters 5** and **6** offer accounts of the students' encounter with some fundamental concepts of advanced mathematics – Functions (across the domains of Analysis, Linear Algebra and Group Theory) and Limits. **Chapter 7** revisits many of the 'learning stories' told in Chapters 3-6 in order to highlight issues of university-level pedagogy. Finally, in **Chapter 8** M and RME, starting from the experience of working together in the context of the studies on which the book is based – and as showcased in Chapters 3-7 – discuss their often fragile relationship as well as the necessary and sufficient conditions for a collaboration between mathematicians and researchers in mathematics education. The book concludes with: a brief **Epilogue** in which I reflect on the experience of engaging with the research behind this book, and with its production, and I outline some steps that, at the final stages of writing, this research was beginning to take; and, with a **Post-script** in which I offer a chronological and reflexive account of the events that led to the production of the book.

    Note to the reader regarding Chapters 1 and 2. Chapter 1 describes the theorerical background and previous studies on which the book is based and Chapter 2 the method through which the dialogues in Chapters 3-8 came to be. Readers less interested in these may wish to skip these chapters and go directly to Chapter 3. For minimal familiarisation there is a one-page summary at their beginning.

    Note to the reader regarding Chapters 3-8. Each of these consists of Episodes which I recommend that you read as follows: engage briefly with the mathematics in the Episode (problem, solution and examples of student response); reflect on the learning/teaching issues these may generate; and, read and reflect on the dialogue between M and RME. There is a more elaborate version of this recommendation between Chapters 2 and 3 and a one-page summary of the issues covered at the beginning of each chapter.

CHAPTER 2
METHOD, PROCESS AND PRESENTATION

SUMMARY

In this chapter I explain how the data samples that M and RME discuss came to be and who 'is' M; the dialogue composition process; and, the style and thematic breakdown of Chapters 3 – 8. The presentation is in three parts.

Part 1. Data samples and M
How the data samples that M and RME discuss came to be; the participating mathematicians that 'became' M.

The dialogues between M and RME in Chapters 3 – 8 originate in half-day focused group interviews with mathematicians of varying experience and backgrounds from across the UK. In the interviews discussion was triggered by data samples consisting of students' written work, interview transcripts and observation protocols collected during (overall typical in the UK) Year 1 introductory courses in Analysis / Calculus, Linear Algebra and Group Theory.

Part 2. The dialogic format
The Narrative Approach adopted in this work; the composition process through which the dialogues between M and RME came to be.

The dialogues between M and RME are fictional, yet data-grounded: they were constructed entirely out of the raw transcripts of the interviews with the mathematicians and then thematically arranged in *Episodes*. For an example of the construction process see p27-28.

Part 3. Style, format and thematic breakdown of Chapters 3 – 8

Chapters were constructed as series of *Episodes* (sometimes also broken in *Scenes*). Each Episode starts with a mathematical problem and (usually) two student responses. A dialogue between M and RME on issues exemplified by the student responses follows. Other examples of relevant student work are interspersed in the dialogue and links with relevant mathematics education research literature are made in the footnotes. *Special Episodes* are episodes that supplement the discussion in the main *Episodes* and *Out-Takes* are slightly peculiar or too specific incidents that stand alone and outside the more 'paradigmatic' material of the main *Episodes* but somehow address the wider theme of a chapter.

Notes to the reader:

- The account is in, more or less, chronological order and it is intended to be as transparent as possible. By revealing – hopefully without too much pedantry – the details of the construction process (of data samples, dialogues and chapters), I am opening up this process to critique, a quintessential element to its validation.
- In what follows I use the same abbreviations for the studies I introduced in Chapter 1, Part 3.

1. DATA SAMPLES AND M

The bulk of the data used for the composition of the dialogues between M and RME in Chapters 3 – 8 originates in the Focused Group Interviews with mathematicians in the course of Study L. Here I return to the brief description of that Study in Chapter 1, Part 3 in order to zoom-in on some of that study's features and provide information that is a necessary prerequisite for understanding the context and background of the dialogues between M and RME in Chapters 3 – 8.

Chapters 3 – 8 consist of a series of Episodes in which M and RME set out from a discussion of a data sample which, in most cases, operates as a trigger for addressing an issue on the learning and teaching of mathematics at the undergraduate level. These data samples are parts of the Datasets used for the same purpose in the interviews during Study L. In Chapter 1 I outline these Datasets as consisting of a short literature review and bibliography; samples of student data (e.g.: students' written work, interview transcripts, observation protocols) collected in the course of Studies D and N; and, a short list of issues to consider. Here I provide a bit more information.

As studies D and N had focused primarily on collecting data from Year 1 mathematics undergraduates, the focus of the Datasets (which built on the data and analyses of those studies) reflects the students' experience in Year 1[1]. The courses, in particular the parts referred to in the dialogues in Chapters 3 – 8, are mostly typical parts of introductory courses in Analysis / Calculus, Linear Algebra and Group Theory[2]. Paola Iannone, who conducted Studies N and L with me, and I were occasionally asked to provide further information about the stage of the students' studies that a particular data sample was exemplifying. However participants

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[1] With the exception of Study N's foray into Year 2 for the Group Theory course that had then recently been moved from Year 1. As Group Theory is a splendid context for exploring the issues of abstraction and formalism that the previous studies had explored extensively, for continuity and coherence, we decided that this 'digression' into Year 2 would be most pertinent.

[2] For Study N the relevant courses were: *Analysis and Algebra*, *Probability* and *Groups and Rings* (content available at http://www.mth.uea.ac.uk/maths/syllabuses/0506/). For Study D the relevant courses were: *Continuity and Differentiability, Linear Algebra* and *Group Theory* (content available at http://www.maths.ox.ac.uk/current-students/undergraduates/lecture-material/). The parts of the courses that the discussion of M and RME focuses in Chapters 3 – 8 have not changed radically since the studies were conducted but I hold details of the courses as offered in the years in which the studies were conducted (2000-1, 2001-2 for Study N; 1993-4 for Study D)

generally recognised that the material discussed in the samples was typical of the students' early experiences of university mathematics. Of course the reader needs to bear in mind that all studies referred to here were conducted in the UK.

There were six Datasets, five 'mathematical' (as listed in the Study L outline in Chapter 1, Part 3) and one 'meta-mathematical' (on the theme of collaboration between mathematicians and mathematics educators and mathematicians' engagement with educational research).

The eleven half-day interviews that constitute the interview material of Study L were conducted as follows: six, one for each Dataset, at the University of East Anglia, where the data for Study N had been collected. The five 'mathematical' Datasets were also used in five analogous events in non-UEA institutions across the UK: two in England, one in Scotland, one in Wales and one[3] in the context of a group-work session at the Annual Conference of the Mathematical Association[4]. In total there were twenty participants, pure and applied mathematicians, all-bar-one male, white and European (but several with significant international experience), of age ranging from early thirties to late fifties and of teaching experience varying from a few years to a few decades.

Interviews were conducted during the academic year 2002 – 2003. They lasted approximately four hours and revolved around the above described Datasets. These had been distributed to participants at least a week prior to the interview. Most participants arrived at the interviews well prepared, with comments and questions scribbled in the margins and keen on a close examination of the Datasets. The interviews were conducted according to the principles of Focused Group Interviews (Wilson, 1997; Madriz, 2001) – see (Iannone & Nardi, 2005a) for a rationale and a description of our use of this tool.

As you can see in Chapters 3 – 8, each Episode starts with the discussion of a data sample that, in most cases, consists of:

- a mathematical problem (including its formulation as well as the suggested solution distributed to the students once they had submitted their written responses to their tutor[5])
- two typical student responses[6]

In the course of the discussion between M and RME, the latter presents M with more student responses[7]. All of the data samples originate in the Datasets and in the extra examples that were used in the course of the interviews. In the next Part of this chapter I explain how the raw material of the interviews was turned into the dialogues in Chapters 3 – 8.

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[3] Replacing one institution which was happy to participate in principle but with which scheduling negotiations failed to result in an arrangement within the timeline of the project's data collection period.

[4] In this we used the Dataset we would have used in the missing institution mentioned above.

[5] See description of the data collection in Study N in Chapter 1, Part 3.

[6] Namely responses that typify the issue that the part of the Dataset aimed to address in the original interviews with the mathematicians.

[7] Replicating the events in the original interviews.

2. THE DIALOGIC FORMAT

In what follows (Part 2(i)) I place the choice of the dialogic format within the increasingly vigorous methodological tradition of Narrative Approaches in Qualitative Research (Denzin & Lincoln, 2001) and zoom in on my particular rationale for this choice. I then explain[8] (Part 2(ii)) how I applied some of the techniques of the Narrative Approach towards the composition of the dialogues presented in Chapters 3 – 8.

(i) The Narrative Approach

> '…all you can do, if you really want to be truthful, is to tell a story.'
> John Mason (1998, p367) consolidating Paul Feyerabend's epistemological position as
> elaborated in his *Three Dialogues on Knowledge* (1991)

Qualitative data analysis aims to produce generalisations embedded in the contextual richness of individual experience. Coding and categorising techniques, for example, a method I use and generally respect, often results in texts sorted out into units of like meaning. Despite evident benefits of this, such as facilitating access to interpretation of the situation in question etc., this sorting out can strip contextual richness away. A more holistic account, based on the rapidly developing Narrative Approach, can often be even more illuminating.

> 'People tell stories about their life experiences. Telling stories helps people to think about, and understand, their personal or another individual's, thinking, actions, and reactions (Bruner, 1986, 1990; Polkinghorne, 1988; Ricoeur, 1991). Thus, it is not surprising that collecting stories has emerged as a popular form of interpretive or qualitative research (Gudmundsdottir, 1997). It has rapidly gained legitimacy in education and has flourished at research conferences and in professional development activities in schools (Connelly & Clandinin, 2000).
> Over the past 20 years, the popularity of narrative research in the social sciences and education is evident from an increase in narrative publications having to do with narrative questions, phenomena, or methods (Lieblich et al, 1998). Narrative brings researchers and educators together collaboratively to construct school experiences (Connelly & Clandinin, 1990). It provides a voice for teachers and students (Errante, 2000), and it places emphasis on the value of stories in all aspects of life (McEwan & Egan, 1995).' (Ollerenshaw & Creswell 2002, p329-330)

My own general understanding of the potency of this approach has come largely from Clandinin & Connelly's *Narrative Inquiry* (2000), an extension of their influential 1990 article in the *Educational Researcher* and their other work at the time.

> 'Connelly and Clandinin's advocacy for this form of qualitative inquiry has deep roots in the social sciences and the humanities (Casey, 1995-1996; Cortazzi, 1993; Polanyi, 1989; Polkinghorne, 1988). Procedures for finding tellers and collecting their stories has

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[8] Sometimes I do so with reference to the process of conceptualising the book as a whole, a process I give an account of in the Post-script. However I have intended the account in this part to be self-contained.

emerged from cultural studies, oral history, folklore, anthropology, literature, sociology, and psychotherapy. Interdisciplinary efforts at narrative research have been encouraged by Sage Publications through their *Narrative Study of Lives* annual series that began in 1993 (Josselson & Lieblich, 1993)'. (Ollerenshaw & Creswell 2002, p331)

Understandings on what constitutes narrative research, or 'narratology' (Connelly & Clandinin, 1990), are very diverse. Ollerenshaw & Creswell outline some common characteristics as follows:

'The inquirer emphasizes the importance of learning from participants in a setting. This learning occurs through individual stories told by individuals, such as teachers or students. For Clandinin and Connelly (2000), these stories report personal experiences in narrative inquiry (what the individual experiences) as well as social experiences (the individual interacting with others). This focus on experience draws on the philosophical thoughts of John Dewey, who saw that an individual's experience was a central lens for understanding a person. One aspect of Dewey's thinking was to view experience as continuous (Clandinin & Connelly, 2000), where one experience led to another experience. The stories constitute the data, and the researcher typically gathers it through interviews or informal conversations. These stories, called field texts (Clandinin & Connelly, 2000), provide the raw data for researchers to analyze as they retell or restory the story based on narrative elements such as the problem, characters, setting, actions, and resolution […]' (Ollerenshaw & Creswell 2002, p332)

Re-storying is the central feature of *holistic-content analysis*, one of Lieblich et al's (1998) four-type attempt to classify narrative analytic approaches, and the one that is most relevant to the work presented in this book.

'[*holistic-content analysis* is] a narrative approach for understanding the meaning of an individual's stories'. […] The holistic-content analysis of field texts (e.g., transcripts, documents, and observational field notes) includes more than description and thematic development [..]. It involves a complex set of analysis steps based on the central feature of "re-storying" a story from the original raw data. The process of re-storying includes reading the transcript, analyzing this story to understand the lived experiences (Clandinin & Connelly, 2000) and then retelling the story.' (Ollerenshaw & Creswell 2002, p330).

Ollerenshaw & Creswell (2002, p332) describe the process of re-storying as follows:

'Re-storying is the process of gathering stories, analyzing them for key elements of the story (e.g., time, place, plot, and scene), and then rewriting the story to place it within a chronological sequence. Often when individuals tell a story, this sequence may be missing or not logically developed, and by re-storying, the researcher provides a causal link among ideas. In the re-storying of the participant's story and the telling of the themes, the narrative researcher includes rich detail about the setting or context of the participant's experiences. This setting in narrative research may be friends, family, workplace, home, social organization, or school—the place in which a story physically occurs.
A story in narrative research is a first-person oral telling or retelling of events related to the personal or social experiences of an individual. Often these stories have a beginning, middle, and an end. Similar to basic elements found in good novels, these aspects involve a predicament, conflict, or struggle; a protagonist or character; and a sequence with implied causality (i.e., a plot) during which the predicament is resolved in some fashion (Carter, 1993). In a more general sense, the story might include the elements

typically found in novels, such as time, place, plot, and scene (Connelly & Clandinin, 1990). In this process, researchers narrate the story and often identify themes or categories that emerge from the story. Thus, the qualitative data analysis may be both descriptions of the story and themes that emerge from it. In addition, the researcher often writes into the reconstituted story a chronology of events describing the individual's past, present, and future experiences lodged within specific settings or contexts. Cortazzi (1993) suggested that it is the chronology of narrative research with an emphasis on sequence that sets narrative apart from other genres of research. Throughout this process of collecting and analyzing data, the researcher collaborates with the participant by checking the story and negotiating the meaning of the database. Within the story may also be the story of the researcher interwoven as she or he gains insight into himself or herself'.

A particularly helpful way of seeing the brand of (re)story-ing I have used is Jerome Bruner's account of how the mind constructs a sense of reality through 'cultural products, like language and other symbolic systems' (p3) in his *Critical Inquiry* article 'The Narrative Construction of Reality' (1991)[9]. Bruner proposes narrative as one of these cultural products and defines it in terms of the following ten characteristics:

1. Diachronicity (narratives deal with events taking place over a period of time)
2. Particularity (narratives deal with particular events)
3. Intentional State Entailment (characters within a narrative have 'beliefs, desires, theories, values, and so on')
4. Hermeneutic Composability (narratives can be interpreted as playing a role in a series of events that constitute a 'story')
5. Canonicity and breach (stories are about something unusual happening that 'breaches' a normal or canonical state)
6. Referentiality (a story references reality although it does not offer verisimilitude in any direct way)
7. Genericness (as a flipside to particularity, this is a characteristic of narrative whereby the story can be classified as representing a genre, as being paradigmatic)
8. Normativeness (as a follow up to 'canonicity and breach' a narrative may also make claims to how one *ought to* act)
9. Context Sensitivity and Negotiability (relating to hermeneutic composability, an understanding of a narrative requires that between the author, the text and the reader there is a negotiation regarding the contextual boundaries within which the narrative works)
10. Narrative accrual (stories are cumulative; new stories follow from older ones).

In this work I use Narrative in at least two senses: in the literal sense of Narrative as the form of processing and presenting the data collected in the studies I am drawing on. In this sense the Episodes in Chapters 3 – 8 are the stories I am constructing as a researcher in order to make sense of how the participants in my

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[9] I would like to thank Lulu Healy for bringing this part of Bruner's work to my attention.

studies experience learning, teaching etc. There is another sense parallel to this one though and one that is perhaps a bit closer to the one Bruner directly refers to in the above. Bruner talked about the stories children, for example, construct in order to make sense of the mathematics they are taught in school. Analogously the mathematicians participating in the studies I draw on here have their own 'stories', their own interpretive frames for making sense of things like their students' learning (in Chapters 3 – 6), their own pedagogical practices (in Chapter 7), the way they relate to mathematics educators and educational researchers (in Chapter 8) etc.. For example, 'landscapes' (an idea that recurs in their description of students' perception of mathematical concepts and that often resembles Vinner & Tall's (1981) 'concept image', a widely used descriptor in mathematics education research) seems to be one of the 'stories' employed in their accounts of their students' learning; 'mathematics as a language to master' seems to be their 'story' for interpreting students' writing; 'communication / interaction / gradual and negotiated induction into the practices of university mathematics' seems to be their 'story' for most of the teaching practices they express preference for; 'us and them' seems to be their 'story' for expressing a cautious attitude towards mathematics education research; etc.

Having placed this work within a certain strand of narrative inquiry I conclude this part with a presentation of my rationale for the use of the dialogic format as a way of presenting the 'stories' in Chapters 3 – 8.

My fascination with the dialogic format can be traced back in the 1980s and my first contact in school with the texts of Plato[10], first master of the format who consolidated earlier attempts at its use into what remain to this day some of the liveliest and most lucid philosophical texts. In his dialogues Plato (c400BC/1999), while tackling some of the hardest questions ever asked, offers perceptively drawn characters, minute contextual detail and a great sense of often deeply ironic humour. His dialectic, an exchange of theses and anti-theses between interlocutors resulting in a syn-thesis, particularly in its *Socratic* version[11], became a major literary form. A form often chosen by philosophers as a means to present their work all the way through to, for instance, Galileo (1638/1991) and Berkeley (1713) – and, in our days, Feyerabend (1991).

Feyerabend playfully dedicated his sometimes seen as incendiary *Against Method* (1975) to the man who brought the dialogic format closer to the preoccupations of mathematics educators than anyone else, Imre Lakatos. *Proofs and Refutations* (1976) is a fictional dialogue set in a mathematics classroom. It depicts students' attempts to prove the formula for Euler's characteristic. Through their successive attempts they re-live the trials and tribulations of the mathematicians who had previously engaged with it – most famously through the successive construction and employment of counterexamples that refute hitherto versions of the conjecture they are trying to prove.

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[10] I was educated in Greece where some of his texts are included in the Ancient Greek syllabus.
[11] The process of refuting or verifying an initial conjecture through searching for the contradictions its acceptance may yield.

The way I use the dialogic format in this work draws inspiration from the illustrative, evocative and suggestive powers of the medium that is evident in the above works. Lakatos, for example, aimed at demonstrating his view of the creative processes through which mathematics comes to be − and on the way, also inspired by Pólya (1945), he challenged our perception of how it is learnt and therefore how it ought to be taught. Plato employed the dialogic format, partly, as a platform to introduce us to the character of his teacher and mentor Socrates (and in his later works his own ideas as transformed by, and gradually independently of, Socrates' teaching). My aim here is to employ the dialogic format as a platform to introduce the character of M and, through his exchanges with the auxiliary, constantly prompting character of RME, showcase the rich canvas of perspectives on learning and teaching that have emerged from my collaborative work with mathematicians. In doing so I aim to demonstrate the potential that lies within this type of collaborative work (the main characteristics of which I describe in Chapter 1, Part 2).

Of course, unlike Plato / Socrates, there is no truth to be reached at the end of my lane and, unlike Lakatos, I am not proving any theorem. Alan Bishop (1998, p33) sums up a part of my rationale deftly in the summary of his involvement with an ICMI Study that aimed to offer a 'state of the nation' look at the field of mathematics education research at the time: '… I could detect certain emphases in the discourses together with some important silences' he observes and lists the silences as follows: syntheses, consensus-building, awareness of other audiences, researched arguing (to convince), over-arching structures, global theory, well-articulated similarities, agreements. This work aspires to address some of these silences − with the exception perhaps of 'global theory' on the teaching and learning of mathematics at the undergraduate level…!

Apart from the above, mostly philosophical, works another source of inspiration in my use of the dialogic format lies in theatre, particularly in a small number of plays where I feel that the subtle and the artful meet effectively. I am constantly fascinated by the capacity of great writers such as Tom Stoppard (particularly *Arcadia*) and Michael Frayn (particularly *Copenhagen* and *Democracy*) to touch on 'big issues' in ways that are accessible but are *'ni récréation ni vulgarisation'*, a difficult and in many ways problematic dual objective [12]. I have learnt a lot from the ways in which these authors manage to weave complex, multi-layered information into their characters' discourse: the superb marriage of the personal, political and scientific in *Copenhagen*; more the obvious, thus perhaps less elegant, way in which, say, Catherine's questions in David Pinner's *Newton's Hooke* prompt exposition on the part of Isaac Newton on his then developing ideas on Planetary Motion, Force, Fluxions / Calculus etc.. Without digressing into too lengthy a waxing lyrical about the virtues of these works let me exemplify with a short excerpt from Tom Stoppard's *Arcadia* (p48-50). In it Thomasina, age 13 and a prodigious

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[12] as reviewer Arnaud Spire observed in *L'Humanité* (http://www.humanite.presse.fr/journal/2001-04-18/2001-04-18-243018) in the context of discussing Isabelle Stengers' 2001 '*scientifiction*' foray into the Newton - Leibniz priority rift about the invention of Calculus.

student of in-house tutor Septimus is amazed to find out he has given her work an A Minus. 'What is the minus for?' she asks.

Septimus:	For doing more than was asked.
Thomasina:	You did not like my discovery?
Septimus:	A fancy is not a discovery.
Thomasina:	A jibe is not a rebuttal. […] I think it is an excellent discovery. Each week I plot your equations dot for dot, xs against ys in all manner of algebraical relation, and every week they draw themselves as commonplace geometry, as if the world of forms were nothing but arcs and angles. God's truth, Septimus, if there is an equation for a curve like a bell, there must be an equation for one like a bluebell, why not a rose? Do we believe nature is written in numbers?
Septimus:	We do.
Thomasina:	Then why do your equations only describe the shapes of manufacture?
Septimus:	I do not know.
Thomasina:	Armed thus, God could only make a cabinet.
Septimus:	He has mastery of equations which lead into infinities where we cannot follow.
Thomasina:	What a faint-heart! We must work outward from the middle of the maze. We will start with something simple *[She picks us an apple leaf.]* I will plot this leaf and deduce its equation […]! *[Septimus firmly orders Thomasina to return to the piece of poetry they were supposed to discuss next in the lesson.]*

Thirst for knowledge 19th century-style? Awakening of a mathematical mind albeit – unthinkable! – in the body of a young girl? Pedagogy as uninspired reduction and drudgery? … whatever discourse one chooses to trace in the above exchange, one thing that comes across clearly is the power of the dialogic format to carry through complex, multi-layered conversation in engaging and thought-provoking ways.

'This is not science, this is story-telling' (p125), Septimus exclaims later at the perplexing sight of Thomasina's writings, whose prose combines well-ahead-of-her-time mathematical preoccupations with grander speculations about the universe. His narrow-minded treatment of his student's work is not pertinent here just for the obvious pedagogical reasons but also, as an acknowledgement by Tom Stoppard of the dangerous waters his own enterprise is attempting to tread. I share Stoppard's qualms but I cannot help but acknowledge that we have been living, often productively, in a literary world of such hybrids for quite some time now, at least since Truman Capote's non-fiction novel *In Cold Blood.* At the end of the day the dialogic format is something that I feel a 'natural affinity for'[13]. It also suits perfectly well that of which M and RME speak of. In Part 2(ii) I describe the genetic process through which their dialogues came to be[14].

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[13] quoting *Copenhagen*'s Margrethe speculation about the reason why Heisenberg 'did Uncertainty', p78 – also see quotation following the title of this book.

[14] I would like to extend my warmest thanks to four friends, one of them also a colleague, with whom at this stage I shared the fledgling idea of using the dialogic format. At this very insecure stage had they cringed, had they insinuated anything like doubt about the idea, I may have never started. They are: Barbara Jaworski who supervised my work in Studies D and PD1, directed Study PD2, was consulted at various stages of Studies N and L, whose judgment I wholly trust and who insisted throughout that the

(ii) From interview transcripts to Dialogue: an application of the Narrative Approach

> *Vanbrugh:* […] The plot already exists… in real life. The play and all its scenes.
> *Cibber:*   A drama documenting facts? […] Will you allow yourself the same
> liberties as Shakespeare? Taking liberties with facts converts facts into plays.
> *Vanbrugh:* No liberties… just facts in this play.
> Carl Djerassi, *Calculus* (Djerassi & Pinner, 2003, aka 'Newton's Whores'), *Scene I*

Study L was completed in January 2004. From February to July 2004 I spent the largest part of a Study Leave on searching for a method to be used towards the composition of the dialogues between M and RME and on composing first drafts of the dialogues. In this part I describe how I arrived at this method[15] and how I used it for producing these first drafts. Then in Part 3 I describe how I turned these first drafts into the episodic Chapters 3 – 8.

Study L's eleven half-day Focused Group Interviews produced material that amounted to about 30,000 - 40,000 words per interview. The order of discussion in these, in most cases, followed the structure of the Datasets. That structure was as follows – see (Sangwin et al 2004) for a complete Dataset:

- two introductory pages: cover, interview scheduling details, a few lines on the theme of the Dataset;
- four sections[16] each entitled Example I, Example II, Example III and Example IV. Each Example was about a couple of pages long and contained scanned images of
    - a mathematical problem and a lecturer's recommended response (both from course materials)
    - usually two student responses (more were used in the course of the interview) from the data collected in the course of Study N (and /or in several occasions Study D)

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practitioners' perspectives and priorities should stay right at the forefront of my priorities as the author; Panos Karnezis and James Ferron Anderson – both writers (of the literary kind!) whose trade, I felt, my choice of the dialogic format would be seen as intruding – for helping me to clarify the distinction between using the format for literary and for academic purposes; and, Margarita Angelou, mathematician, for her overall encouragement and for reminding me that, even though the focus of the dialogue is deliberately on M, 'whoever reads the book they will want to know what *you* make of M as well'. Within the dialogues this is kept to a minimum – in symmetry with how the original interviews were conducted. But in the choice of themes of the Episodes and Chapters, in highlighting of M's characteristic 'behaviour' and in the overall synthesis of the character, I believe the balance has been redressed towards my not fleeing the responsibility Margarita's comment assigns to me.

[15] In a February 2004 entry of my research diary I have scribbled the words 'Re-conceptualising Our Discipline As Conversation'. The scribbling looks like the title of something; it maybe a part of a sentence from something I read at the time. I have failed to locate the origin – so, with apologies, could the owner, if any, please stand up?! – but it seems to encapsulate my thinking and soul-searching at the time with regard to the direction this work was going to take. So I feel it is worth mentioning here.

[16] Except Datasets 3 and 4 for which there were five Examples.

- a mini literature review and bibliography on the theme of the Dataset
- concluding page with thanks to the participants (and, in the case of UEA interviews, arrangements for the next interview)

In the five 'mathematical' Datasets there were twenty two Examples and in the one 'meta-mathematical' Dataset there were three. For ease of access – and a much needed at this stage sense of preliminary structure! – I created twenty five folders, one for each Example, in which I filed the following materials:

- Transcripts
- Narrative Accounts, the descriptive summaries of the interviews produced in the course of Study L's data analysis,
- Scanned images of relevant materials and student writing or other student-related data

that had been used towards the creation and discussion of the Example in the two half-day interviews (one at UEA, one at an institution outside UEA). These folders were labeled Narratives X.Y, where X stood for the Dataset they came from and Y for the number of the Example they came from. So, for instance, the folder labeled 'Narrative 2.III' contains the materials revolving around Example III in Dataset 2. The materials within each folder formed the basis for a text, symmetrically labeled Narrative X.Y, in which the 'story' of this Example's discussion was told, in most cases[17], as follows. First I presented the mathematical problem and its recommended solution; then the student responses that had been used as triggers of the discussion in the original interviews; then a list of issues that the interviewees had been asked to consider (copied from the original Dataset); finally, and most significantly, a dialogue between two characters, M and RME, each consolidating the contributions in the interviews by the participating mathematicians (for M) and the researchers conducting the interviews (for RME).

In the previous section I explained the appeal that the dialogic format has exerted on me and my rationale for using it in this work. Here is how I presented this rationale in my very first attempt at composing a dialogue from Narrative 1.I. The outcome was intended as a chapter (that later became the opening scene of Chapter 4 – see Post-script) in a book aimed for mathematics undergraduates in Rio De Janeiro, Brazil where I spent the first weeks of my Study Leave in 2004 and where much of the conceptualisation described above took place[18]. The following is from the chapter submitted to the Brazilian colleagues responsible for the volume in April 2004:

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[17] This process differs slightly for the three Narrative Folders from the 'meta-mathematical' Dataset.

[18] I would like to thank my friend and colleague Victor Giraldo for his hospitality and support during this period, a perhaps not surprisingly muddled period during which I started the writing. Combining the purely academic parts of the visit with the unadulterated fun – that the city of Rio De Janeiro and its inhabitants seem to have a unique and magic recipe for! – succeeded entirely because of him. With the hindsight of the amount of work that followed that visit, the experience further supported a dearly-held conjecture of mine, that there are times when there is nothing more productive than a break away!

'The presentation is in the somewhat unconventional format of a dialogue (My fascination with the dialogic format dates back to my school days in Greece and the experience of the works of Plato. However, within mathematics education literature, an early and defining influence was Imre Lakatos' *Proofs and Refutations* (1976)) between a mathematician and a researcher in mathematics education – M and RME respectively. The dialogue is fictional but based on data collected in a series of studies that the author and her associates have been conducting in recent years. The set up of the conversation is as follows. M is presented with: a mathematical question given to Year 1 students in the early weeks of their course; the suggested answer expected by the lecturer of the course; two examples of students' written responses to the question; a list of issues to consider in preparation for the discussion with RME. […] Relevant research literature is referred to within the dialogue as footnotes – as is information on the students' mathematical background. The studies where the data that formed the basis for the dialogue originate from are introduced briefly in [an] Appendix.

The exchange between M and RME sets out from the concrete context of a specific mathematical question and examples of student writing. However it soon becomes about an issue that is commonly known to cause some difficulty amongst mathematics undergraduates in the beginning of their studies: […]

The rationale for using the dialogic format is then explained as follows:

'The dialogic format of the presentation is not a merely stylistic choice, even though the idea of improving its readability is appealing. After all it is not an easy task to represent the various layers of data and analysis (a mathematical question, students' responses to the question, researchers' analyses of these responses and distillation of cognitive and pedagogical issues from these analyses, university teachers' reflections on the student data, on the analyses and on pedagogical practices relevant to these issues and researchers' analyses of the teachers' reflections…!) that form the basis of the work presented here. Beyond a stylistic choice, the dialogic format of the presentation is above all intended as a reminder of the need of the worlds of M and RME – both intrigued by and having a commitment to improving mathematical learning – to meet and confer more often. It is intended also as a response to stereotypical views that see researchers as irrelevant theorisers with a suspiciously loose commitment to the cause of mathematics (RME) and practitioners as non-reflective actors who insensitively rush through content-coverage and have no pedagogical ambition other than that related to success in exams and audits (M). In the realm of these stereotypes M and RME, deaf and blind to each other's needs, skills but also idiosyncrasies of their respective epistemological worlds, have no choice other than also of being mute, remain silent, indifferent and even hostile to each other's presence. Even though there is little pretense of constructing M and RME in a naturalistic way – after all their words are consolidations of those of the numerous mathematicians and researchers who participated in the studies that form the basis of the dialogue presented here – the effect is intended to be as close as possible to a realistic proposition: one of partnership.'

Between then and now I have had two more opportunities to present succinctly the rationale for (and way of) my using of the dialogic format. One was the CERME4 paper I mention in the Post-script; the other the Delphi Summer School sessions I also mention in the Post-script. The following is from the script I used for my introduction to those sessions (July 2005):

'The dialogue consists […] of M and RME's utterances, let's call them 'quotations' – please note the inverted commas. The text within M's 'quotations' is a consolidation of verbatim quotations from across the board of the twenty participants in the last study I

> mentioned. […]. The text within RME's 'quotations' is a consolidation of the minimally leading interventions of the researchers in the group interviews on which the dialogues are based.'

The links of the material in the dialogue with relevant works in the field is then described as follows:

> 'The references to literature, attached to the dialogue in the form of footnotes, aim at highlighting places where I believe there is resonance between the views expressed in the text and other relevant works. To suggest that one unified perspective on M, RME and the literature is possible – or even desirable – would be facile and deprive the conversation this work wishes to contribute to of the richness that often emerges from difference. I invite you to see the aim of this exercise as two-fold and approach it having this multi-layered set of intentions in mind: to contribute to the substantive conversation regarding student learning and pedagogical practice at university level by bringing to the fore M's views on these issues; and, to represent the complexity and sensitivity of the pedagogical perspectives demonstrated by M […]'.

The rationale for using the dialogic format varies slightly but largely repeats the points made in its April 2004 version:

> 'The presentation in the form of a dialogue is not merely a stylistic choice: it serves as a reminder of the overall intention of the study to contribute to the highly needed rapprochement of the worlds of mathematics and research in mathematics education. These two worlds – whose members are both intrigued by and having a commitment to the improvement of mathematical learning – need to meet, confer and generate negotiated, mutually acceptable perspectives more often. Through a demonstration of the rich pedagogical perspectives that are evident in these 'quotations' this heavy-on-data presentation is intended also as a response to stereotypical views that see practitioners as non-reflective actors who rush through content-coverage in ways often insensitive to their students' needs and have no pedagogical ambition other than that related to success in examinations and audits (analogously these stereotypes also see researchers as irrelevant theorisers with a suspiciously loose commitment to the cause of mathematics and incapable of 'connecting' with practitioners). In the realm of these stereotypes M and RME, oblivious to each other's needs, skills but also idiosyncrasies of their respective epistemological worlds, have no choice other than to remain indifferent, and even hostile, to each other. Through the presentation in the form of a dialogue the effect is intended to be a hopefully not too unrealistic proposition: that of partnership.'

Let me now exemplify the process through which the dialogues in Chapters 3 – 8 came to be. I will do so by picking a dialogue excerpt and juxtaposing it to the pieces of transcript it originates from. The following excerpt is from E3.5, Scene I (first Scene of the fifth Episode in Chapter 3) in which M comments on a student's response to a question, Student L's (see details within Chapter 3):

> M: …there are issues in the ways students engage with constructing mathematical arguments that need attention: often students will write down the thing that they are asked to prove and manipulate it. I see that they can't avoid doing this at some point in a contradiction proof but I would be much happier if the word *suppose* figured firmly in the beginning of their sentence. To allow this to go without comment would be doing the student a disservice. I would still like to stress though the originality of Student L's thought. That's something she thought up herself, not something she copied from a tutor

or her lecture notes or a book. She believes in her claim totally! It is the type of informal, intuitive claim students may have been conditioned to find satisfying at school level, this idea that *well, we cannot keep on cancelling forever, can we?* The descending argument, by the way, is an approach I am perfectly happy with. Historically the habit of choosing a minimum counterexample, *m* and *n* having no common factors and reaching contradiction because of that, is a modern habit.

This originates in Narrative 5.I, namely in Example 1 of Dataset used in the fifth Cycle of Data Collection (*Students' Enactment of Proving Techniques*). The original piece of transcript used to produce this brief monologue is the following[19]:

M1:	…[I am quite pleased with what is written here] apart from one or two things that need rescuing like the initial declaration. And what slightly depresses me about her is that often the students will write down the thing that they are asked to prove and manipulate it.
R1:	Hum…
M2:	Hum…
M3:	But they can't avoid doing this at some point in a contradiction proof.
M4:	Yes, but there isn't the word "suppose"…
M1:	Yes, it is all in that first line, isn't it? She is guessing a lot more… So on the face of it this is very good but of course to allow this to go without comment it would be doing the student a disservice so… lots of comments about that first line, I didn't believe…
M3:	Yes… I think that L's solution … she didn't get it from her advisor or a book, she actually thought it up…you see what I mean? I think that they believe it.
M2:	But isn't this the sort of thing that would have appeared on the blackboard in an A-level class?
R2:	Yes, this could be the only example of a proof by contradiction they see …
M4:	Yes, but how do they do it? Do they do it by m and n being co-prime or do they do it by this method of saying, well, you cannot keep cancelling forever and raise their hands and say…?
R2:	I don't know but I know that this is in a lot of cases the only example of formal proof that they would have of one sort of another…
M3:	And this descending thing is fine. I mean it is a modern habit to say, ok, choose a minimum counterexample to get the contradiction. Whereas I would have just said: this is the descent argument, so we are done.

By the end of June 2004[20] twenty two Narratives, one for each Example had been produced[21]. In the time that followed these Narratives turned into the themed episodic entities that constitute Chapters 3 – 8. In Part 3 I tell this part of the story.

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[19] For the sake of simplicity for this demonstration I have chosen a piece of dialogue that originates in one interview transcript only. I believe however that it is illustrative as several participants, denoted M1, M2, … express views in it. These views are consolidated into this monologue by M. The researchers facilitating the group interview are denoted R1 and R2. Their contributions did not seem to influence the course of discussion amongst M1,…, M4 therefore their voices have been removed – hence the monologue in the final product is, in this case, a mini-monologue.

[20] The twenty two Narratives, the preliminary and major groundwork for the text presented in Chapters 3 – 8, was done largely during eight weeks of my 2004 Study Leave in our family home in Thessaloniki, Greece. I deeply thank my mother Nicole and my father Christophoros for providing the tranquil and nurturing environment that was so important at that stage.

[21] The three Narratives from the sixth Dataset were produced later as at this stage I did not plan to use this material in the way I ended up using it in Chapter 8.

3. STYLE, FORMAT AND THEMATIC BREAKDOWN OF CHAPTERS 3 – 8

The twenty-two Narratives contained the first attempts at converting the material from each Example into a dialogue between M and RME. Having created the Narratives an increasingly precise understanding began to emerge of the themes and issues the dialogues were revolving around. Having selected the foci of the Datasets for the original interviews with the mathematicians on the evidence of relevant literature and findings from the previous studies, it was now a good moment to revisit, and update, this literature in order to start drawing tighter connections between it and the dialogues. Thus followed a period of searching, reading and summarising of relevant literature in July, August and September 2004[22]. I quote from my research diary in order to illustrate my thinking at the time on how the dialogues in the Narratives – which, in the natural course of conversation in the interviews, ebbed and flowed across many different issues – could be handled from now on so that the strength of the material (authenticity, richness and naturalistic flow) could be maintained while offering the reader a sense of focus, structure and direction(s) towards which the conversation is heading. The following is from the November 15th, 2004 diary entry and documents initial ideas on how to structure the Narratives internally as well as organise them under chapter headings:

> 'Narratives: sharpen the focus on each of the 22 Narratives until each is about 'something', a focal point. Write each Narrative with sharpened focus in mind as follows:
> - Introduce the focal point with reference to previous studies (mine and others')
> - Main perspective is M on student learning but M perspectives on teaching, on own thinking, on cultural / institutional issues and M/RME can be brief (abbreviated perhaps?) digressions. Signal these digressions or 'other' perspectives?
> - Insert as footnotes further references (mainly on focal point but on digressions too)
> - Conclude with a brief analysis of M-perspective (e.g. 'judging' the sensitivity of the discourse etc.)
>
> Chapters: within each chapter introduce the issue generally, justify the selection of the focal points. Title of each of the five[23] chapters: poetic plus 'On some issues regarding the learning of…'.
> - Five mathematical chapters: mathematical reasoning I (necessity of proof) and II (techniques of proving), mathematical language (notation and graphs), mathematical concepts I (limits) and II (functions).
> - One introductory chapter on aims and methodology
> - One chapter on M's discourse and on M/RME'

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[22] Having collected material for reading in July, a substantial part of the reading took place on the Aegean island of Kythnos in August 2004. I warmly thank my friend Alexis Spanos for sharing the slightly surreal experience of delving into piles of journal papers whilst inhabiting some of the most breathtaking Aegean landscapes and in a locale (a Greek island!) that, unsurprisingly, he and I, and most of our friends, had associated till then with total escape from the clutter of our lives in the city.

[23] Reminder: 'five' at this stage refers to the themes of the five datasets (see list in Part 1 and Chapter 1, Part 3).

After a period of minimal engagement with the material (largely the first semester of academic year 2004-5) further formation of the chapters took place from the spring of 2005 onwards. In what follows I illustrate these stages of the process. Landmarks of this process were the versions of the chapters in May, July and August 2005 and the work towards the submission of the manuscript for reviewing in February and March 2006.

A first crucial shift from the thinking that the above quotation reveals occurred in mid-March 2005. Following a re-reading of the material – at the state it was left in November 2004 when it had been last engaged with – a March 15th, 2005 diary entry lists the contents of the Narratives (apparently in the order that I read them at the time) as follows:

> 'In terms of what the Narratives so far are about:
>
> 1.I   What is a mathematical argument to students?
> 1.II Students' mathematical reasoning: resort to familiarity of numbers, example construction and the tension between the specific and the general, [section on Question Setter's Intentions, QSI], [teaching and curriculum], implication / deduction, [against examples]
> 1.III Use of definition towards building an argument or not, concept images (det(A)), use of deduction (e.g. substitution), [mathematics], [QSI], writing
> 1.IV [QSI], role of definition in building an argument, writing random mathematics, the non-linear and sudden nature of mathematical understanding, premature compression in students argument?, the limiting process, the meta-theme
> 5.I   Difficulties with Proof by Contradiction: spotting and logical leaps (the syndrome of the obvious); proof in school (UK): perceptions of proof in school mathematics, of √2, proof as algebraic manipulation, spotting contradiction at all cost; the contextual meaning of *prove*; on the significance of √2∈R-Q; a critique of the Proof by Contradiction that √2∈R-Q; on counter-examples
> 5.II Difficulties with Proof by Mathematical Induction; Syndrome of the Obvious: premature compression; difficulties with inequalities; school mathematics: lamentable state of proof, rationale for, attracting mathematicians in current cultural ambience, teacher recruitment (quality of degree, social status of the profession), value of articulating ideas (writing / speaking), (mathematics as one case in the school curriculum)
> 5.III Difficulty with applying the general to the particular (solve a Group Theory question on symmetries by resorting to the relevant axioms and theorems), [QSI]
> 5.IV In trouble with some properties in Group Theory, on counterexamples, the meta-theme
> 3.I   to-ing and fro-ing between mathematics and language, [teaching]
> 3.II the use of graphs and graphic calculators in mathematical reasoning, [teaching: is absolute rigour pedagogically viable?]
> 3.III [QSI] an attempt to link matrices, vectors and linear equations, to connect the apparently distinct worlds of Algebra and Geometry.
> 3.IV Use of group tables to construct meaning about groups
> 3.V   [teaching: pros and pitfalls in the use of pictures to convey meaning in Group Theory; teacher – student communication issues]
> 2.I   Students' use of (and problems with) the definition of convergence, ≥ versus > in the definition of convergence, convergence of a series, [teaching: definition of convergence, numerical experiments, the tension between formal and informal, use of pictures, foundational issues in the teaching of Analysis], students and pictures, [QSI]
> 2.II an episode in the learning about convergence: ignoring the head of a sequence, also on writing, symbols etc.

2.III students' logical misuse of LCT, [teaching: theorems as a toolbox, provoking student participation, the art of clever choice, lecturing], students' strategies on determining convergence

2.IV [QSI], students' perceptions of the definition of limit, symbolic writing, [teaching: interaction]

4.I [QSI], students' conceptions of the concept of function, [teaching: names for new concepts], using representations, students' misuse of logic, symbolic writing

4.II students' perceptions of function, [teaching: via rhymes and memory triggers, acquisition of mental representations, against indefinite integration, symbolism for Fourier series]

4.III students' perceptions of function, [teaching: fostering new definitions, context-bound usefulness of graphs, abstraction, critique of insensitivity of tutor, Oxford tutorials], [QSI]

4.IV [QSI], polynomials, [teaching new objects], [teaching: use of libraries and books]

4.V student difficulty with Group Theory concepts, [teaching: Group Theory, abstraction, attracting students, student participation, coping with content, the price of a vision].

And an idea for chapter headings as follows:

'proof (necessity and technique); function across topics; limiting process; mediating meaning (words, symbols, diagrams); methodology and introduction; meta-theme'

This chapter breakdown differs to the one up until then in that Proof is not dealt with in two (necessity, enactment) but in one chapter, Mediation of Mathematical Meaning is similarly not dealt with in two (words / symbols, diagrams) but in one and the idea for a 'meta-theme' chapters has also emerged. Obviously the 'data spoke' at this stage and dictated this reconsideration which is further consolidated in a March 17th, 2005 diary entry as follows:

'NEW CHAPTER STRUCTURE:
  Reasoning (based on Narratives 1, 5)
  Communicating (Narratives 3)
  Limits (Narratives 2)
  Functions (Narratives 4)
  Teaching (across)*
  M-RME relationship (across)
  Intro
  Methodology
Plus spin-off episodes? E.g. school, the equals sign
* e.g. sections on: [QSI] and mismatch between M and student perspectives, tutorials, writing, rigour as pedagogically viable?, pictures in Group Theory etc.'

In this I observe several developments:

- The new chapter breakdown specifies which of the 22 Narratives will form the basis for which chapter. So the four Narratives 1 and the four Narratives 5 will form the basis for the Reasoning Chapter etc.

- The idea for a separate chapter on teaching makes a first appearance – apparently because of the substantial number of [teaching] entries across the March 15th breakdown.
- The 'meta-theme' chapter is now called 'M-RME relationship' chapter and, similarly to the one on teaching will draw on material from across the Narratives[24].
- There is a first reference to 'spin-off episodes', an acknowledgement that there is substantial part of the material that doesn't fit neatly under the headings used at the time. This material later the Special Episodes and Out-Takes in Chapters 3 – 8.

Next day's entry, March 18th 2005, consolidates the above even further[25]:

'- A new structure: one chapter on reasoning (both necessity for proof and reasoning techniques); one on mediating mathematical meaning through writing in words, diagrams and symbols); then two chapters where we see all of the previous stuff in the concrete context of two fundamental concepts: function and the limiting process; one chapter on the meta-theme and one chapter on teaching (before the meta-theme chapter). Finally the introduction should tell the story of the book (from PhD onwards) and there should be one methodology chapter where the construction of the dialogues is made transparent. In there, there needs to be a section on failed / unfocused (e.g. Narratives 4) dialogues where the interviewees get distracted or changed course. This would stress the value of the successful ones and strengthen our faith in them[26].
- Regarding the chapter on teaching: put in there all the digressions from talking about students in the main narratives in the form of tactics etc.
- On the [QSI] digressions: put them as vignettes in the main narratives' chapters or the teaching one.
- In the introduction: say as much as possible about the background of the course, the students. There needs to be enough anchoring into the reality of the course but enough for the reader to take off into the more general.
- When the narrative takes a turn to discuss thesis extracts, use [Study PD1] data to enrich the discussion[27].'

Subsequent work on 'chopping' the Narratives and forming the Chapters took place in May 2005 as follows.

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[24] But soon, in May 2005, Narratives 6, based on the material from Dataset 6, was added to this database.

[25] A month after this entry an updated proposal was submitted to the publisher (see Post-script).

[26] A worthy idea which I have not followed through in Part 2(ii) because its illustration is too lengthy. It requires use of extensive parts of the data so that we can have a macroscopic view of the trajectory the discussion between M and RME is taking, substantiate the 'failure' that the diary entry perhaps a bit too non-challantly highlights, and reflect on possible causes. It is an idea worth pursuing in another, more methodologically-geared piece of writing. The starting point for this could be diary entries such as this from May 9th, 2005: 'Narrative 4.I, where RME makes genuine attempts to steer the conversation towards the concept of function but the conversation keeps 'slipping' into the other themes (language, writing, logic, history, etc.!), is a good example of a dialogue that didn't work very well. For the methodology chapter: show and speculate why… Narrative 4.II fluctuates too… (and so do III, IV and V…).'

[27] Another worthy idea that never materialised. One of the reasons was that as time went by the material inflated to an extent that adding more data in it seemed less and like a good idea less and less.

'…each Narrative should be broken to pieces[28], each labelled content-wise and with a reference where each piece belongs to (namely which chapter). The 'Mathematical Problem' page would of course only appear once and will be accompanied by a text where the choice of this problem / student examples / issue would be explained. This explanation will come from the analysis in the previous studies and the bibliography'. *Diary entry May 3rd, 2005*

'…created the files called Chapters 1-8. I am putting the chunks from last week's etc chunks in each of the files for Chapters 3-7 [as numbered in the April 2005 outline for the publisher]'. *Diary entry May 17th, 2005*

'Now that I am planning a chapter on the M/RME relationship, Chapter 8, I need to go back to the data from Dataset 6 and produce Narratives 6.I, 6.II and 6.III.' *Diary entry May 18th, 2005*

In the weeks that followed work on the material now within each one of the data chapters, Chapters 3 – 8, is recorded in the diary as follows:

'…breakdown each chapter in episodes according to the issues raised; relate each issue to relevant literature; write for each chapter (or for conclusions chapter? Epilogue?) my analysis of what the mathematicians say'. *Diary entry June 8th, 2005*

One outcome of this work was Content Maps for each of Chapters 3 – 8 followed by a construction of Chapter Summaries / Flow Texts (translations of the Maps into rough prose). The process of constructing the Summaries is recorded in the diary as follows:

'… how to construct the summary – maps: I need an episode breakdown. Start from a smaller chapter, e.g. Limits, put all on the floor, read map, see clusters, reorganize material accordingly, title them…' *Diary entry June 27th, 2005*

At that stage I had also collected the material that I had earlier labelled 'Spin-Off Episodes' (see March 17th diary entry quoted above) under the heading Other Matters and was toying with ideas on how to handle this material from now on. One of these ideas was to insert them as separate Vignettes across the text. One example of this material, as recorded in a June 18th entry of the diary, was what has now become Out-Take 7.1. Analogously most of this material is now in Special Episodes and Out-Takes. Further rumination on the different status of the Episodes, the main carrier of the themes in each chapter, and this other material is in the same entry of the diary, June 27th, 2005:

'A note on the narrative approach of the book (and the studies it draws on): this is not a comprehensive coverage of all topics (e.g. why just limit, function etc?) and I often cover 'peculiar' cases (Head of a Sequence? Students and Bikes?[29]) BUT what these 'peculiarities' do have is a capacity to throw light on a paradigmatic issue [dealt with in the Episodes]. Is this then where the narrative becomes paradigmatic?'[30]

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[28] A preliminary breakdown into these 'pieces' is suggested in the March 15th, 2005 diary entry quoted earlier in this section.

[29] referring to two examples from the material that has now become an Episode (E4.1, Scene II) and a Special Episode (SE6.1) respectively.

[30] The distinction uses the terminology referred to in Part 2(i) of this chapter.

The arrangement of the material within Chapters proved to be an arduous experience as the 'taming' of naturalistic data typically tends to be. It is recorded in the diary entries between June 27th and July 5th, 2005 as 'slow', as being necessarily 'much rougher way than the June 27th, 2005 notes suggest' and 'very rough' at this stage. My feelings are recorded as needing to 'check whether this way works', as 'an even stronger urge to start seeing the final episode breakdown of the chapters' etc.. To this aim I keep 'rearranging the episodes until some flow/coherence started to emerge'. By July 5th I appear to have achieved some clarification on what is now called Special Episodes, namely different from the main Episodes but supplementing them in some way:

> '…and today I […] put these chapters in order. The main text, Special Episodes (they look special but in fact extend the discussion in the main text) and the Out-takes (special as well but somewhat outside the main flow – still interesting enough to keep). As the richness of the discussion makes it inevitable that there will be some digressions – and for helping the reader – I am thinking about numbering the text as follows: x.y (x for the chapter, y for the section) and insert [x.y] anywhere else in the text where the same issue is touched upon[31].' *Diary entry July 5th, 2005*

Crucially at this time the opportunity to trial some first outcomes of this work appeared in the shape of the Delphi Summer School sessions (see Post-script). For the three sessions I selected one Episode from each of the six Chapters. The preparation of the six Delphi Summer School Excerpts is described in the diary as follows:

> '…trim to the essentials, insert missing info (such as questions and answers etc), weave in relevant literature and write a brief chapter overview based on the Flow Texts. I will introduce each excerpt with such overviews to give a flavour of where each excerpt comes from' *Diary entry July 8th, 2005*
> ' Start with short [excerpts] for practice… of course I am doing this slightly differently to how I initially planned. I have trimmed the Functions [excerpt] one but then started writing little introductions to each of the six episodes: what went on before, which larger issue they maybe coming from, what we could be expecting from this episode. They are intended to be written in a relaxed way. I am also figuring out what the reader (and the students in the Summer School) are expected to do when faced with these dialogues. Given time / space constraints I can only insert literature indicatively, not exhaustively, in the footnotes of the text.' *Diary entry July 9th, 2005*
> 'It takes a lot longer to work on these excerpts and I am still figuring out the status of the footnotes: links to literature, analytical comments etc. It's hard but enjoyable.' *Diary entry for period July 14th – 18th, 2005*

The exercise was significant in several ways: it helped me clarify what 'episode' was starting to mean; and, it highlighted issues around the status and content of the footnotes (an essential component of the Episodes as the footnotes play the role of

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[31] These evolved into Ex.y, SEx.y and OTx.y for Episodes, Special Episodes and Out-Takes from Chapter x and y standing for the number of Episode, Special Episode or Out-Take within the chapter.

associating the very context-specific discussion in the dialogue with its more general addressing in the literature[32]).

The experience of and feedback from the sessions would, I hoped, indicate ways in which these issues could be resolved.

And largely it did. Most of the decisions regarding the state of the text as it currently stands were finalised in the aftermath of these sessions[33] and in the work that took place in August 2005[34].

Alongside work on Chapters 3 – 8 ideas about the structuring of the *other* story-telling in the book (background, methodology etc.) are also beginning to emerge. I quote from the August 13th, 2005 diary entry:

> '… this diary […] tells, I believe, the Making of… story […]. [Tell the story…] <u>also</u> with scanned excerpts of the diary […]. The to-ing and fro-ing, the fluctuations, indecision, the 'worth noting' attempts all show […] in here'

This subsequent work on Chapters 3 – 8 consisted largely of the following: defining the boundaries of the Episodes and separating these from the Special Episodes and the Out-Takes; and, taking cue from the way I had worked on the Delphi Summer School Excerpts, starting to weave in preliminary indications for where Footnotes need to be inserted. In the light of the feedback on the Delphi material and my own further reading, observations, such as the ones recorded below in the diary, led to the next steps the refinement of the text needed to take.

On flow of the dialogues and the status of footnotes:

> 'I notice, now that a few weeks passed since I wrote [the Delphi Excerpts], where the text jars, where faithfulness to the transcript compromises flow or readability, where repetition may annoy or confuse the reader (is it repetition? Or does M or RME try to say something different?). Same applies to their footnotes: their status fluctuates…' *Diary entry August 16th, 2005*
> '…the type of footnote that simply says 'I say this and this other paper says that too' is kind of cheating, it's not enough. Add what that 'other paper' actually says that's relevant to the discussion here'. *Diary entry August 18th, 2005 (reminder repeated in the February 8th, 2006 entry)*

And on M's 'character':

> '…reading all the Delphi excerpts – perhaps experiencing these excerpts en masse similarly to how the students and faculty in Delphi did – I see now that M is a particularly 'nice' pedagogue. Part of the metaphor I think is that this kind of richness, while not plausibly attainable by each one of us, is however communally obtainable: it's the kind of richness we can achieve as a field if we learn from each other, practitioners and researchers alike'. *Diary entry August 18th, 2005*

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[32] Several of these references originate in the literature surveys carried out in the course of the studies introduced in Chapter 1, Part (iii) and the summaries distributed to the participants in Study L. Most were added in the course of constructing the dialogue.

[33] For this I would like to thank the students and faculty who participated in these sessions as well as Paola Iannone for their constructive feedback on the Delphi Excerpts.

[34] This work took place in two locations: on the Greek island of Alonissos in the company of my sister Danai Nardi and in the beach house of my friend Margarita (also mentioned in Part 2(i) of this chapter) in Chalkidiki, Greece. I warmly thank them both for their support.

By the end of August 2005 the structure of Chapters 3 – 8 had stabilised to one that is quite close to the current. The next level of refinement[35] took place in the eight weeks leading to the submission of the manuscript on March 31st, 2006. I now list observations and decisions made in this period (February and March 2006) through quotations from the diary[36].

On the Episode / Special Episode / Out-Take structure:

> '… in a naturalistic enquiry tidying up things in boxes doesn't work. There is always the spill over effect and what spills over is often equally interesting to what's inside. There are two spill over types in this book: the Special Episode and the Out-take. The former is just a slightly peculiar (too specific, at face value too distant from the mainstream of topics and themes frequenting our discussion in the field but still revolving around the Episodes' themes. The second is the type that doesn't fit the themes but it still says something about participants' point of view that I thought it deserves to be seen. Whereas what's in the E and SE represents or synthesises the point of view of the majority of participants, the OT is more of a lone-ranger category.' *Diary entry February 3rd, 2006*

On cross-referencing of Episodes and breaking particularly long or complex Episodes in Scenes:

> '…the same piece of data can appear in different Episodes and Chapters with a different function, in a different capacity, illustrating a different issue. However I cannot afford repeating the same piece of data: certain Episodes, e.g. E3.1 is already too long. So here are two suggestions: first, to avoid repetition, make cross-references, e.g. in Episode 3.x to Episode 6.y. Chapters 3 (Reasoning) and 4 (Writing) will then be introducing cross-topical issues and Chapters 5, 6 will be putting some of these within the topical context of Function and Limit. Also, across Chapters 3 and 4, where the dynamic between thought and language is often so intense that it becomes almost impossible to separate the discussion of one from the discussion of the other, I will need to exercise judgement about where every piece of data fits and apply cross-referencing, relentlessly if I have to. Second, as in Chapter 3, for example, Episodes are very long and move across mathematical problems and occasions, it may be a good idea to break them into Scenes [as I have already experimented in Chapter 6 back in August 2005]'. *Diary entry February 9th, 2006*

On strengthening the coherent flow of the text:

> 'I read and insert footnote signs where I feel there is a necessity for a comment or a reference. I also insert the connective sentences (including references) and the introduction so that the text flow is improved [and goes beyond] a collation of Episodes / Scenes with a synopsis at the front.' *Diary entry February 27th, 2006*
> 'Write one continuous text as the opening page of each data chapter. Start with a statement of the overall issue with a seminal reference in the field, preferably a quotation. In Chapter 3, for example, this would be Mathematical Reasoning as the defining activity of mathematics (in Chapter 4 then would start with a quotation on putting this thinking in communicable mode and this thinking being influenced by this

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[35] The first semester of the academic year (2005-6) allowed minimal engagement with the material.

[36] In this intensive and isolated period of Writing Leave contact with the … outside world was mostly through the telephone. To the list of friends I have already thanked in other parts of the text I need to add here my sister Anthi Nardi, resident of the USA, who spent much of her valuable time on expensive long-distance phone-calls during this period. Her positive spirit and encouragement are priceless.

mode of expression - *if we didn't have these symbols we may have never achieved the level of abstraction mathematics has achieved... etc*). Within each *Setting the Scene* section of Episodes / Scenes start similarly with highlighting the issue that takes centre-stage. […] I need to explain the non exhaustive, selective nature of the Episodes as they emerged from the data: M's preference guided by the datasets that were in turn guided by our and other studies. *Diary entry February 28ᵗʰ, 2006*

On the nature of footnotes in Chapters 3 – 8:

'Indicative, non exhaustive references to literature. The range of topics addressed in the book is so broad […] that the aim of exhaustiveness is unattainable'. *Diary entry March 7ᵗʰ, 2006*

This phase also included 'injecting references and cross-references in brackets or footnotes', 'inserting missing images, ensuring flow, cutting repetition' (all February 2006 entries)[37] from Chapters 3 – 8. Particularly, given the scope of issues raised in these Chapters, the task of choosing the location for the footnotes, identifying a salient piece or relevant literature to refer to and composing its brief but dense text was almost at times overwhelming:

'Sometimes I feel that my reading which I never really stop doing seems to be happening in a parallel universe to that of my writing. I am sure these two connect but I do need to find the connections, and fluently so…' *Diary entry March 8ᵗʰ, 2006*

Any 'proof' of whether the above targets have been hit can only be, as the saying goes, 'in the pudding' – Chapters 3 to 8.

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[37] The diary records the trials and tribulations of this phase with extended examples that I omit here. Perhaps they are more pertinent to a more methodologically inclined piece of writing.