75. *Topics in ergodic theory*

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Preface

Ergodic theory is difficult to characterise, as it stands at the junction of so many areas, drawing on the techniques and examples of probability theory, vector fields on manifolds, group actions on homogeneous spaces, number theory, statistical mechanics, etc. A comprehensive account of the subject today would necessarily lack the formal unity which one expects of fields wholly contained within one of the main disciplines of mathematics.

Nevertheless, there are excellent general accounts in Hopf [1], Halmos [1], Friedman [1], Jacobs [1], and Walters [1], which display a unity where there might seem to be none, and the survey by Mackey [1] demonstrates conclusively that the seeming chaos of the subject camouflages a very real order – albeit a complex, organic, rather than mechanical one. Of course most books and surveys on ergodic theory reflect their authors' special interests as does the present volume.

The main reason for my writing this monograph is a simple desire to put together some of my favourite topics. I believe, however, that it might serve more than a personal whim. There are many directions a researcher in ergodic theory might take and the chapters in this book could provide the first steps for each of these various journeys. Following the main body of this treatise I have included an appendix on the spectral multiplicity theory of unitary operators. This appendix should help students in so far as the material presented is either left out of most texts or is imbedded in treatments which are unnecessarily exhaustive for our purposes.

I have aimed neither for the utmost generality in the theorems presented nor for scholarly comprehensiveness in my acknowledgement of authorship or of modern trends; in this connection I must beg the indulgence of mathematicians whose contributions, though relevant, have been glossed over or not mentioned at all.

The introduction includes a brief account of the origins of ergodic theory and an outline of present trends. It is included merely to give the reader some feeling for the place of this selection in a rapidly developing and important subject and in the hope that the coherence of history will compensate for the subjectivity of my choice. For some readers it may be advisable to pass over this section until some familiarity with ergodic problems is gained.

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Reference symbols Lemmas and theorems are labelled successively 1, 2, . . . in each chapter; statements for reference are labelled (1.1), (1.2), . . . in Chapter 1 with a similar system for other chapters; in each chapter exercises are numbered successively 1, 2, ... References to books and papers are given by name and a number.

University of Warwick WILLIAM PARRY
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