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77. *Approaches to the theory of optimization*

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Approaches to the theory of optimization

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Preface

Perhaps the title of this book should have been, somewhat in the style of past times, 'Some approaches towards the theory of optimization, with an emphasis on the topological aspects, ignoring combinatorial problems and almost ignoring combinatorial tools; not going into the algorithmic and numeric problems of effectively finding solutions to problems, yet meant as a contribution to applied and even practical mathematics.'

Even this does not make it sufficiently clear what is going on in the book and what is not. It is worth saying a few words about the omissions. Convex processes are not treated, problems involving more than one objective to be optimized are only touched on lightly. Of game theory, only the simplest model is considered. Since, as far as Lagrangian duality is concerned, any practitioner wants a nonzero multiplier attached to the objective function, theorems where this multiplier is allowed to be zero do not receive any attention. Similarly the reader will not find anything about regularity conditions, which apart from being sufficient are also necessary for a whole class of problems (rather than for a single one). This is because when we try to solve any one problem out of such a class, those conditions may well be too strong for practical purposes. This is not to say that everything in the book is so 'practical': some basic theorems rest on the axiom of choice, for example, and sometimes we are satisfied to establish the equality of an infimum and a supremum rather than that of the corresponding minimum and maximum.

Although in practical situations we can often make do with decision variables which are elements of a Banach space, or even a Euclidean space, part of the theory includes a generalization to locally convex topological vector spaces. This provides us with the important possibility of considering, in Banach spaces, topologies weaker than the strong one.

If we single out for competition the results dealt with in the appendixes, the prize for the most beautiful result should perhaps go to Theorem 6.2.2 on the equality of *inf sup* and *sup inf*, because of its generality and its natural and elegant proof; or perhaps to the general fixed point theorem (6.1.18) because of its combinatorially ingenious proof; or to the theory of conjugate duality (chapter 4) because of its power and symmetry. The reader should judge for himself.

The main text does not contain references to the literature. These, together with comments, have been combined in a separate section entitled 'Comments on the text and related literature'. Internal cross-references are indicated either by two numerals, such as 3.14 referring to section 14 of chapter 3, or by three numerals, such as 3.14.19 referring to an item in 3.14.

The text assumes a basic knowledge of topology as well as functional analysis.

Explicitly, J. W. Nieuwenhuis contributed by generalizing some basic theorems, and implicitly he and W. K. Klein Haneveld contributed through many discussions on all kinds of subjects involved. The number of these discussions is only countable, but their effect was invaluable.

Symbols

Latin

A usually a matrix
B usually a matrix
b usually a right-hand side
C usually a subset of X
cl closure of
$\mathrm{D}h(z, \tau)$ see (4.4.4)
dom effective domain
epi epigraph
f objective function
F bifunction
F^{d} dual bifunction
F^{dd} bidual bifunction
g in $g(x) \leqslant 0$
g_{a} (includes) the active part of g (in $g(x) \leqslant 0$)
G set (feasible region)
h in $h(x) = 0$ or a function
h^* conjugate of h
h^{**} biconjugate of h
$h'(z_{\mathrm{o}}, z)$ one-sided directional derivative
H hyperplane or Hamiltonian
int interior
$K(\gamma)$ cone defined by (3.8.6)
L Lagrangian
l_1, l_∞ see Example 3.2.9
$L(V)$ see Definition 3.3.4
N negative cone or nullspace
N^* dual negative cone
p perturbation function
p^{d} dual perturbation function
p^{dd} bidual perturbation function
P positive cone
P^* dual positive cone
q' usually Fréchet derivative of q
R real axis
ri relative interior
R_n n-dimensional Euclidean space
T set defined in (3.5.3) or point-to-set mapping defined by (6.1.7)
U a set or a neighbourhood
u control variable, or in $u \in U$
V a set or a neighbourhood, or as defined by (3.5.4)
W a set or a neighbourhood
X decision space
x in $x \in X$, decision variable
X^* dual perturbation space
x^* in $x^* \in X^*$, dual perturbation
Y constraint space = perturbation space

y in $y \in Y$, (primal) perturbation
Y^* multiplier space
y^* in $y \in Y^*$, multiplier
Z a (perturbation) space
z in $z \in Z$
Z^* dual of Z
z^* in $z^* \in Z^*$

Greek

α infimum as in (3.5.1)
β supremum as in (3.5.2)
δ indicator function
δ^* support function

Other

∇ gradient
∂ subgradient