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*A handbook of Fourier theorems*

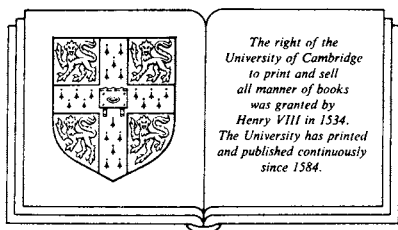
A HANDBOOK OF

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# *Fourier theorems*

D. C. CHAMPENEY

*School of Mathematics and Physics, University of East Anglia*



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*To Julie, John and Anna*

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## *PREFACE*

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This handbook is intended to assist those scientists, engineers and applied mathematicians who are already familiar with Fourier theory and its applications in a non-rigorous way, but who wish to find out the exact mathematical conditions under which particular results can be used. A reader is assumed whose mathematical grounding in other respects goes no further than the traditional first year university course in mathematics taken by physical scientists or engineers. Advances in mathematical sophistication have led to a growing divide between those books intended for mathematics specialists and those intended for others, and this handbook represents a conscious effort to bridge this gap.

The core of the book consists of rigorous statements of the most important theorems in Fourier theory, together with explanatory comments and examples, and this occupies chapters 6–16. This is preceded, in chapters 1–5, by an introduction to the terminology and the necessary ideas in mathematical analysis including, for instance, the interpretation of Lebesgue integrals. Proofs of theorems are not provided, and the first part is not intended as a complete grounding in mathematical analysis; however, it is intended that the book should be self contained and that it should provide a background which will assist the interested reader to follow proofs in standard mathematical texts.

In the chapters on Fourier theorems, those classical theorems dealing with locally integrable functions are covered first in a way that leads naturally to the later sections covering generalized functions. The more modern extensions in the direction of abstract harmonic analysis are not covered.