PREFACE TO THE SECOND EDITION

It is with great pleasure that we are presenting to the community the second edition of this extraordinary handbook. It has been over 15 years since the publication of the first edition and there have been great changes in the landscape of philosophical logic since then.

The first edition has proved invaluable to generations of students and researchers in formal philosophy and language, as well as to consumers of logic in many applied areas. The main logic article in the Encyclopaedia Britannica 1999 has described the first edition as 'the best starting point for exploring any of the topics in logic'. We are confident that the second edition will prove to be just as good!

The first edition was the second handbook published for the logic community. It followed the North Holland one volume *Handbook of Mathematical Logic*, published in 1977, edited by the late Jon Barwise. The four volume *Handbook of Philosophical Logic*, published 1983–1989 came at a fortunate temporal junction at the evolution of logic. This was the time when logic was gaining ground in computer science and artificial intelligence circles.

These areas were under increasing commercial pressure to provide devices which help and/or replace the human in his daily activity. This pressure required the use of logic in the modelling of human activity and organisation on the one hand and to provide the theoretical basis for the computer program constructs on the other. The result was that the *Handbook of Philosophical Logic*, which covered most of the areas needed from logic for these active communities, became their bible.

The increased demand for philosophical logic from computer science and artificial intelligence and computational linguistics accelerated the development of the subject directly and indirectly. It directly pushed research forward, stimulated by the needs of applications. New logic areas became established and old areas were enriched and expanded. At the same time, it socially provided employment for generations of logicians residing in computer science, linguistics and electrical engineering departments which of course helped keep the logic community thriving. In addition to that, it so happens (perhaps not by accident) that many of the Handbook contributors became active in these application areas and took their place as time passed on, among the most famous leading figures of applied philosophical logic of our times. Today we have a handbook with a most extraordinary collection of famous people as authors!

The table below will give our readers an idea of the landscape of logic and its relation to computer science and formal language and artificial intelligence. It shows that the first edition is very close to the mark of what was needed. Two topics were not included in the first edition, even though

D. Gabbay and F. Guenthner (eds.),
Handbook of Philosophical Logic, Volume 7, vii ix.

they were extensively discussed by all authors in a 3-day Handbook meeting. These are:

- a chapter on non-monotonic logic
- a chapter on combinatory logic and λ-calculus

We felt at the time (1979) that non-monotonic logic was not ready for a chapter yet and that combinatory logic and λ-calculus was too far removed.[1] Non-monotonic logic is now a very major area of philosophical logic, alongside default logics, labelled deductive systems, fibring logics, multi-dimensional, multimodal and substructural logics. Intensive re-examinations of fragments of classical logic have produced fresh insights, including at time decision procedures and equivalence with non-classical systems.

Perhaps the most impressive achievement of philosophical logic as arising in the past decade has been the effective negotiation of research partnerships with fallacy theory, informal logic and argumentation theory, attested to by the Amsterdam Conference in Logic and Argumentation in 1995, and the two Bonn Conferences in Practical Reasoning in 1996 and 1997.

These subjects are becoming more and more useful in agent theory and intelligent and reactive databases.

Finally, fifteen years after the start of the Handbook project, I would like to take this opportunity to put forward my current views about logic in computer science, computational linguistics and artificial intelligence. In the early 1980s the perception of the role of logic in computer science was that of a specification and reasoning tool and that of a basis for possibly neat computer languages. The computer scientist was manipulating data structures and the use of logic was one of his options.

My own view at the time was that there was an opportunity for logic to play a key role in computer science and to exchange benefits with this rich and important application area and thus enhance its own evolution. The relationship between logic and computer science was perceived as very much like the relationship of applied mathematics to physics and engineering. Applied mathematics evolves through its use as an essential tool, and so we hoped for logic. Today my view has changed. As computer science and artificial intelligence deal more and more with distributed and interactive systems, processes, concurrency, agents, causes, transitions, communication and control (to name a few), the researcher in this area is having more and more in common with the traditional philosopher who has been analysing

[1] I am really sorry, in hindsight, about the omission of the non-monotonic logic chapter. I wonder how the subject would have developed, if the AI research community had had a theoretical model, in the form of a chapter, to look at. Perhaps the area would have developed in a more streamlined way!

such questions for centuries (unrestricted by the capabilities of any hardware).

The principles governing the interaction of several processes, for example, are abstract an similar to principles governing the cooperation of two large organisation. A detailed rule based effective but rigid bureaucracy is very much similar to a complex computer program handling and manipulating data. My guess is that the principles underlying one are very much the same as those underlying the other.

I believe the day is not far away in the future when the computer scientist will wake up one morning with the realisation that he is actually a kind of formal philosopher!

The projected number of volumes for this Handbook is about 18. The subject has evolved and its areas have become interrelated to such an extent that it no longer makes sense to dedicate volumes to topics. However, the volumes do follow some natural groupings of chapters.

I would like to thank our authors are readers for their contributions and their commitment in making this Handbook a success. Thanks also to our publication administrator Mrs J. Spurr for her usual dedication and excellence and to Kluwer Academic Publishers for their continuing support for the Handbook.

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Logic	IT			
	Natural language processing	**Program control specification, verification, concurrency**	**Artificial intelligence**	**Logic programming**
Temporal logic	Expressive power of tense operators. Temporal indices. Separation of past from future	Expressive power for recurrent events. Specification of temporal control. Decision problems. Model checking.	Planning. Time dependent data. Event calculus. Persistence through time—the Frame Problem. Temporal query language. temporal transactions.	Extension of Horn clause with time capability. Event calculus. Temporal logic programming.
Modal logic. Multi-modal logics	generalised quantifiers	Action logic	Belief revision. Inferential databases	Negation by failure and modality
Algorithmic proof	Discourse representation. Direct computation on linguistic input	New logics. Generic theorem provers	General theory of reasoning. Non-monotonic systems	Procedural approach to logic
Non-monotonic reasoning	Resolving ambiguities. Machine translation. Document classification. Relevance theory	Loop checking. Non-monotonic decisions about loops. Faults in systems.	Intrinsic logical discipline for AI. Evolving and communicating databases	Negation by failure. Deductive databases
Probabilistic and fuzzy logic	logical analysis of language	Real time systems	Expert systems. Machine learning	Semantics for logic programs
Intuitionistic logic	Quantifiers in logic	Constructive reasoning and proof theory about specification design	Intuitionistic logic is a better logical basis than classical logic	Horn clause logic is really intuitionistic. Extension of logic programming languages
Set theory, higher-order logic, λ-calculus, types	Montague semantics. Situation semantics	Non-well-founded sets	Hereditary finite predicates	λ-calculus extension to logic programs

Imperative vs. declarative languages	Database theory	Complexity theory	Agent theory	Special comments: A look to the future
Temporal logic as a declarative programming language. The changing past in databases. The imperative future	Temporal databases and temporal transactions	Complexity questions of decision procedures of the logics involved	An essential component	Temporal systems are becoming more and more sophisticated and extensively applied
Dynamic logic	Database updates and action logic	Ditto	Possible actions	Multimodal logics are on the rise. Quantification and context becoming very active
Types. Term rewrite systems. Abstract interpretation	Abduction, relevance	Ditto	Agent's implementation rely on proof theory.	
	Inferential databases. Non-monotonic coding of databases	Ditto	Agent's reasoning is non-monotonic	A major area now. Important for formalising practical reasoning
	Fuzzy and probabilistic data	Ditto	Connection with decision theory	Major area now
Semantics for programming languages. Martin-Löf theories	Database transactions. Inductive learning	Ditto	Agents constructive reasoning	Still a major central alternative to classical logic
Semantics for programming languages. Abstract interpretation. Domain recursion theory.		Ditto		More central than ever!

JOHN P. BURGESS

BASIC TENSE LOGIC

1 WHAT IS TENSE LOGIC?

We approach this question through an example:

(1)	*Smith*:	Have you heard? Jones is going to Albania!
	Smythe:	He won't get in without an extra-special visa. Has he remembered to apply for one?
	Smith:	Not yet, so far as I know.
	Smythe:	Then he'll have to do so soon.

In this bit of dialogue the argument, such as it is, turns on issues of temporal order. In English, as in all Indo-European and many other languages, such order is expressed in part through changes in verb-form, or tenses. How should the logician treat such tensed arguments?

A solution that comes naturally to mathematical logicians, and that has been forcefully advocated in [Quine, 1960], is to regiment ordinary tensed language to make it fit the patterns of classical logic. Thus Equation 1 might be reduced to the quasi-English Equation 1 below, and thence to the 'canonical notation' of Equation 3:

(2) Jones/visits/Albania at some time later than the present.

At any time later than the present, if Jones/visits/Albania then, then at some earlier time Jones/applies/for a visa.

At no time earlier than or equal to the present it is the case that Jones/applies/for a visa.

Therefore, Jones/applies/for a visa at some time later than the present.

(3)
$$\begin{array}{l} \exists t(c < t \wedge P(t)) \\ \forall t(c < t \wedge P(t) \rightarrow \exists u(u < t \wedge Q(u))) \\ \neg\exists t((t < c \vee t = c) \wedge Q(t)) \\ \therefore \exists t(c < t \wedge Q(t)). \end{array}$$

Regimentation involves introducing quantification over instants $t, u, \ldots$ of time, plus symbols of the present instant c and the earlier- later relation $<$. Above all, it involves treating such a linguistic item as 'Jones is visiting Albania' *not* as a complete sentence expressing a proposition and having a truth-value, to be symbolised by a sentential variable $p, q, \ldots$, but rather as a predicate expressing a property on instants, to be symbolised by a one-place predicate variable $P, Q, \ldots$. Regimentation has been called *detensing*

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Handbook of Philosophical Logic, Volume 7, 1–42.

since the verb in, say, 'Jones/visits/Albania at time t', written here in the grammatical present tense, ought really to be regarded as *tenseless*; for it states not a present fact but a timeless or 'eternal' property of the instant t. Bracketing is one convention for indicating such tenselessness. The knack for regimenting or detensing, for reducing something like Equation 1 to something like Equation 3, is easily acquired. The analysis, however, cannot stop there. For a tensed argument like that above must surely be regarded as an *enthymeme*, having as unstated premises certain assumptions about the structure of Time. Smith and Smythe, for instance, probably take it for grated that of any two distinct instants, one is earlier than the other. And if this assumption is formalised and added as an extra premise, then Equation 3, invalid as it stands, becomes valid.

Of course, it is the job of the cosmologist, not the logician, to judge whether such an assumption is physically or metaphysically correct. What *is* the logician's job is to formalise such assumptions, correct or not, in logical symbolism. Fortunately, most assumptions people make about the structure of Time go over readily into first- or, at worst, second-order formulas.

1.1 Postulates for Earlier-Later

(B0)	Antisymmetry	$\forall x \forall y \neg(x < y \wedge y < x)$
(B1)	Transitivity	$\forall x \forall y \forall z (x < y \wedge y < z \to x < z)$
(B2)	Comparability	$\forall x \forall y (x < y \vee x = y \vee y < x)$
(B3)	(a) Maximum	$\exists x \forall y (y < x \vee y = x)$
	(b) Minimum	$\exists x \forall y (x < y \vee x = y)$
(B4)	(a) No Maximals	$\forall x \exists y (x < y)$
	(b) No Minimals	$\forall x \exists y (y < x)$
(B5)	Density	$\forall x \forall y (x < y \to \exists z (x < z \wedge z < y))$
(B6)	(a) Successors	$\forall x \exists y (x < y \wedge \neg \exists z (x < z \wedge z < y))$
	(b) Predecessors	$\forall x \exists y (y < x \wedge \neg \exists z (y < z \wedge z < x))$
(B7)	Completeness	$\forall U ((\exists x U(x) \wedge \exists x \neg U(x) \wedge$ $\forall x \forall y (U(x) \wedge$ $\wedge \neg U(y) \to x < y)) \to$ $(\exists x (u(x) \wedge$ $\wedge \forall y (x < y \to \neg U(y))) \vee$ $\exists x (\neg U(x) \wedge$ $\wedge \forall y (y < x \to U(y))))$
(B8)	Wellfoundedness	$\forall U (\exists x U(x) \to \exists x (U(x) \to$ $\wedge \forall y (y < x \to \neg U(y)))$
(B9)	(a) Upper Bounds	$\forall x \forall y \exists z (x < z \wedge y < z)$
	(b) Lower Bounds	$\forall x \forall y \exists z (z < x \wedge z < y)$.

For more on the development of the logic of time as a branch of applied first- and second-order logic, see [van Benthem, 1978].

The alternative to regimentation is the development of an autonomous *tense logic* (also called *temporal* logic or *chronological* logic), first undertaken in [Prior, 1957] (though several precursors are cited in [Prior, 1967]). Tense logic takes seriously the idea that items like 'Jones is visiting Albania' are already complete sentences expressing propositions and having truth-values, and that they should therefore by symbolised by sentential variables $p, q, \ldots$. Of course, the truth-value of a sentence in the present tense may well differ from that of the corresponding sentence in the past or future tense. Hence, tense logic will need some way of symbolising the relations between sentences that differ only in the tense of the main verb. At its simplest, tense logic adds for this purpose to classical truth-functional sentential logic just two one-place connectives: the future-tense or 'will' operator F and the past-tense or 'was' operator P. Thus, if p symbolises 'Jones is visiting Albania', then Fp and Pp respectively symbolise something like 'Jones is sooner or later going to visit Albania' and 'Jones has at least once visited Albania'. In reading tense-logical symbolism aloud. F and P may be read respectively as 'it will be the case that' and 'it was the case that'. Then $\neg F \neg$, usually abbreviated G, and $\neg P \neg$, usually abbreviated H, may be read respectively as 'it is always going to be the case that' and 'it has always been the case that'. Actually, for many purposes it is preferable to take G and H as primitive, defining F and P as $\neg G \neg$ and $\neg H \neg$ respectively. Armed with this notation, the tense-logician will reduce Equation 1 above to the stylised Equation 1.1 and then to the tense-logical Equation 5:

(4) Future-tense (Jones visits Albania)

Not future-tense (Jones visits Albania and not past-tense (Jones applies for a visa)).

Not past-tense (Jones applies for a visa) and not Jones applies for a visa.

Therefore, future-tense (Jones applies for a visa)

(5)
$$Fp$$
$$\neg F(p \wedge \neg Pq)$$
$$\neg Pq \wedge \neg q$$
$$\therefore Fq.$$

Of course, we will want some axioms and rules for the new temporal operators F, P, g, H. All the axiomatic systems considered in this survey will share the same standard format.

1.2 Standard Format

We start from a stock of sentential *variables* $p_0, p_2, p_2, \ldots$, usually writing p for p_0 and q for p_1. The (well-formed) *formulas* of tense logic are built up from the variables using negation ($\neg$), and conjunction ($\wedge$), and the strong future (G) and strong past (H) operators. The *mirror image* of a formula is the result of replacing each occurrence of G by H and vice versa. Disjunction ($\vee$), material conditional ($\rightarrow$), material biconditional ($\leftrightarrow$), constant true ($\top$), constant false ($\bot$), weak future (F), and weak past (P) can be introduced as abbreviations.

As *axioms* we take all substitution instances of truth-functional tautologies. In addition, each particular system will take as axioms all substitution instances of some finite list of extra axioms, called the *characteristic* axioms of the system. As *rules* of inference we take Modus Ponens (MP) plus the specifically tense-logical:

Temporal Generalisation(TG): From α to infer $G\alpha$ and $H\alpha$

The *theses* of a system are the formulas obtainable from its axioms by these rules. A formula is *consistent* if its negation is not a thesis; a set of formulas is *consistent* if the conjunction of any finite subset is. These notions are, of course, relative to a given system.

The systems considered in this survey will have characteristic axioms drawn from the following list:

1.3 Postulates for a Past-Present-Future

(A0)	(a)	$G(p \rightarrow q) \rightarrow (Gp \rightarrow Gq)$	(b)	$H(p \rightarrow q) \rightarrow (Hp \rightarrow Hq)$
	(c)	$p \rightarrow GPp$	(d)	$p \rightarrow HFp$
(A1)	(a)	$Gp \rightarrow GGp$	(b)	$Hp \rightarrow HHp$
(A2)	(a)	$Pp \wedge Fq \rightarrow F(p \wedge Fq) \vee F(p \wedge q) \vee F(Fp \wedge q)$		
	(b)	$Pp \wedge Pq \rightarrow P(p \wedge Pq) \vee P(p \wedge q) \vee P(Pp \wedge q)$		
(A3)	(a)	$G\bot \vee FG\bot$	(b)	$H\bot \vee PH\bot$
(A4)	(a)	$Gp \rightarrow Fp$	(b)	$Hp \rightarrow Pp$
(A5)	(a)	$Fp \rightarrow FFp$	(b)	$Pp \rightarrow PPp$
(A6)	(a)	$p \wedge Hp \rightarrow FHp$	(b)	$p \wedge Gp \rightarrow PGp$
(A7)	(a)	$Fp \wedge FG\neg p \rightarrow F(HFp \wedge G\neg p)$		
	(b)	$Pp \wedge PH\neg p \rightarrow P(GPp \wedge H\neg p)$		
(A8)		$H(Hp \rightarrow p) \rightarrow Hp$		
(A9)	(a)	$FGp \rightarrow GFp$	(b)	$PHp \rightarrow HPp$.

A few definitions are needed before we can state precisely the basic problem of tense logic, that of finding characteristic axioms that 'correspond' to various assumptions about Time.

1.4 Formal Semantics

A *frame* is a nonempty set C equipped with a binary relation R. A *valuation* in a frame (X, R) is a function V assigning each variable p_i a subset of X. Intuitively, X can be thought of as representing the set of instants of time, R

the earlier-later relation, V the function telling us *when* each p_i is the case. We extend V to a function defined on *all* formulas, by abuse of notation still called V, inductively as follows:

$$
\begin{aligned}
V(\neg\alpha) &= X - V(\alpha)\\
V(\alpha \wedge \beta) &= V(\alpha) \cap V(\beta)\\
V(G\alpha) &= \{x \in X : \forall y \in X(xRy \to y \in V(\alpha))\}\\
V(H\alpha) &= \{x \in X : \forall y \in X(yRx \to y \in V(\alpha))\}.
\end{aligned}
$$

(Some writers prefer a different notion. Thus, what we have expressed as $x \in V(\alpha)$ may appear as $||\alpha||_x^V = \text{TRUE}$ or as $(X, R, V) \vDash \alpha[x]$.) A formula α is *valid* in a frame (X, R) if $V(\alpha) = X$ for every valuation V in (X, R), and is *satisfiable* in (X, R) if $V(\alpha \neq \varnothing$ for some valuation V in (X, R), or equivalently if $\neg\alpha$ is not valid in (X, R). Further, α is *valid* over a class $\mathcal{K}$ of frames if it is valid in every $(X, R) \in \mathcal{K}$, and is *satisfiable* over $\mathcal{K}$ if it is satisfiable in some $(X, R) \in \mathcal{K}$, or equivalently if $\neg\alpha$ is not valid over $\mathcal{K}$. A system **L** in standard format is *sound* for $\mathcal{K}$ if every thesis of **L** is valid over $\mathcal{K}$, and a sound system **L** is *complete* for $\mathcal{K}$ if conversely every formula valid over $\mathcal{K}$ is a thesis of **L**, or equivalently, if every formula consistent with **L** is satisfiable over $\mathcal{K}$. Any set (let us say, finite) Φ of first- or second-order axioms about the earlier-later relation $<$ determines a class $\mathcal{K}(\Phi)$ of frames, the class of its *models*. The basic correspondence problem of tense logic is, given Φ to find characteristic axioms for a system **L** that will be sound and complete for $\mathcal{K}(\Phi)$. The next two sections of this survey will be devoted to representing the solution to this problem for many important Φ.

1.5 Motivation

But first it may be well to ask, why bother? Several classes of motives for developing an autonomous tense logic may be cited:

(a) *Philosophical* motives were behind much of the pioneering work of A. N. Prior, to whom the following point seemed most important: whereas our ordinary language is tensed, the language of physics is mathematical and so untensed. Thus, there arise opportunities for confusions between different 'terms of ideas'. Now working in tense logic, what we learn is precisely how to avoid confusing the tensed and the tenseless, and how t clarify their relations (e.g. we learn that essentially the same thought can be formulated tenselessly as, 'Of any two distinct instants, one /is/ earlier and the other /is/ later', and tensedly as, 'Whatever is going to have been the case either already has been or now is or is sometime going to be the case). Thus, the study of tense logic can have at least a 'therapeutic' value. Later writers have stressed other philosophical applications, and some of these are treated elsewhere in this *Handbook.*

M. FINGER, D. GABBAY AND M. REYNOLDS

ADVANCED TENSE LOGIC

1 INTRODUCTION

In this chapter we consider the tense (or temporal) logic with until and since connectives over general linear time. We will call this logic US/LT. This logic is an extension of Prior's original temporal logic of F and P over linear time [Prior, 1957], via the introduction of the more expressive connectives of Kamp's U for "until" and S for "since" [Kamp, 1968b]. U closely mimics the natural language construct "until" with $U(A, B)$ holding when A is constantly true from now up until a future time at which B holds. S is similar with respect to the past. We will see that U and S do indeed extend the expressiveness of the temporal language.

In the chapter we will also be looking at other related temporal logics. The logics differ from each other in two respects. Logics may differ in the kinds of structures which they are used to describe. Structures vary in terms of their underlying model of time (or frame): this can be like the natural numbers, or like the rationals or like the reals or some other linear order or some non-linear branching or multi-dimensional shape. Logics are defined with respect to a class of structures. Considering a logic defined by the class of all linear structures is a good base from which to begin our exploration. Temporal logics also vary in their language. For various purposes, until and since may be not expressive enough. For example, if we want to be able to reason about alternative avenues of development then we may want to allow branches in the flow of time and, in order to represent directly the fact of alternative possibilities, we may need to add appropriate branching connectives. Equally, until and since may be too strong: for simple reasoning about the forward development of a mechanical system, using since may not only be unnecessary, but may require additional axioms and complexity of a decision procedure.

In this chapter we will not be looking at temporal logics based on branching. See the handbook chapter by Thomason for these matters. We will also avoid consideration of temporal logics incorporating quantification. Instead, see the handbook chapter by Garson for a discussion of predicate temporal and modal logics and see the reference [Gabbay *et al.*, 1994] for a discussion of temporal logics incorporating quantification over propositional atoms.

So we will begin with a tour of the many interesting results concerning US/LT including axiom systems, related logics, decidability and complexity. In section 3 we sketch a proof of the expressive completeness of the logic.

D. Gabbay and F. Guenthner (eds.),
Handbook of Philosophical Logic, Volume 7, 43–203.

Then, in section 4 we investigate combinations of logics with a temporal element. In section 5, we develop the proof theory for temporal logic within the framework of labelled deductive systems. In section 6, we show how temporal reasoning can be handled within logic programming. In section 7, we survey the much studied temporal logic of the natural numbers and consider the powerful automata technique for reasoning about it. Finally, in section 8, we consider the possibility of treating temporal logic in an imperative way.

2 U, S LOGIC OVER GENERAL LINEAR TIME

Here we have a close look at the US logic over arbitrary linear orders.

2.1 The logic

Frames for our logic are linear. Thus we have a non-empty set T and a binary relation $< \subseteq T \times T$ which is:

1. *irreflexive*, i.e. $\forall t \in T$, we do not have $t < t$;
2. *total*, i.e. $\forall s, t \in T$, either $s < t$, $s = t$ or $t < s$;
3. *transitive*, i.e. $\forall s, t, u \in T$, if $s < t$ and $t < u$ then $s < u$.

The underlying model of time for a temporal logic is captured by the frame $(T, <)$.

Any use of a temporal logic will involve something happening over time. The simplest method of trying to capture this formally is to use a propositional temporal logic. So we fix a countable set $\mathcal{L}$ of atoms. The truth of a particular atom will vary in time. For example, points of time (i.e. $t \in T$) may correspond to days and the truth of the atom r on a particular day may correspond to the event of rain on that day.

A structure is a particular history of the truth of all the atoms over the full extent of time. Structures $(T, <, h)$ are linear so we have a linear frame $(T, <)$ and we have a valuation h for the atoms, i.e. for each atom $p \in \mathcal{L}$, $h(p) \subseteq T$. The set $h(p)$ is the set of all time points at which p is true.

The language $L(U, S)$ is generated by the 2-place connectives U, S along with classical $\neg$ and $\wedge$. That is, we define the set of formulas recursively to contain the atoms and $\top$ (i.e. truth) and for formulas A and B we include $\neg A$, $A \wedge B$, $U(A, B)$ and $S(A, B)$, We read $U(A, B)$ as "until A, B" corresponding to B being true until A is. Similarly S is read as "since".

Formulas are evaluated at points in structures. We write $\mathcal{T}, x \models A$ when A is true at the point $x \in T$. This is defined recursively as follows. Suppose that we have defined the truth of formulas A and B at all points of $\mathcal{T}$. Then for all points x:

$\mathcal{T}, x \models p$	iff	$x \in h(p)$, for p atomic;
$\mathcal{T}, x \models \top$;		
$\mathcal{T}, x \models \neg A$	iff	$\mathcal{T}, x \not\models A$;
$\mathcal{T}, x \models A \wedge B$	iff	both $\mathcal{T}, x \models A$ and $\mathcal{T}, x \models B$;
$\mathcal{T}, x \models U(A, B)$	iff	there is a point $y > x$ in T such that $\mathcal{T}, y \models A$ and for all $z \in T$ such that $x < z < y$ we have $\mathcal{T}, z \models B$;
$\mathcal{T}, x \models S(A, B)$	iff	there is a point $y < x$ in T such that $\mathcal{T}, y \models A$ and for all $z \in T$ such that $y < z < x$ we have $\mathcal{T}, z \models B$;

Often definitions and results involving S can be given by simply exchanging U and S and swapping $<$ and $>$. In that situation we just mention that a *mirror image* case exists and do not go into details.

There are many abbreviations that are commonly used in the language. As well as the classical $\perp$ (i.e. $\neg\top$ for falsity), $\vee$, $\rightarrow$ and $\leftrightarrow$, we have the following temporal abbreviations:

FA	$= U(A, \top)$	A will happen (sometime);
GA	$= \neg F \neg A$	A will always hold;
PA	$= S(A, \top)$	A was true (sometime);
HA	$= \neg P \neg A$	A was always true;
$K^+(A)$	$= \neg U(\top, \neg A)$	A will be true arbitrarily soon;
$K^-(A)$	$= \neg S(\top, \neg A)$	A was true arbitrarily recently.

Notice that Prior's original connectives F and P appear as abbreviations in this logic. The reader should check that their original semantics (see [Burgess, 2001]) are not compromised.

A formula ϕ is *satisfiable* if it has a model: i.e. there is a structure $\mathcal{T} = (T, <, h)$ and $x \in T$ such that $\mathcal{T}, x \models \phi$. A formula is *valid* iff it is true at all points of all structures. We write $\models A$ iff A is a validity. Of course, a formula is valid iff its negation is not satisfiable.

We can also define (semantic) consequence in the logic. Suppose that Γ is a set of formulas and A a formula. We say that A is a consequence of Γ and write $\Gamma \models A$ iff whenever we have $\mathcal{T}, t \models C$ for all $C \in \Gamma$, for some point t from some structure $\mathcal{T}$, then we also have $\mathcal{T}, t \models A$.

First-Order Monadic Logic of Order

For many purposes such as assessing the expressiveness of temporal languages or establishing their decidability, it is useful to be able to move from the internal tensed view of the world to an external untensed view. In doing so we can also make use of logics with more familiar syntax. In the case of our linear temporal logics we find it convenient to move to the first-order monadic logic of linear order which is a sub-logic of the full second-order monadic logic of linear order.

The language of the full second-order monadic logic of linear order has formulas built from $<$, $=$, quantification over individual variable symbols and quantification over monadic (i.e. 1-ary) predicate symbols. To be more formal, suppose that $X = \{x_0, x_1, ...\}$ is our set of individual variable symbols and $Q = \{P_0, P_1, ...\}$ is our set of monadic predicates. The formulas of the language are $x_i < x_j$, $x_i = x_j$, $P_i(x_j)$, $\neg\alpha$, $\alpha \wedge \beta$, $\exists x_i \alpha$, and $\exists P_j \alpha$ for any $i, j < \omega$ and any formula α. We use the usual abbreviations $x_i > x_j$, $x_i \leq x_j$, $x_i < x_j < x_k$, $\forall x_i \alpha$ and $\forall P_i \alpha$ etc.

As usual we define the concept of a free individual variable symbol in a formula. We similarly define the set of free monadic variables of a formula. Write $\phi(x_1, ..., x_m, P_1, ..., P_n)$ to indicate that all the free variables (of both sorts) in the formula ϕ are contained in the lists $x_1, ..., x_m$ and $P_1, ..., P_n$.

The language is used to describe linear orders. Suppose that $(T, <)$ is a linear order. As individual variable symbols we will often use t, s, r, u etc, instead of $x_1, x_2, ...$.

An individual variable assignment V is a mapping from X into T. A predicate variable assignment W is a mapping from Q into $\wp(T)$ (the set of subsets of T). For an individual variable assignment V, an individual variable symbol $x \in X$ and an element $t \in T$, we define the individual variable assignment $V[x \mapsto t]$ by:

$$V[x \mapsto t](y) = \begin{cases} V(y) & y \neq x \\ t & y = x. \end{cases}$$

Similarly for predicate variable assignments and subsets of T.

For a formula ϕ, variable assignments V (individual) and W (predicate), we define whether (or not resp.) ϕ under V and W is true in $(T, <)$, written $(T, <), V, W \models \phi$ by induction on the quantifier depths of ϕ.

Given some ϕ, suppose that for all its subformulas ψ, for all variable assignments V and W, we have defined whether or not $(T, <), V, W \models \psi$. For variable assignments V and W we define:

$$\begin{array}{lll}
(T,<),V,W \models x_i < x_j & \text{iff} & V(x_i) < V(x_j); \\
(T,<),V,W \models x_i = x_j & \text{iff} & V(x_i) = V(x_j); \\
(T,<),V,W \models P_i(x_j) & \text{iff} & V(x_j) \in W(P_i); \\
(T,<),V,W \models \neg\alpha & \text{iff} & (T,<),V,W \not\models \alpha; \\
(T,<),V,W \models \alpha \wedge \beta & \text{iff} & (T,<),V,W \models \alpha \text{ and } (T,<),V,W \models \beta; \\
(T,<),V,W \models \exists x_i \alpha & \text{iff} & \text{there exists some } t \in T \text{ such that} \\
& & (T,<),V[x_i \mapsto t],W \models \alpha. \\
(T,<),V,W \models \exists P_j \alpha & \text{iff} & \text{there is some } S \subseteq T \text{ such that} \\
& & (T,<),V,W[P_j \mapsto S] \models \alpha.
\end{array}$$

This is standard second-order semantics. Note that it is easy to show that the truth of a formula does not depend on the assignment to variables which do not appear free in the formula.

Mostly we will be interested in fragments of the full second-order monadic logic. In particular, we refer to the first-order monadic logic of linear order which contains just those formulas with no quantification of predicate variables. We will also mention the universal second-order monadic logic of linear order which contains just those formulas which consist of a first-order monadic formula nested under zero or more universal quantifications of predicate variables.

The important correspondence for us is that between temporal logics such as US/LT and the first-order monadic logic. Most of the temporal logics which we will consider allow a certain equivalence between their formulas and first-order monadic formulas. To define this we need to use a fixed one-to-one correspondence between the propositional atoms of the temporal language and monadic predicate variables. Let us suppose that $p_i \in \mathcal{L}$ corresponds to $P_i \in Q$.

The translation will propagate upwards through the full temporal language provided that each of the connectives have a first-order translation. In particular we require for any n-ary temporal connective C some first-order monadic formula $\phi_C(t, P_1, ..., P_n)$ which corresponds to $C(p_1, ..., p_n)$. We say that ϕ_C is the *(first-order) table* of C iff for every linear order $(T,<)$, for all $h : \mathcal{L} \to \wp(T)$, for all $t_0 \in T$, for all variable assignments V and W,

$$(T,<,h),t_0 \models C(p_1,...,p_n)$$
$$\text{iff } (T,<),V[t \mapsto t_0],W[P_1 \mapsto h(p_1),...,P_n \mapsto h(p_n)] \models \phi_C.$$

U and S have first-order tables as follows: the table of U is
$\phi_U = \exists s((t < s) \wedge P_1(s) \wedge \forall r((t < r \wedge r < s) \to P_2(r))).$
The table of S is the mirror image.

If we have a temporal logic with first-order tables for its connectives then it is straightforward to define a meaning-preserving translation (to the first-order monadic language) of all formulas in the language. The translation is

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COMBINATIONS OF TENSE AND MODALITY

1 INTERACTIONS WITH TIME

Physics should have helped us to realise that a temporal theory of a phenomenon X is, in general, more than a simple combination of two components: the statics of X and the ordered set of temporal instants. The case in which all functions from times to world-states are allowed is uninteresting; there are too many such functions, and the theory has not begun until we have begun to restrict them. And often the principles that emerge from the interaction of time with the phenomena seem new and surprising. The most dramatic example of this, perhaps, is the interaction of space with time in relativistic space-time.

The general moral, then, is that we shouldn't expect the theory of time $+X$ to be obtained by mechanically combining the theory of time and the theory of X.[1]

Probability is a case that is closer to our topic. Much ink has been spilled over the evolution of probabilities: take, for instance, the mathematical theory of Markov processes (Howard [1971a; 1971b] make a good text), or the more philosophical question of rational belief change (see, for example, Chapter 11 of Jeffrey [1990] and Harper [1975].) Again, there is more to these combinations than can be obtained by separate reflection on probability measure and the time axis.

probability shares many features with modalities and, despite the fact that (classical) probabilities are numbers, perhaps in some sense probability *is* a modality. It is certainly the classic case of the use of possible worlds in interpreting a calculus. (Sample points in a state space are merely possible worlds under another name.) But the literature on probability is enormous, and almost none of it is presented from the logician's perspective. So, aside from the references I have given, I will exclude it from this survey. However, it seems that the techniques we will be using can also help to illuminate problems having to do with probability; this is illustrated by papers such as D. Lewis [1981] and Van Fraassen [1971]. For lack of space, these are not discussed in the present essay.

[1]For a treatment that follows this procedure, see [Woolhouse, 1973]; [Werner, 1974] may also fit into this category, but I have not been able to obtain a copy of it. The tense logic of Woolhouse's paper is fairly crude: e.g. moments of time appear both in models and in the object language. The paper seems mainly to be of historical interest.

D. Gabbay and F. Guenthner (eds.),
Handbook of Philosophical Logic, Volume 7, 205–234.

2 INTRODUCTION TO HISTORICAL NECESSITY

Modern modal logic began with necessity (or with things definable with respect to necessity), and the earliest literature, like C. I. Lewis [1918], confuses this with validity. Even in later work that is formally scrupulous about distinguishing these things, it is sometimes difficult to tell what concepts are really metalinguistic. Carnap, for instance [1956, p. 10], begins his account of necessity by directing our attention to l- truth; a sentence of a semantical system (or language) is L-true when its truth follows form the semantical rules of the language, without auxiliary assumptions. This, of course, is a metalinguistic notion. But later, when he introduces necessity into the object language [Carnap, 1956, p. 174], he stipulates that $\Box\varphi$ is true if and only if φ is L-true.

Carnap thinks of the languages with which he is working as fully determinate; in particular, their semantical rules are fixed. This has the consequence that whatever is L-true in a language is eternally L-true in that language. (See [Schlipp, 1963, p. 921], for one passage in which Carnap is explicit on the point: he says 'analytic sentences cannot change their truth-value'.) Combining this consequence with Carnap's explication of necessity, we see that[2]

(1) $\Box\varphi \rightarrow HG\Box\varphi$

will be valid in languages containing both necessity and tense operators: necessary truths will be eternally true. The combination of necessity with tense would then be trivialised.

But there are difficulties with Carnap's picture of necessity; indeed, it seems to be drastically misconceived.[3] For one thing, many things appear to be necessary, even though the sentences that express them can't be derived from semantical rules. In Kripke [1982], for instance, published 26 years after *Meaning and Necessity*, Saul Kripke argues that it is necessary that Hesperus is Phosphorous, though 'Hesperus' and 'Phosphorous' are by no means synonymous. Also at work in Kripke's conception of necessity, and that of many other contemporaries, is the distinction between φ expressing a necessary truth, and φ necessarily expressing a truth. In a well-known defence of the analytic-synthetic distinction, Grice and Strawson [1956] write as follows:

[2] I use the tense logical notation of the first Chapter in this volume.

[3] For an early appreciation of the philosophical importance of making necessity time-dependent (the point I myself am leading up to), see [Lehrer and Taylor, 1965]. The puzzles they raise in this paper are genuine and well presented. But the solution they suggest is very implausible, and the considerations that motivate it seem to confuse semantic and pragmatic phenomena. This is a good example of a case in which philosophical reflections could have been aided by an appeal to the technical apparatus of model theory (in this case, to the model theory of tense logic).

> Any form of words at one time held to express something true may, no doubt, at another time come to be held to express something false. but it is not only philosophers who would distinguish between the case where this happens as the result of a change of opinion solely as to matters of fact, and the case where this happens at least partly as a result of shift in the sense of the words (p. 157).

This distinction, at lest in theory, makes it possible that a sentence φ should necessarily (perhaps, because of semantical rules) express a truth, even though the truth that it expresses is contingent. This idea is developed most clearly in [Kaplan, 1978].

On this vie of necessity, it attaches not primarily to sentences, but to propositions. A sentence will express a proposition, which may or may not be necessary. This can be explicated using possible worlds: propositions take on truth values in these worlds, and a proposition is necessary if and only if it is true in all possible worlds.[4]

This conception can be made temporal without trivialising the results. Probably the simplest way of managing this is to begin with nonempty sets T of times and W of worlds;[5] T is linearly ordered by a relation $<$. I will call this the $T \times W$ approach.

Recall that a tensed formula, say $F\varphi$, is true at $\langle w, t \rangle$, where $w \in W$ and $t \in T$, if and only if φ is true at $\langle w, t' \rangle$, for some t' such that $t < t'$.[6] We now want to ask under what conditions $\Box\varphi$ is true at $\langle w, t \rangle$. (In putting it this way we are suppressing propositions; this is legitimate, as long as we treat propositional attitudes as unanalysed, and assume that sentences express the same proposition everywhere.)

If we appeal to intuitions about languages like English, it seems that we should treat formulas like $\Box\varphi$ as nontrivially tensed. This is shown most clearly by sentences involving the adjective 'possible', such as 'In 1932 it was possible for Great Britain to avoid war with Germany; but in 1937 it was impossible'. This suggests that when $\Box\varphi$ is evaluated at $\langle w, t \rangle$ we

[4]To simplify matters, I confine the discussion to the absolute necessity of S5. But perhaps I should mention in passing that in explicating the relative breeds of necessity, such as that of S4, it is easy to confuse modal relations with temporal relations, relative necessity with evanescent necessity. And, of course, tense logic was inspired in part by work on relative necessity. But the two notions are separate; an S5 breed of necessity, for instance, can be evanescent. And when tense and modality are combined, it is very important to attend to the distinction.

[5]I dislike this way of arranging things for philosophical reasons. it doesn't strike me as a logical truth that all worlds have the same temporal orderings: some may have an earliest time, for instance, and others not. Also, the notion of different worlds sharing the same time is philosophically problematic; it is hard to reconcile with a plausible theory of time, when the possible worlds differ widely. Finally, I like to think of possible worlds as overlapping, so that at the same moment may have alternative futures. This requires a more complicated representation. However, the $T \times W$ arrangement will do for now.

[6]See the discussion of the interpretation of tense in Chapter 1 of this volume.

are considering what is *then necessary*; what is true in all worlds at that particular time, t.

The rule then is that $\Box\varphi$ is true at $\langle w, t\rangle$ if and only if φ is true at $\langle w', t\rangle$ for all $w' \in W$. If we like, we can make this relational. Let $\{\approx_t : t \in T\}$ be a family of equivalence relations on W, and let $\Box\varphi$ be true at $\langle t, w\rangle$ if and only if φ is true at $\langle t, w'\rangle$ for all $w' \in W$ such that $w \approx_t w'$.

The resulting theory generates some validities arising from the assumption that the worlds share a common temporal ordering. Formulas (2) and (3) are two such validities, corresponding to the principle that one world has a first moment if and only if all worlds do.

(2) $P[\varphi \vee \neg\varphi] \leftrightarrow \Box P[\varphi \vee \neg\varphi]$

(3) $H[\varphi \wedge \neg\varphi] \leftrightarrow \Box H[\varphi \wedge \neg\varphi]$

In case $\approx_t$ is the universal relation for every t (or the relations $\approx_t$ are simply omitted from the satisfaction conditions) there are other validities, such as (4) and (5).

(4) $P\Box\varphi \rightarrow \Box P\varphi$

(5) $F\Box\varphi \rightarrow \Box F\varphi$

As far as I know, the general problem of axiomatising these logics has not been solved. But I'm not sure that it is worth doing, except as an exercise. The completeness proofs should not be difficult, using Gabbay's techniques (described in Section 4, below). and these logics do not seem particularly interesting from a philosophical point of view.

But a more interesting case is near to hand. The tendency we have noted to bring Carnap's metalinguistic notion of necessity down to earth has made room for the reintroduction of one of the most important notions of necessity: practical necessity, or historical necessity.[7] This is the sort of necessity that figures in Aristotle's discussion of the Sea Battle (*De Int.* $18^{b}25$–$19^{b}4$), and that arises when free will is debated. It also seems to be an important background notion in practical reasoning. Jonathan Edwards, in his usual lucid way, gives a very clear statement of the matter.

> Philosophical necessity is really nothing else than the full and fixed connection between the things signified by the subject and predicate of a proposition, which affirms something to be true .... [This connection] may be fixed and made certain, because the existence of that thing is already come to pass; and either

[7] I am note sure if there are personal, ore relational varieties of inevitability; it seems a bit peculiar to my ear to speak of an accident John caused as inevitable for Mary, but not inevitable for John. If there are such sorts of inevitability I mean to exclude them, and to speak only of impersonal inevitability. Thus 'inevitable' does not belong to the same modal family as 'able', since the latter is personal.

> now is, or has been; and so has as it were made sure of existence. And therefore, the proposition which affirms present and past existence of it, may be this means be made certain, and necessarily and unalterably true; the past event has fixed and decided that matter, as to its existence; and has made it impossible but that existence should be truly predicted of it. Thus the existence of whatever is already come to pass, is not become necessity; 'tis become impossible it should be otherwise than true, that such a thing has been. [Edwards, 1957, pp. 152–3]

Historical necessity can be fitted into the $T \times W$ framework; it is merely a matter of adjusting the relations $\approx_t$ so that if $w \approx_t w'$, then w and w' share the same past up to and including t. So for $t' < t$, atomic formulas must be treated the same way in w and w'. furthermore, we have to stipulate that historical possibilities diminish monotonically with the passage of time: if $t < t'$, then $\{w' : w \approx_{t'} w'\} \subseteq \{w;: w \approx_t w'\}$. This interaction between time and relative necessity creates distinctive validities, such as (6) and (7).

(6) $\varphi \leftrightarrow \Box\varphi$, if φ contains no occurrences of F.

(7) $P\Box\varphi \rightarrow \Box P\varphi$

Formula (8), on the other hand, is clearly invalid.

(8) $\Box Pp \rightarrow P\Box p$

These correspond to rather natural intuitions relating the flow of time to the loss of possibilities.

There is another way of representing historical necessity, which perhaps will seem less straightforward to logicians steeped in possible worlds. Time can be treated as non-linear (branching only towards the future), and worlds represented as branches on the resulting ordered structure. This corresponds very closely to the $T \times W$ account: (6) and (7) remain valid, and (8) invalid. But the validities are not the same. This matter will be taken up below, in Section 4.

So much for necessity; I will deal more briefly with 'ought' and conditionals.

As Aristotle points out, we don't deliberate about just anything; in particular, we deliberate only about what is in our power to determine. [Ne 1112[a] 19f.] But the past, and the instantaneous present, are not in our power: deliberation is confined to future alternatives.

This suggests that deontic logic, insofar as it investigates practical oughts, should identify its possibilities with the ones of historical necessity. Unfortunately, this conception played little or no role in the early interpretation of deontic logic; those who developed the deontic applications of possible worlds semantics seemed to think of deontic possibilities ahistorically, as

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PHILOSOPHICAL PERSPECTIVES ON QUANTIFICATION IN TENSE AND MODAL LOGIC

INTRODUCTION

The trouble with modal logic, according to its critics, is quantification into modal contexts—i.e. *de re* modality. For on the basis of such quantification, it is claimed, essentialism ensues, and perhaps a bloated universe of *possibilia* as well. The essentialism is avoidable, these critics will agree, but only by turning to a Platonic realm of individual concepts whose existence is no less dubious or problematic than mere *possibilia*. Moreover, basing one's semantics on individual concepts, it is claimed, would in effect render all identity statements containing only proper names either necessarily true or necessarily false—i.e. there would then be no contingent identity statements containing only proper names.

None of these claims is true quite as it stands, however; and in what follows we shall attempt to separate the chaff from the grain by examining the semantics of (first-order) quantified modal logic in the context of different philosophical theories. Beginning with the primary semantics of logical necessity and the philosophical context of logical atomism, for example, we will see that essentialism not only does not ensue but is actually rejected in that context by the validation of the modal thesis of anti-essentialism, and that in consequence all *de re* modalities are reducible to *de dicto* modalities.

Opposed to logical atomism, but on a par with it in its referential interpretation of quantifiers and proper names, is Kripke's semantics for what he properly calls metaphysical necessity. Unlike the primary semantics of logical necessity, in other words, Kripke's semantics for metaphysical necessity is in direct conflict with some of the basic assumptions of logical atomism; and in the form which that conflict takes, which we shall refer to here as the form of a *secondary* semantics for necessity, Kripke's semantics amounts to the initial step toward a proper formulation of Aristotelian essentialism. (A secondary semantics for necessity stands to the primary semantics in essentially the same way that non-standard models for second-order logic stand to standard models.) The problem with this initial step toward Aristotelian essentialism, however, is the problem of all secondary semantics; viz. that of its objective, as opposed to its merely formal, significance—a problem which applies all the more so to Kripke's deepening of his formal semantics by the introduction of an accessibility relation between possible worlds. This, in fact, is the real problem of essentialism.

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There are no individual concepts, it will be noted in what follows, in either logical atomism or Kripke's implicit philosophical semantics, and yet in both contexts proper names are rigid designators; that is, in both there can be no contingent identity statements containing only proper names. One need not, accordingly, turn to a Platonic realm of individual concepts in order to achieve this result. Indeed, quite the opposite is the case. That is, it has in fact been for the defence of contingent identity, and not its rejection, that philosophical logicians have turned to a Platonic realm of individual concepts, since, on this view, it is only through the mere coincidence of the denotations of the individual concepts expressed by proper names that an identity statement containing those names can be contingent. Moreover, unless such a Platonic realm is taken as the intensional counterpart of logical atomism (a marriage of dubious coherence), it will not validate the modal thesis of anti-essentialism. That is, one can in fact base a Platonic or logical essentialism—which is not the same thing at all as Aristotelian essentialism—upon such a realm. However, under suitable assumptions, essentialism can also be avoided in such a realm; or rather it can in the weaker sense in which, given these assumptions, all *de re* modalities are reducible to *de dicto* modalities.

Besides the Platonic view of intensionality, on the other hand, there is also a socio-biologically based conceptualist view according to which concepts are not independently existing Platonic forms but cognitive capacities or related structures of the human mind whose realisation in thought is what informs a mental act with a predicable or referential nature. This view, it will be seen, provides an account in which there can be contingent identity statements, but not such as to depend on the coincidence of individual concepts in the platonic sense. Such a conceptualist view will also provide a philosophical foundation for quantified tense logic and paradigmatic analyses thereby of metaphysical modalities in terms of time and causation. The problem of the objective significance of the secondary semantics for the analysed modalities, in other words, is completely resolved on the basis of the nature of time, local or cosmic. The related problem of a possible ontological commitment to *possibilia*, moreover, is in that case only the problem of how conceptualism can account for direct references to past or future objects.

1 THE PRIMARY SEMANTICS OF LOGICAL NECESSITY

We begin by describing what we take to be the primary semantics of logical necessity. Our terminology will proceed as a natural extension of the syntax and semantics of standard first-order logic with identity. Initially, we shall assume that the only singular terms are individual variables. As primitive *logical constants* we take $\rightarrow, \neg, \forall, =$, and $\Box$ for the material conditional sign, the negation sign, the universal quantifier, the identity sign and the necessity sign, respectively. (The conjunction, disjunction, biconditional, existential quantifier and possibility signs—$\wedge, \vee, \leftrightarrow, \exists$ and $\Diamond$, respectively—are understood to be defined in the usual way as metalinguistic abbreviatory devices.) The only non-logical or *descriptive constants* at this point are predicates of arbitrary (finite) degree. We call a set of such predicates a *language* and understand the well-formed formulas (*wffs*) of a language to be defined in the usual way.

A *model* $\mathfrak{A}$ indexed by a language L, or for brevity, an L-*model*, is a structure of the form $\langle D, R\rangle$, where D, the *universe* of the model, is a non-empty set and R is a function with L as domain and such that for each positive integer n and each n-place predicate F^n in L, $R(F^n) \subseteq D^n$, i.e. $R(F^n)$ is a set of n -tuples of members of D. An *assignment in D* is a function A with the set of individual variables as domain and such that $A(x) \in D$, for each variable x. Where $d \in D$, we understand $A(d/x)$ to be that assignment in D which is exactly like A except for its assigning d to x. The *satisfaction* of a wff φ of L *in* $\mathfrak{A}$ *by* an assignment A in D, in symbols $\mathfrak{A}, A \models \varphi$, is recursively defined as follows:

1. $\mathfrak{A}, A \models (x = y)$ iff $A(x) = A(y)$;
2. $\mathfrak{A}, A \models P^n(x_1, \ldots, x_n)$ iff $\langle A(x_1), \ldots, A(x_n)\rangle \in R(P^n)$;
3. $\mathfrak{A}, A \models \neg\varphi$ iff $\mathfrak{A}, A \not\models \varphi$;
4. $\mathfrak{A}, A \models (\varphi \rightarrow \psi)$ iff either $\mathfrak{A}, A \not\models \varphi$ or $\mathfrak{A}, A \models \psi$;
5. $\mathfrak{A}, A \models \forall x\varphi$ iff for all $d \in D, \mathfrak{A}, A(d/x) \models \varphi$; and
6. $\mathfrak{A}, A \models \Box\varphi$ iff for all R', if $\langle D, R'\rangle$ is an L-model, then $\langle D, R'\rangle$, $A \models \varphi$.

The *truth* of a wff in a model (indexed by a language suitable to that wff) is as usual the satisfaction of the wff by every assignment in the universe of the model. *Logical truth* is then truth in every model (indexed by any appropriate language). One or another version of this primary semantics for logical necessity, it should be noted, occurs in [Carnap, 1946]; [Kanger, 1957]; [Beth, 1960] and [Montague, 1960].

2 LOGICAL ATOMISM AND QUANTIFIED MODAL LOGIC

These definitions, as already indicated, are extensions of essentially the same semantical concepts as defined for the modal free wffs of standard first-order predicate logic with identity. The clause for the necessity operator has a particularly natural motivation within the framework of logical atomism. In such a framework, a model $\langle D, R\rangle$ for a language L represents a *possible world* of a *logical space* based upon (1) D as the universe of objects of that space and (2) L as the predicates characterising the *atomic states of affairs* of that space. So based, in other words, a logical space consists of the totality of atomic states of affairs all the constituents of which are in D and the characterising predicates of which are in L. A possible world of such a logical space then amounts in effect to a partitioning of the atomic states of affairs of that space into two cells: those that obtain in the world in question and those that do not.

Every model, it is clear, determines both a unique logical space (since it specifies both a domain and a language) and a possible world of that space. In this regard, the clause for the necessity operator in the above definition of satisfaction is the natural extension of the standard definition and interprets that operator as ranging over *all* the possible worlds (models) of the logical space to which the given one belongs.

Now it may be objected that logical atomism is an inappropriate framework upon which to base a system of quantified modal logic; for if any framework is a paradigm of anti-essentialism, it is logical atomism. The objection is void, however, since in fact the above semantics provides the clearest validation of the modal thesis of anti-essentialism. Quantified modal logic, in other words, does not in itself commit one to any non-trivial form of essentialism (cf. [Parsons, 1969]).

The general idea of the modal thesis of anti-essentialism is that if a predicate expression or open wff φ *can be true of* some individuals in a given universe (satisfying a given identity- difference condition with respect to the variables free in φ), then φ *can be true of* any individuals in that universe (satisfying the same identity -difference conditions). In other words, no conditions are essential to some individuals that are not essential to all, which is as it should be if necessity means logical necessity.

The restriction to identity-difference conditions mentioned (parenthetically) above can be dropped, it should be noted, if nested quantifiers are interpreted exclusively and not (as we have done) inclusively where, e.g. it is allowed that the value of y in $\forall x \exists y \varphi(x, y)$ can be the same as the value of x. (Cf. [Hintikka, 1956] for a development of the exclusive interpretation.) Indeed, as Hintikka has shown, when nested quantifiers are interpreted exclusively, identity and difference wffs are superfluous—which is especially apropos of logical atomism where an identity wff does not represent an atomic state of affairs. (Cf. Wittgenstein's *Tractatus Logico-Philosophicus* 5.532– 5.53 and [Cocchiarella, 1975a, Section V].)

Retaining the inclusive interpretation and identity as primitive, however, an *identity-difference condition* for distinct individual variables $x_1, \ldots, x_n$ is a conjunction of one each but not both of the wffs $(x_i = x_j)$ or $(x_i \neq x_j)$, for all i, j such that $1 \leq i < j \leq n$. It is clear of course that such a conjunction specifies a complete identity-difference condition for the variables $x_1, \ldots, x_n$. Since there are only a finite number of non-equivalent such conditions for $x_1, \ldots, x_n$, moreover, we understand $\mathrm{ID}_j(x_1, \ldots, x_n)$, relative to an assumed ordering of such non-equivalent conjunctions, to be the jth conjunction in the ordering . *The modal thesis of anti-essentialism* may now be stated as the thesis that every wff of the form

$$\exists x_1 \ldots \exists x_n(\mathrm{ID}_j(x_1, \ldots, x_n) \wedge \Diamond\varphi) \\ \rightarrow \forall x_1 \ldots \forall x_n(\mathrm{ID}_j(x_1, \ldots, x_n) \rightarrow \Diamond\varphi)$$

is to be logically true, where $x_1, \ldots, x_n$ are all the distinct individual variables occurring free in φ. (Where $n = 0$, the above wff is understood to be just $(\Diamond\varphi \rightarrow \Diamond\varphi)$; and where $n = 1$, it is understood to be just $\exists x\Diamond\varphi \rightarrow \forall x\Diamond\varphi$).) The validation of the thesis in our present semantics is easily seen to be a consequence of the following lemma (whose proof is by a simple induction on the wffs of L).

LEMMA If L is a language, $\mathfrak{A}, \mathfrak{B}$ are L-models, and h is an isomorphism of $\mathfrak{A}$ with $\mathfrak{B}$, then for all wffs φ of L and all assignments A in the universe of $\mathfrak{A}$, $\mathfrak{A}, A \models \varphi$ iff $\mathfrak{B}, A/h \models \varphi$.

One of the nice consequences of the modal thesis of anti-essentialism in the present semantics, it should be noted, is the reduction of all *de re* wffs to *de dicto* wffs. (A *de re* wff is one in which some individual variable has a free occurrence in a subwff of the form $\Box\psi$. A *de dicto* wff is a wff that is not *de re*.) Naturally, such a consequence is a further sign that all is well with our association of the present semantics with logical atomism.

THEOREM (De Re Elimination Theorem) For each *de re* wff φ, there is a *de dicto* wff ψ such that $(\varphi \leftrightarrow \psi)$ is logically true.[1]

These niceties aside, however, another result of the present semantics is its essential incompleteness with respect to any language containing at least one relational predicate. (It is not only complete but even decidable when restricted to monadic wffs—of which more anon.) The incompleteness is easily seen to follow from the following lemma and the well-known fact that the modal free non-logical truths of a language containing at least one relational predicate is not recursively enumerable (cf. [Cocchiarella, 1975b]). (It is also for the statement of the infinity condition of this lemma that a relational predicate is needed.)

LEMMA If ψ is a sentence which is satisfiable, but only in an infinite model, and φ is a modal and identity-free sentence, then $(\psi \rightarrow \neg\Box\varphi)$ is logically true iff φ is not logically true.

[1]A proof of this theorem can be found in [McKay, 1975]. Briefly, where $x_1, \ldots, x_n$ are all the distinct individual variables occurring free in φ and $ID_1(x_1, \ldots, x_n), \ldots, ID_k(x_1, \ldots x_n)$ are all the non-equivalent identity-difference conditions for $x_1, \ldots, x_n$, then the equivalence in question can be shown if ψ is obtained from φ by replacing each subwff $\Box\chi$ of φ by:

$$[ID_1(x_1, \ldots x_n) \wedge \Box\forall x_1 \ldots \forall x_n(ID_1(x_1, \ldots, x_n) \rightarrow \chi)] \vee \ldots \\ \vee[ID_k(x_1, \ldots, x_n) \wedge \Box\forall x_1 \ldots \forall x_n(ID_k(x_1, \ldots, x_n) \rightarrow \chi)].$$

STEVEN T. KUHN AND PAUL PORTNER

TENSE AND TIME

1 INTRODUCTION

The semantics of tense has received a great deal of attention in the contemporary linguistics, philosophy, and logic literatures. This is probably due partly to a renewed appreciation for the fact that issues involving tense touch on certain issues of philosophical importance (viz., determinism, causality, and the nature of events, of time and of change). It may also be due partly to neglect. Tense was noticeably omitted from the theories of meaning advanced in previous generations. In the writings of both Russell and Frege there is the suggestion that tense would be absent altogether from an ideal or scientifically adequate language. Finally, in recent years there has been a greater recognition of the important role that all of the so-called indexical expressions must play in an explanation of mental states and human behavior. Tense is no exception. Knowing that one's friend *died* is cause for mourning, knowing that he *dies* is just another confirmation of a familiar syllogism.

This article will survey some attempts to make explicit the truth conditions of English tenses, with occasional discussion of other languages. We begin in Section 2 by discussing the most influential early scholarship on the semantics of tense, that of Jespersen, Reichenbach, and Montague. In Section 3 we outline the issues that have been central to the more linguistically-oriented work since Montague's time. Finally, in Section 4 we discuss recent developments in the area of tense logic, attempting to clarify their significance for the study of the truth-conditional semantics of tense in natural language.

2 EARLY WORK

2.1 Jespersen

The earliest comprehensive treatment of tense and aspect with direct influence on contemporary writings is that of Otto Jespersen. Jespersen's *A Modern English Grammar on Historical Principles* was published in seven volumes from 1909 to 1949. Jespersen's grammar includes much of what we would call semantics and (since he seems to accept some kind of identification between meaning and use) a good deal of pragmatics as well. The

D. Gabbay and F. Guenthner (eds.),
Handbook of Philosophical Logic, Volume 7, 277–346.

aims and methods of Jespersen's semantic investigations, however, are not quite the same as ours.[1]

First, Jespersen is more interested than we are in cataloging and systematizing the various uses of particular English constructions and less interested in trying to characterize their meanings in a precise way. This leads him to discuss seriously uses we would consider too obscure or idiomatic to bother with. For example, Jespersen notes in the *Grammar* that the expressions of the form *I have got A* and *I had got A* are different than other present perfect and past perfect sentences. *I have got a body*, for example, is true even though there was no past time at which an already existent me received a body. Jespersen suggests *I have in my possession* and *I had in my possession* as readings for *I have got* and *I had got.* And this discussion is considered important enough to be included in his *Essentials of English Grammar*, a one volume summary of the *Grammar.*

Jespersen however does *not* see his task as being merely to collect and classify rare flora. He criticizes Henry Sweet, for example, for a survey of English verb forms that includes such paradigms as *I have been being seen* and *I shall be being seen* on the grounds that they are so extremely rare that it is better to leave them out of account altogether. Nevertheless there is an *emphasis* on cataloging, and this emphasis is probably what leads Jespersen to adhere to a methodological principle that we would ignore; viz., that example sentences should be drawn from published literature wherever possible rather than manufactured by the grammarian. Contemporary linguists and philosophers of language see themselves as investigating fundamental intuitions shared by all members of a linguistic community. For this reason it is quite legitimate for them to produce a sentence and assert without evidence that it is well-formed or ill-formed, ambiguous or univocal, meaningful or unmeaningful. This practice has obvious dangers. Jespersen's methodological scruples, however, provide no real safety. On the one hand, if one limits one's examples to a small group of masters of the language one will leave out a great deal of commonly accepted usage. On the other hand, one can't accept *anything* as a legitimate part of the language just because it has appeared in print. Jespersen himself criticizes a contemporary by saying of his examples that 'these three passages are the only ones adduced from the entire English literature during nearly one thousand years'.

A final respect in which Jespersen differs from the other authors discussed here is his concern with the recent history of the language. Although the *Grammar* aims to be a compendium of contemporary idiom, the history of a construction is recited whenever Jespersen feels that such a discussion might be illuminating about present usage. A good proportion of the discussion of the progressive form, for example, is devoted to Jespersen's

[1] By 'ours' we mean those of the authors discussed in the remainder of the article. Some recent work, like that of F. Palmer and R. Huddleston, is more in the tradition of Jespersen than this.

thesis that *I am reading* is a relatively recent corruption of *I am a-reading* or *I am on reading*, a construction that survives today in expressions like *I am asleep* and *I am ashore*. This observation, Jespersen feels, has enabled him to understand the meaning of the progressive better than his contemporaries.[2] In discussing Jespersen's treatment of tense and aspect, no attempt will be made to separate what is original with Jespersen from what is borrowed from other authors. Jespersen's grammar obviously extends a long tradition. See Binnick for a recent survey.[3] Furthermore there is a long list of grammarians contemporaneous with Jespersen who independently produced analyses of tenses. See, for example, Curme, Kruisinga and Poutsma. Jespersen, however, is particularly thorough and insightful and, unlike his predecessors and contemporaries, he continues to be widely read (or at least cited) by linguists and philosophers. Jespersen's treatment of tense and aspect in English can be summarized as follows:

2.1.1 Time

It is important to distinguish *time* from *tense*. Tense is the linguistic device which is used (among other things) for expressing time relations. For example, *I start tomorrow* is a present tense statement about a future time. To avoid time-tense confusion it is better to reserve the term *past* for time and to use *preterit* and *pluperfect* for the linguistic forms that are more commonly called past tense and past perfect. Time must be thought of as something that can be represented by a straight line, divided by the present moment into two parts: the past and the future. Within each of the two divisions we may refer to some point as lying either before or after the main point of which we are speaking. For each of the seven resulting divisions of time there are *retrospective* and *prospective* versions. These two notions are not really a part of time itself, but have rather to do with the perspective from which an event on the time line is viewed. The prospective present time, for example, is a variety of present that looks forward into the future. In summary, time can be pictured as in Figure 2.1.1. The three divisions marked with A's are past; those marked with C's are future. The short pointed lines at each division indicate retrospective and prospective times.

2.1.2 Tense morphology

The English verb has only two tenses proper, the present tense and the preterit. There are also two tense phrases, the perfect (e.g., *I have written*) and the pluperfect or anteperfect (e.g., *I had written*). (Modal verbs,

[2] A similar claim is made in Vlach [1981]. For the most part, however, the history of English is ignored in contemporary semantics.

[3] Many of the older grammars have been reprinted in the series *English Linguistics: 1500–1800 (A Collection of Facsimile Reprints)* edited by R.C. Alston and published by Scholar Press Limited, Menston, England in 1967.

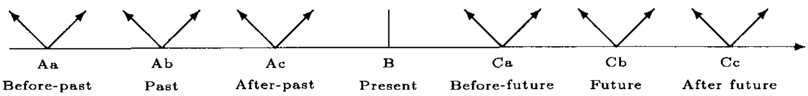

Figure 1.

including *can, may, must, ought, shall,* and *will,* cannot form perfects and pluperfects.) Corresponding to each of the four tenses and tense phrases there is an *expanded* (what is more commonly called today the *progressive*) form. For example, *had been writing* is the expanded pluperfect of *write.* It is customary to admit also future and future perfect tenses, as in *I will write* and *I shall have written.* But these constructions lack the fixity of the others. On the one hand, they are often used to express nontemporal ideas (e.g., volition, obstinacy) and on the other hand future time can be indicated in many other ways.

The present tense is primarily used about the present time, by which we mean an interval containing the present moment whose length varies according to circumstances. Thus the time we are talking about in *He is hungry* is shorter than in *None but the brave deserve the fair.* Tense tells us nothing about the duration of that time. The same use of present is found in expressions of intermittent occurrences (*I get up every morning at seven* and *Whenever he calls, he sits close to the fire*). Different uses of the present occur in statements of what might be found at all times by all readers *Milton defends the liberty of the press in his Areopagitica*) and in expressions of feeling about what is just happening or has just happened (*That's capital!*). The present can also be used to refer to past times. For example, the *dramatic* or *historical* present can alternate with the preterit: *He perceived the surprise, and immediately pulls a bottle out of his pocket, and gave me a dram of cordial.* And the present can play the same role as the perfect in subordinate clauses beginning with *after*: *What happens to the sheep after they take its kidney out?* Present tense can be used to refer to future time when the action described is considered part of a plan already fixed: *I start for Italy on Monday.* The present tense can also refer to future events when it follows *I hope, as soon as, before,* or *until.*

The perfect is actually a kind of present tense that seems to connect the present time with the past. It is both a retrospective present, which looks upon the present as a result of what happened in the past and an inclusive present, which speaks of a state that is continued from the past into the present time (or at least one that has results or consequences bearing on the present time).

The preterit differs from the perfect in that it refers to some time in the past without telling anything about its connection with the present moment. Thus *Did you finish?* refers to a past time while *Have you finished?* is a question about present status. It follows that the preterit is appropriate with words like *yesterday* and *last year* while the perfect is better with *today, until now* and *already. This morning* requires a perfect tense when uttered in the morning and a preterit in the afternoon. Often the correct form is determined by context. For example, in discussing a schoolmate's Milton course, *Did you read Samson Agonistes?* is appropriate,
whereas in a more general discussion *Have you read Samson Agonistes?* would be better. In comparing past conditions with present the preterit may be used (*English is not what it was*), but otherwise vague times are not expressed with the preterit but rather by means of the phrase *used to* (*I used to live at Chelsea*). The perfect often seems to imply repetition where the preterit would not. (Compare *When I have been in London*, with *When I was in London*).

The pluperfect serves primarily to denote before-past time or retrospective past, two things which cannot easily be kept apart. (An example of the latter use is *He had read the whole book before noon.*) After *after, when,* or *as soon as*, the pluperfect is interchangeable with the preterit.

The expanded tenses indicate that the action or state denoted provides a temporal frame encompassing something else described in the sentence or understood from context. For example, if we say *He was writing when I entered*, we mean that his writing (which may or may not be completed now) had begun, but was not completed, at the moment I entered. In the expanded present the shorter time framed by the expanded time is generally considered to be *very recently.* The expanded tenses also serve some other purposes. In narration simple tenses serve to carry a story forward while expanded tenses have a retarding effect. In other cases expanded tense forms may be used in place of the corresponding simple forms to indicate that a fact is already known rather than new, than an action is incomplete rather than complete or that an act is habitual rather than momentary. Finally, the expanded form is used in two clauses of a sentence to mark the simultaneity of the actions described. (In that case neither really frames the other.)

In addition to the uses already discussed, all the tenses can have somewhat different functions in passive sentences and in indirect speech. They also have uses apparently unrelated to temporal reference. For example, forms which are primarily used to indicate past time are often used to denote unreality, impossibility, improbability or non-fulfillment, as in *If John had arrived on time, he would have won the prize.*[4]

[4]From the contemporary perspective we would probably prefer to say here that *had*