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W.T. Tutte 1917-2002

Norman Biggs

Bill Tutte will take his place in history for two reasons. First, as part of the now-famous codebreaking team at Bletchley Park, he made a significant individual contribution to the outcome of the Second World War. Secondly, he formulated and proved many of the theorems that form the foundations of Graph Theory. In both cases his achievements resulted from very deep insights into matters that, at first sight, might be thought simple. In 1977 Paul Erdős recalled [13]:

> I first heard about Tutte in early September 1939 .... But of the real powers I only learned later. T. Gallai and I as freshmen in 1930 took the course on graph theory by D. Kőnig – he mentioned the conjecture of Tait and the extension of Petersen's theorem on factorisation of graphs as important outstanding problems – we tried both unsuccessfully. As is well known, Tutte settled both problems – and many others.

William Thomas Tutte was born on May 14, 1917 at Fitzroy House, a horseracing establishment in Newmarket, England. Around 1921 the family settled in Chevely, a village near Newmarket, where his father was gardener at the Rutland Arms Hotel. He attended the village school until, at the age of 11, he won a scholarship to the Cambridge and County High School. The school was 15 miles from his home and the daily journey was difficult, but well worth the effort. He won many prizes, and in the school library he discovered W.W. Rouse Ball's book of *Mathematical Recreations and Essays*, in which he read about the Five-Colour Theorem and Petersen's Theorem. Both these results were to figure largely in his life's work.

In 1935 he went up to Trinity College, Cambridge, supported by a State Scholarship and a College Scholarship. He read Natural Sciences, specialising in Chemistry, and getting a First Class Honours degree. He also joined the Trinity Mathematical Society, where he met R.L. Brooks, C.A.B. Smith, and A.H. Stone. Together they worked on the problem of dividing a square into squares of different sizes. The story of their work on this problem has been told many times, including by Tutte himself in the *Scientific American* [6]. It is always a surprise to find just how many important ideas arose first in this work on a problem in 'recreational' mathematics. The ideas about spanning trees can be traced back to Kirchhoff, but many of their algebraic results were new, as were the insights into planarity and duality.

By the time that the paper on squaring the square was published [1], Tutte had started his research in Chemistry, and had produced two papers on his experimental results. His progress was interrupted when he was called up for

national service in the Second World War, and after initial training he arrived at Bletchley Park, the British cryptographic HQ, in 1941. He was one of many who regarded signing the Official Secrets Act as a lifelong obligation, and when stories of the great deeds done at Bletchley began to leak out, often in a garbled fashion, he did not immediately leap on the bandwagon. It was probably a relief to him when, in the 1990s, it became clear that at least some of the secrets were no longer official. At his eightieth birthday celebrations in 1997 he felt able to talk informally about some of the details, and in 1998 he was persuaded to give a talk at the opening of the Center for Applied Cryptographic Research at the University of Waterloo.

In this talk [10], entitled *Fish and I*, he tells how, others having failed, he was asked to work on the cipher system known at Bletchley as Tunny. This was one of the 'Fish' codes used by the German High Command. He had an idea and, although not optimistic, he 'thought it best to seem busy'. So he copied out some ciphertext onto sheets of squared paper, using chunks of various lengths, noticed certain patterns, and was able to infer the structure of the system. Indeed, he achieved a virtual reconstruction of an extremely complex machine using only scraps of information – an amazing feat that must rank as one of his greatest intellectual achievements.

The success of Bletchley as an institution was partly due to the fact that the powers-that-be were not stupid, and soon many people were helping to work out the implications of Tutte's discovery. This continued throughout 1942 and 1943, with regular upgrading of the techniques to deal with improvements in the system. Eventually it became necessary to use a form of number-crunching statistical analysis, and Tutte saw how this could be done. He reported his ideas and, in his own words, 'there were rapid developments'. The outcome was that the famous Colossus computer was deployed on these problems.

At the end of the War, Trinity College elected Tutte to a Research Fellowship in Mathematics. Although less prestigious in the public eye than the awards given to other civil servants, it was probably more appreciated by the recipient. Exactly how it came about is unclear. C.A.B. Smith recalled being stopped in the street by a Fellow of Trinity, who said 'we've just elected Tutte to a Fellowship but we don't know what he has done or where he lives' [14].

The period at Trinity was a highly productive one. His Ph.D. thesis on 'An Algebraic Theory of Graphs' contained many seminal ideas, and these were published in papers that quickly established graph theory as a significant area of mathematics, with Tutte as its master builder. Among the papers that were published at that time there are several classics. In a paper published in 1946 [2] he disproved Tait's conjecture by constructing a planar cubic graph that has no Hamilton cycle. His paper on the symmetry of cubic graphs [3] contains a truly unexpected bound on the order of a vertex-stabilizer, a fact that was to resurface twenty years later in the work of permutation-group-theorists. Perhaps the most influential paper is the one on factorization of graphs [4], in which he obtains the canonical form of the basic result on this

topic, with Petersen's theorem as a simple corollary.

Much later, in his book *Graph Theory As I Have Known It* [11], he gave a fascinating account of how he arrived at some of these fundamental results. Perhaps not surprisingly, it was often by a process that offered an intellectual challenge rather than a guarantee of success. The use of Pfaffians in the proof of the factorization theorem was marvellous, even if it was later shown to be unnecessary. As well as graph theory, his thesis also contained important results about matroids, a subject that had been inaugurated by Hassler Whitney. Many of these results were published about ten years later [7], but their significance was not fully recognized until they appeared in a series of lectures in 1965 [8].

In 1948 he took up a post at the University of Toronto. Here he continued to produce a stream of new ideas and, rather unexpectedly, for he was a very shy man, he got married. His wife Dorothea would bemoan the fact that weekends had to be spent on research, because Bill feared that mathematical inspiration would dry up before he was 40 (at least, that's what he told her). Some of his Toronto papers discussed aspects of the chromatic polynomial and its two-variable generalization, now known (justifiably) as the Tutte polynomial [5]. Several famous conjectures, such as the conjecture that every bridgeless graph has a 5-flow, also appeared in print at this time.

In 1962 he was persuaded to move to the newly-established University of Waterloo. By this time he had been appointed a Fellow of the Royal Society of Canada, and his eminence was being recognised internationally. The university created around him a world-famous Department of Combinatorics and Optimisation, and it was instrumental in the foundation of the *Journal of Combinatorial Theory*. He himself was not an administrator, but he supported and encouraged people whose talents lay in that direction, and his placid temperament helped to calm the troubled waters that sometimes threatened his Department.

By the 1970s the growth of air travel meant that Bill and Dorothea were able to travel extensively, and they returned to England on several occasions. In 1971 he was the principal guest at a small meeting held in Royal Holloway College. The success of that meeting led to the establishment of the continuing series of British Combinatorial Conferences, and Bill spoke at the first one to be organised on a regular basis, the Fourth BCC in Aberystwyth (1973).

His work at this period centred on the enumeration of planar graphs, and specifically four-colourable planar graphs. There was a slight chance that the four-colour conjecture could be settled 'asymptotically', but he did not have great hopes for the method. He greeted the Appel-Haken resolution of the conjecture enthusiastically, agreeing that the strategy was sound, even if the calculations could not be checked by hand [9].

He retired formally in 1985, but continued to be active in mathematics. In his quiet way he enjoyed the recognition that accompanied the growth in popularity and status of Graph Theory, the subject he had built. Outstanding

mathematicians were attracted to work in this field, many of them inspired by Tutte's earlier results.

After Dorothea's death in 1994 he lived in England for a while, but he did not settle, and eventually returned to his adopted home in Ontario. It was proper that his eightieth birthday should be marked by a celebration in Waterloo where he was able to talk about his work to an audience that fully appreciated what he had achieved. In Britain we were fortunate to have him as the Rado Lecturer at the BCC in Canterbury in 1999, where he spoke about *The Coming of the Matroids* [12]. In this talk he explained how some of his work at Bletchley had helped him to understand the properties of linear dependence, and how this led to some of the fundamental theorems of matroid theory.

In 2001 his eminence was recognised by the award of the Order of Canada, which he received with characteristic humour and humility. At that time he was in good health, but in March 2002 he was diagnosed with a serious medical condition, and he died on May 2, in his 85th year.

Author's Note This is an extended version of the obituary published in *The Independent* on 9 May 2002. I am grateful to several people, in particular Dan Younger, for additional information. A full appreciation of Tutte's mathematical work is planned to appear in the *Bulletin of the London Mathematical Society.*

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