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This authoritive volume shows how modern dynamical systems theory can help us in understanding the evolution of cosmological models. It also compares this approach with Hamiltonian methods and numerical studies. A major part of the book deals with the spatially homogeneous (Bianchi) models and their isotropic subclass, the Friedmann–Lemaître models, but certain classes of inhomogeneous models (for example 'silent universes') are also examined. The analysis leads to an understanding of how special (high symmetry) models determine the evolution of more general families of models; and how these families relate to real cosmological observations.

This is the first book to relate modern dynamical systems theory to both cosmological models and cosmological observations. It provides an invaluable reference for graduate students and researchers in relativity, cosmology and dynamical systems theory.

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Dynamical Systems in Cosmology

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Preface

This book had its origins in a workshop held in Cape Town from June 27 to 2 July 1994, with participants from South Africa, USA, Canada, UK, Sweden, Germany, and India. The meeting considered in depth recent progress in analyzing the evolution and structure of cosmological models from a dynamical systems viewpoint, and the relation of this work to various other approaches (particularly Hamiltonian methods). This book is however not a conference report. It was written by some of the conference participants, based on what they presented at the workshop but altered and extended after reflection on what was learned there, and then extensively edited so as to form a coherent whole. This process has been very useful: a considerable increase in understanding has resulted, particularly through the emphasis on relating the results of the qualitative analysis to possible observational tests. Apart from describing the development of the subject and what is presently known, the book serves to delineate many areas where the answers are still unknown. The intended readers are graduate students or research workers from either discipline (cosmological modeling or dynamical systems theory) who wish to engage in research in the area, tackling some of these unsolved problems.

The role of the two editors has been somewhat different. One (GFRE) has played a standard editorial role in terms of reviewing and editing material; he also initiated and organized the meeting, which took place in South Africa shortly after the transition to democracy there through the remarkable elections of 1994. The meeting was a suitable scientific celebration of those events. The other editor (JW) has been responsible for the main burden of shaping the volume, giving it its overall coherence. We thank the authors for their cooperation in this process, and for reading and commenting on other chapters, which has resulted in a far more useful volume than if it had been just a record of the workshop proceedings. We also thank Richard Matzner,

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one of the participants in the workshop, for his contributions concerning cosmological observations, which were very useful to us in writing Chapter 3. We would like to thank Alan Coley, Malcolm MacCallum and David Siegel for their comments on some of the chapters; Henk van Elst for discussions concerning evolution and constraint equations, and for detailed comments on the manuscript; and research students Robert Bridson and Derek Harnett at the University of Waterloo for reading parts of the manuscript and checking calculations. We are particularly indebted to Conrad Hewitt and Claes Uggla for continuing discussions over the past eighteen months.

We thank the Foundation for Research and Development (South Africa) and the Research Committee of the University of Cape Town for financial support of the meeting; our secretary Di Loureiro for her unfailing good humour, efficiency, and helpfulness in its organization; and graduate students Bruce Bassett, Tim Gebbie, Nico Christodoulides, and particularly Conrad Mellin, for the assistance they gave to conference participants. One of us (J. Wainwright) would like to thank the Department of Mathematics and Applied Mathematics at the University of Cape Town and the School of Mathematical Sciences at Queen Mary and Westfield College for their hospitality during several visits while the book was being edited.

As regards the preparation of the manuscript we thank Ann Puncher and Helen Warren at the University of Waterloo for their excellent and efficient work, and unfailing good humour when faced with successive rounds of changes. We also thank Lorraine Kritzer for help with the figures.

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