

Cambridge University Press  
0521265037 - A Handbook of Fourier Theorems  
D. C. Champeney  
Frontmatter  
[More information](#)

---

*A handbook of Fourier theorems*

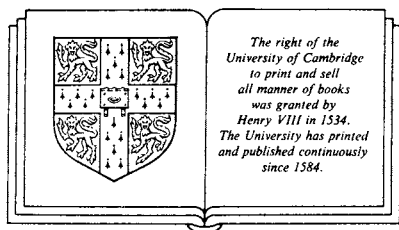
A HANDBOOK OF

---

# *Fourier theorems*

D. C. CHAMPENEY

*School of Mathematics and Physics, University of East Anglia*



CAMBRIDGE UNIVERSITY PRESS

Cambridge

New York Port Chester Melbourne Sydney

Cambridge University Press  
0521265037 - A Handbook of Fourier Theorems  
D. C. Champeney  
Frontmatter  
[More information](#)

---

Published by the Press Syndicate of the University of Cambridge  
The Pitt Building, Trumpington Street, Cambridge CB2 1RP  
40 West 20th Street, New York, NY 10011, USA  
10 Stamford Road, Oakleigh, Melbourne 3166, Australia

© Cambridge University Press 1987.

First published 1987  
First paperback edition 1989  
Reprinted 1990

*British Library cataloguing in publication data*

Champeney, D. C.  
A handbook of Fourier theorems.  
1. Fourier analysis  
I. Title  
515'.2433 QA403.5

*Library of Congress cataloguing in publication data*

Champeney, D. C.  
A handbook of Fourier theorems.  
Bibliography  
Includes index.  
1. Fourier analysis. I. Title.  
QA403.5.C47 1987 515'.2433 86-32694

ISBN 0 521 26503 7 hard covers  
ISBN 0 521 36688 7 paperback

Transferred to digital printing 2000

Cambridge University Press  
0521265037 - A Handbook of Fourier Theorems  
D. C. Champeney  
Frontmatter  
[More information](#)

---

*To Julie, John and Anna*

## CONTENTS

---

<i>Preface</i>	xi
<b>1 Introduction</b>	1
<b>2 Lebesgue integration</b>	4
2.1 Introduction	4
2.2 Riemann integration	4
2.3 Null sets	5
2.4 The Lebesgue integral	6
2.5 Nomenclature	8
2.6 Conditions for integrability; measurability	9
2.7 Functions in $L^p$	12
2.8 Integrals in several dimensions	13
2.9 Alternative approaches	14
<b>3 Some useful theorems</b>	15
3.1 The Minkowski inequality	15
3.2 Hölder's theorem	16
3.3 Young's theorem	17
3.4 The Fubini and Tonelli theorems	18
3.5 Two theorems of Lebesgue	19
3.6 Absolute and uniform continuity	20
3.7 The Riemann–Lebesgue theorem	23
<b>4 Convergence of sequences of functions</b>	24
4.1 Introduction	24
4.2 Pointwise convergence	24
4.3 Bounded, dominated and monotone convergence	25
4.4 Uniform convergence	26
4.5 Convergence in the mean	27
4.6 Cauchy sequences	29

viii *Contents*

<b>5</b>	<b>Local averages and convolution kernels</b>	30
5.1	Introduction	30
5.2	Lebesgue points	31
5.3	Approximate convolution identities	33
5.4	The Dirichlet kernel and Dirichlet points	35
5.5	The functions of du Bois-Reymond and of Fejér	36
5.6	Carleson's theorem	37
5.7	Kolmogoroff's theorem	37
5.8	The Dirichlet conditions	38
5.9	Jordan's theorem	39
5.10	Dini's theorem	41
5.11	The de la Vallée-Poussin test	43
<b>6</b>	<b>Some general remarks on Fourier transformation</b>	44
6.1	Introduction	44
6.2	The definition of the Fourier transform	44
6.3	Sufficient conditions for transformability	47
6.4	Necessary conditions for transformability	49
<b>7</b>	<b>Fourier theorems for good functions</b>	51
7.1	Introduction	51
7.2	Inversion, differentiation and convolution theorems	52
7.3	Good functions of bounded support	55
<b>8</b>	<b>Fourier theorems in <math>L^p</math></b>	60
8.1	Basic theorems and definitions	60
8.2	More inversion theorems in $L^p$	63
8.3	Convolution and product theorems in $L^p$	71
8.4	Uncertainty principle and bandwidth theorem	75
8.5	The sampling theorem	77
8.6	Hilbert transforms and causal functions	78
<b>9</b>	<b>Fourier theorems for functions outside <math>L^p</math></b>	81
9.1	Introduction	81
9.2	Functions in class $K$	82
9.3	Convolutions and products in $K$	85
9.4	Functions outside $K$	87
<b>10</b>	<b>Miscellaneous theorems</b>	91
10.1	Differentiation and integration	91
10.2	The Gibbs phenomenon	93
10.3	Complex Fourier transforms	95
10.4	Positive-definite and distribution functions	99
<b>11</b>	<b>Power spectra and Wiener's theorems</b>	102
11.1	Introduction	102
11.2	The autocorrelation function	104

<i>Contents</i>	ix
11.3 The spectrum and spectral density	106
11.4 Discrete spectra	109
11.5 Continuous spectra	113
11.6 Miscellaneous theorems	114
<b>12 Generalized functions</b>	<b>118</b>
12.1 Introduction	118
12.2 The definition of functionals in $S'$	119
12.3 Basic theorems	123
12.4 Examples of generalized functions	127
<b>13 Fourier transformation of generalized functions I</b>	<b>135</b>
13.1 Definition of the transform	135
13.2 Simple properties of the transform	136
13.3 Examples of Fourier transforms	137
13.4 The convolution and product of functionals	139
<b>14 Fourier transformation of generalized functions II</b>	<b>145</b>
14.1 Functionals of types $D'$ and $Z'$	145
14.2 Fourier transformation of functionals in $D'$	149
14.3 Transformation of products and convolutions in $D'$	152
<b>15 Fourier series</b>	<b>155</b>
15.1 Fourier coefficients of a periodic function	155
15.2 The convergence of Fourier series	156
15.3 Summability of Fourier series	158
15.4 Mean convergence of Fourier series	159
15.5 Sampling theorems	162
15.6 Differentiation and integration of Fourier series	164
15.7 Products and convolutions	166
<b>16 Generalized Fourier series</b>	<b>170</b>
16.1 Introduction	170
16.2 Generalized Fourier coefficients	171
16.3 The Fourier formulae	172
16.4 Differentiation, repetition and sampling	173
16.5 Products and convolutions	175
<b>Bibliography</b>	<b>177</b>
<b>Index</b>	<b>181</b>

## *PREFACE*

---

This handbook is intended to assist those scientists, engineers and applied mathematicians who are already familiar with Fourier theory and its applications in a non-rigorous way, but who wish to find out the exact mathematical conditions under which particular results can be used. A reader is assumed whose mathematical grounding in other respects goes no further than the traditional first year university course in mathematics taken by physical scientists or engineers. Advances in mathematical sophistication have led to a growing divide between those books intended for mathematics specialists and those intended for others, and this handbook represents a conscious effort to bridge this gap.

The core of the book consists of rigorous statements of the most important theorems in Fourier theory, together with explanatory comments and examples, and this occupies chapters 6–16. This is preceded, in chapters 1–5, by an introduction to the terminology and the necessary ideas in mathematical analysis including, for instance, the interpretation of Lebesgue integrals. Proofs of theorems are not provided, and the first part is not intended as a complete grounding in mathematical analysis; however, it is intended that the book should be self contained and that it should provide a background which will assist the interested reader to follow proofs in standard mathematical texts.

In the chapters on Fourier theorems, those classical theorems dealing with locally integrable functions are covered first in a way that leads naturally to the later sections covering generalized functions. The more modern extensions in the direction of abstract harmonic analysis are not covered.