INTRODUCTION

This is the second volume of *Collected Papers of Stig Kanger with Essays on his Life and Work.* The first volume contains Kanger's own published papers, most of which have become virtually inaccessible even in his own country, together with one previously unpublished manuscript: 'Choice based on preference'. In this second volume we have collected critical essays on the various aspects of Kanger's work as well as some biographical sketches.

Stig Kanger made groundbreaking contributions to a broad range of areas within both mathematical and philosophical logic:

(i) *General proof theory*: In 1955–57, several logicians – Beth, Hintikka, Kanger and Schütte, independently of each other – brought about a kind of synthesis between the proof-theoretic methods of Gentzen and the model-theoretic ones of Tarski. Exploiting the close correspondence between the rules of inference of Gentzen's calculus of sequents and the semantic clauses of Tarski's definition of truth, they obtained very natural and simple proofs of Gödel's completeness theorem for classical first-order predicate logic. The fundamental idea was to view a proof of a logically valid formula as an unsuccessful attempt to find a counter-model to it. Kanger's completeness proof in his 1957 dissertation *Provability in Logic* – perhaps the most elegant – established in a direct way the connection between Gentzen's sequent calculus and Tarski's model theory. As an immediate corollary, Kanger's completeness proof yielded a simple (but non-constructive) proof of Gentzen's *Hauptsatz.*

Kanger's work in general proof theory is described in Göran Sundholm's contribution to this volume: 'The proof theory of Stig Kanger: a personal recollection'. Sundholm also describes how the Beth-Hintikka-Kanger-Schütte proof method has been extended beyond elementary logic after Kanger. In addition, Sundholm's article contains information about Kanger's early work in mathematical logic.

Additional light is thrown on Kanger's proof theory and semantics by Kaj Børge Hansen in his 'Kanger's ideas on non-well-founded sets'. Hansen describes how, at one point in his dissertation, Kanger outlines a theory of non-wellfounded sets, and makes use of it in a proof of a version of his completeness theorem for predicate logic. Hansen gives a careful and thorough analysis of Kanger's proof and points out that the particular version of the

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completeness theorem does not hold without the assumption of non-wellfounded sets. Nowadays, non-wellfounded sets have of course become a topical research subject due especially to the work of Peter Aczel. This was far from the case when Kanger wrote his book.

(ii) *Efficient proof procedures and automated deduction*: As a by-product, Kanger's completeness proof yields a *proof procedure* that is *effective* in the sense of providing an algorithm for finding a proof of any given logically valid sequent. To construct a proof of a valid sequent $\Gamma \Rightarrow \Delta$, we start from below with the given sequent and construct a tree of sequents above it by means of repeated backwards applications of the rules of the cut-free sequent calculus. We continue until the process terminates and we have reached an axiom at the top of each branch in the tree. The resulting tree is then a proof of the valid sequent that we started with. Kanger's completeness proof guarantees that the process terminates after finitely many applications of the rules, provided, of course, that the sequent we started with was indeed valid. In the paper 'A simple proof procedure for elementary logic', Kanger describes how the proof procedure can be extended to predicate logic with identity and how it can be made more efficient for actual implementation on a computer.

Kanger's work on efficient proof procedures was carried further by Dag Prawitz. In 'A note on Kanger's work on efficient proof procedures' Prawitz gives a personal account of Kanger's and his own work to make proof procedures more efficient. He describes how in the late 1950s Kanger's first proof procedure was implemented on a computer and the difficulties that then arose. Prawitz gives a fascinating account of the genesis of the so-called "dummy method" at one of Kanger's seminars. The subsequent fate of the Kanger–Prawitz method of "dummies" for making proofs more efficient is also described in Prawitz's article.

In 'Kanger's choices in automated reasoning' Anatoli Degtyarev and Andrei Voronkov discuss how Kanger's classical 1963 paper 'A simple proof procedure for elementary logic' fares compared to modern work on automated deduction. They write: "Now, when we are equipped with the impressive amount of techniques developed in this are, we are amazed by the incredible intuition of Kanger that allowed him to choose elegant, interesting (and correct) solutions among many possible choices. This article explains these choices and their place in modern automated deduction".

(iii) *Algebraic logic.* In the 1960s, Kanger came into contact with the group of logicians around Tarski at Berkeley and the work that they pursued on the algebraic study of first-order predicate logic by means of so-called cylindric algebras. Intuitively, cylindric algebras play a role in the study of predicate logic that is analogous to that played by Boolean algebras in the study of sentential logic. Kanger was very impressed by this work and it inspired him to develop an *algebraic logic calculus*, where the ordinary sentences of predicate logic are treated as terms, the statements are equations between terms, and the only rule of inference is substitution of equals by equals.

In his contribution to this volume, 'The proper treatment of quantifiers in ordinary logic', Jaakko Hintikka reviews Kanger's algebraic approach to standard first-order logic and discusses whether it can be applied more generally, in particular to Hintikka's own generalization of standard first-order logic, so-called *independence-friendly first-order logic* (*IF-logic*).

There are several reasons why Kanger's equational approach does not seem well suited for the study of IF-logic. First of all, the set of valid formulas of IF-logic is not recursively enumerable. Hence, IF-logic does not admit of a complete *proof procedure*. On the other hand, there exists a complete *disproof procedure*: If A is an unsatisfiable formula of IF-logic, then there is a taubleaux-type (i.e., Gentzen-type) demonstration of this fact. In ordinary two-valued logic, the existence of a complete disproof procedure is tantamount to the existence of a complete proof procedure. Ordinarily, if a formula is irrefutable, i.e., lacks a counter-model, then it is valid. Due to the failure of the law of excluded middle, this implication does not hold in IF-logic.

But couldn't Kanger's equational approach still be applied to IF-logic – simply by formulating the rules of logical disproof as an equational calculus? This is not a simple matter either, due to the apparent failure in IF-logic of the principle of compositionality. Intuitively speaking, the semantic interpretation of a formula of IF-logic depends, not only on the semantic interpretations of its subformulas, but also on the context in which the formula occurs. Consequently, substitution of equals by equals (applied to formulas) does not, in general, preserve satisfiability.

Hintikka points to a way around this problem. Kanger's algebraic methods can still be used, once predicate symbols and quantifiers have been eliminated in favor of so-called Skolem functions. By means of this technique, the problem of testing a finite set of formulas of IF-logic for satisfiability can be reduced to the problem of testing whether a certain Boolean combination of equations is derivable in an equational calculus à la Kanger. Hintikka remarks: "Kanger's calculus of functional equations can handle more than he himself pointed out".

(iv) *Semantics for modal logic*. In Kanger's dissertation from 1957, appears, for the first time in print, a detailed exposition of a Tarski-style model-theoretic semantics for quantified modal logic. A crucial innovation was the use of accessibility relations in the semantic evaluation clauses for modal operators. Kanger points out that by imposing various formal requirements on the accessibility relation one can make the operator satisfy corresponding well-known axioms of modal logic. In this way, the introduction of accessibility relations made it possible to apply semantic and model-theoretic methods to the study of a variety of modal notions.

Kanger's early semantics for modal logic differs in interesting ways from the semantic frameworks developed, at about the same time, by Hintikka, Kripke and Montague. Kanger's work on modal logic is discussed in Sten Lindström's paper 'An exposition and development of Kanger's early semantics for modal logic'.

(v) *Deontic logic.* In 'New Foundations for Ethical Theory' from 1957, Kanger developed a model-theoretic semantics also for normative concepts, the so-called *deontic modalities* "It ought to be that ...", "It is right that ...", and to *imperatives*: "Let it be the case that ... !". Kanger's formal language contains quantifiers as well and he discusses the interplay between these and deontic operators. It is noteworthy that Kanger already in this early paper discusses the notion of agency. In terms of the deontic operators and the notion of agency, Kanger, already in 1957, takes the first steps in developing a theory of rights.

Kanger's contributions to deontic logic are discussed in Hilpinen's paper 'Stig Kanger on deontic logic'.

(vi) *Theory of rights and actions.* Kanger's work in deontic logic led him to develop a *typology of rights*, inspired by the work of the American jurist W. N. Hohfeld, within the framework of a formal language containing among its primitive concepts, in addition to deontic operators, the *action operator* "X sees to it that ...". Kanger's theory of rights is arguably his most substantial and influential contribution outside of the field of pure logic.

In this volume, Kanger's theory of rights is dealt with in Lars Lindahl's 'Stig Kanger's theory of rights' and in Lennart Åqvist's 'Stig Kanger's theory of rights: bearers and counterparties, sources-of-law, and the Hansson Petaluma example'. Kanger's contributions to the theory of action are described in Ghita Holmström-Hintikka's 'Stig Kanger's actions and influence'. Holmström-Hintikka also discusses Kanger's attempts at developing a typology of different kinds of influence that is analogous to his typology of rights.

(vii) *Theory of preference and choice.* The theory of preference and rational choice occupied Kanger intermittently during the last 20 years of his life. A comprehensive overview of Kanger's contributions to this area is given by Sven Ove Hansson in 'Kanger's theory of preference and choice'. Hansson discusses Kanger's attempts to develop a preference logic in the tradition of Halldén, his so-called *paradox of exclusive disjunction* (more extensively treated by Rabinowicz, see below), as well as Kanger's contribution to the theory of rational choice (more extensively treated by Sen, see below).

In his contribution 'Preference logic and radical interpretation: Kanger meets Davidson', Wlodek Rabinowicz discusses a paradox in preference logic (referred to by Hansson as 'the paradox of exclusive disjunction') that was formulated by Kanger and that led Donald Davidson to modify his theory of radical interpretation. Rabinowicz argues that although Kanger's paradox can be dissolved, Davidson's theory of radical interpretation still confronts serious difficulties.

Finally, Amartya Sen in 'Non-binary choice and preference: a tribute to Stig Kanger' discusses Kanger's contribution to the theory of rational choice in 'Choice based on preference'. In this paper Kanger generalizes the standard theory of preference and choice to choice functions that select a set of alternatives from a "menu" of available alternatives against a "background set" of alternatives. As the background set varies, the selected set may vary as well, even if the menu of available alternatives is kept fixed. Sen compares Kanger's approach to the standard theory of rational choice and discusses the reasons that Kanger might have had for adopting his alternative approach.

The process of publishing these two volumes dedicated to the work of Stig Kanger has been a genuinely joint venture in which many people have contributed in essential ways. We are grateful to them all, including the contributors of essays on Kanger's life and work. We owe special thanks to Jaakko Hintikka and Krister Segerberg for their enthusiastic and steadfast support of the project as well as for their inspiration and good advice. We also wish to thank Jaakko Hintikka for including the two volumes in the Synthese Library Series and Krister Segerberg for editing the section of biographical sketches. We are grateful to Ms. Annie Kuipers and Mr. Rudolf Rijgersberg at Kluwer for their patience and cooperation, Ms. Kaipainen at the Department of Philosophy of the University of Helsinki for doing an excellent work in transforming Kanger's typographically difficult texts into camera-ready copy, Sharon Rider and Kaj Børge Hansen for translating some of Kanger's Swedish texts into English, and Sven Ove Hansson and Lars Lindahl for valuable editorial assistance. Kaj Børge Hansen helped us prepare the indexes for the two volumes and Anders Berglund assisted us with proof reading.

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KRISTER SEGERBERG

STIG KANGER (1924–1988)

The Mission Covenant Church of Sweden — *Svenska Missionsförbundet*, a free church not part of the Swedish State Church — performed missionary work in China 1890–1951. Its effort was concentrated in Hupei, a province in central China through which flows the great Yangtse. In 1919 two young missionaries joined the mission, Gustav Karlsson, a farm labourer from the south of Sweden, and Sally Svensson, a nurse from Stockholm. Gustav's only education beyond elementary school (*folkskolan*) was four years of mission school; nevertheless he later gained, by correspondence, two academic degrees in theology from Webster University, Atlanta, Georgia: a bachelor's degree in 1936 and a doctorate in 1948. Sally had obtained a midwife's certificate before her two years of mission school.

Gustav and Sally were engaged to be married already before their departure from Sweden, but only three years later were they actually married. Their marriage was blessed with two sons of whom the elder, Stig Gustav, was born on 10 July 1924 in Kuling (short for Ku Niu-ling, the Mountain of the Wild Ox) in the province of Kiangsi, a popular summer resort in the mountains. Stig began school when he was six, and he had six years of schooling before the family returned to Sweden. The first year he was taught by his mother, then by two missionaries: for four years by Ida Pettersson and then one year by Lisa Björkdahl. During Ida Pettersson's time the class consisted first of two (Stig and another boy) and later of four (when their brothers had joined). But during the final year, Stig and his brother, Rune, were the only students.

In 1930 Gustav and Sally Karlsson decided to adopt *Kanger* as their family name. The most common traditional Swedish surnames are of the *Karlsson* type — literally, "Karl's son". So common have these names been that, especially at the end of the nineteenth century and the beginning of the twentieth, many have preferred to change to a more distinctive, often made-up, name. *Kanger* is such a name, a combination of *Karlsson* and *Hånger*. (The latter was the name of Gustav Karlsson's birth place, a village in Småland in the south of Sweden, where his forebears had been peasants for generations and his father still operated a small farm. Later Stig inherited and used as a

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holiday home his grandfather's cottage at Erikslund in a forest a few miles from Hånger, usually referred to by Stig as "the Middle of Nowhere".)

Except for a visit to Sweden in 1927–29, the family remained in China until 1936. In 1939 the parents left for a final sojourn in China, which was to last through World War II until 1946; the sons were left at a home for missionaries' children in Stockholm. (The parents, both born in 1893, both died in 1954.) Stig attended Palmgrenska Samskolan and then Tekniska Läroverket, passing *teknisk studentexamen* in May 1942 and *studentexamen på reallinjen* in December 1944. The latter examination was the formal prerequisite for university entrance, but by the time he sat it, he had already begun informal studies at the University of Stockholm. In 1945 there was a brief interlude of military service: Stig was called up but was discharged after only a short time.

At the university, Kanger followed a normal path, gaining the degree of *filosofie kandidat* in 1949. In those days the requirement was at least seven units of courses in at least three subjects. Kanger's degree consisted of three units of theoretical philosophy, three units of practical philosophy and one unit of statistics. Two years later, in 1951, he received the degree of *filosofie licentiat*, a higher degree for which a thesis was required. In his thesis, entitled "En studie i modal logik, med särskild hänsyn till 'böra'-satser" ["A Study in Modal Logic, with Special Attention to 'Ought'-sentences"], Kanger showed how, in a certain sense, deontic logic is reducible to modal logic plus a new primitive constant. After the thesis was accepted — it received the highest grade — Kanger asked his professor, Anders Wedberg, whether he thought that publication was warranted. Wedberg thought not, and the thesis was never published. But when a few years later an idea equivalent to Kanger's was published by Alan Ross Anderson, it attracted a good deal of attention from philosophical logicians. Unfortunately, no copy of Kanger's thesis seems to have survived. In 1957 he defended his doctoral dissertation, *Provability in Logic*. At that time dissertations were graded: Kanger's was given the second highest grade.

The dissertation earned Kanger a position as *docent* in theoretical philosophy 1957–1963 at the University of Stockholm. The *docentur* was a much coveted research position of a kind unfortunately no longer existing; just about the only obligation was to lecture seventy-five hours a year — thus between two or three hours a week — on subjects freely chosen. The idea was of course to leave the *docent* ample time to develop as a researcher. Kanger made good use of this freedom (even though he spent several terms acting in place of professors on leave, something that was better paid). As a formal logician he may have been limited in his methods, but the applications of his work spanned an impressive array of subjects: meaning theory, measurement theory, ethics, theory of action, theory of rights, theory of preference, phonematics and even (unpublished) æsthetics. The academic year 1965–66 he spent as a visiting associate professor at the University of Michigan, Ann Arbor, and in the summer of 1966 he taught at Stanford. In 1968–69 he was a visiting professor at the University of California, Berkeley.

The drawback of the position of *docent* was that it was for a limited period only. The future was uncertain, a fact that became even more pressing when the years as *docent* were up. By luck, this coincided with the appointment in 1963 of Erik Stenius to the Swedish language chair of philosophy at the University of Helsinki. Stenius left vacant the chair of philosophy at the Swedish language university in Finland, Åbo Academy in Turku (Åbo), a chair originally created for Edward Westermarck. Here Kanger became acting professor for several years before finally being appointed *professor ordinarius* on 9 February 1968. But by that time his appointment to the chair of theoretical philosophy at the University of Uppsala from 1 July 1968 was well under way.

When after his Berkeley year Kanger took up his duties as the new professor at Uppsala in the fall of 1969, he inaugurated a new era. This was at a time when there was money around, and Kanger was good at getting hold of it. For some years, the Uppsala philosophy department became a thriving hive of activity with visitors coming and going in numbers unprecedented in Sweden. Student numbers, too, rose at all levels. The list of PhDs who wrote their dissertations under Kanger is long by Uppsala standards. Sören Stenlund, Lennart Nordenfelt, Paul Needham, Lars Lindahl, Ingmar Pörn, Lars Gustafsson, Craig Dilworth and Bengt Molander received their degrees during Kanger's lifetime, while Jan Odelstad, Patrick Sibelius, Ghita Holmström-Hintikka, Sven Ove Hansson and Kaj Børge Hansen finished later. Yet another dissertation influenced by Kanger was one written in political science by Helle Kanger. Furthermore, Kanger dispatched several students to Stanford, notably Ingrid Lindström, Sten Lindström and Patrick Sibelius, who received their PhDs there. (Thus Sibelius holds two doctorates, as does Pörn who earned his first doctoral degree from Birmingham.)

Of the many initiatives that Kanger took during his two decades as professor and head of the department, some are worth mentioning here. One was the Hägerström Lectures to be given annually by a philosopher of international repute; the lecturer would spend a week in Uppsala, giving one lecture on each of five days but also being available to meet faculty and graduate students. The first Hägerström lecturer, in 1970, was Konrad Marc-Wogau, Kanger's immediate predecessor as professor in Uppsala, followed by von Wright in 1972 and by Quine in 1973; the number of distinguished philosophers following in their footsteps is still growing. A second initiative was the Adolf Phalén Annual Memorial Picnic, an informal, three-day affair involving at times considerable numbers of staff and students in the philosophy departments of Uppsala and the two universities of Turku: Åbo Academy and University of Turku (in the early years) as well as Helsinki (later). (Phalén and Hägerström were nationally famous philosophy professors in Uppsala in the first part of the twentieth century.) A third initiative was the Scandinavian Logic Symposium; today it is languishing, but the first few meetings — Turku (Åbo) 1968, Oslo 1970, Uppsala 1973 — were remarkably successful. A fourth initiative, still flourishing, was the revival of the Uppsala Philosophical Studies, an in-house monograph series that was considerably enlarged.

As the years passed, money became less plentiful. Operations at Villavägen 7, later Villavägen 5, lost some of their momentum. Personal problems began to develop. Kanger's output, never massive, dwindled. Even though his personal situation improved during the last few years, he produced little. However, his enthusiasm for philosophy and logic never ceased. He had a repertoire of pet problems that he would bring up in conversation, as a challenge to himself as much as to his listeners. When he died on 13 March 1988, he still had not solved them all.

Kanger was married three times: to Neita Petrini 1949–1960, to Helle Kornerup 1961–1978 and to Dagmar Söderberg from 1980. He had two children with Neita (Elisabeth (Li) and Thomas, born 1950 and 1951, respectively) and one son with Helle (Kim, born 1963). He is buried in the cemetery at Hånger.

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Stig Kanger was a hard man to figure out, a mixture of many contrary qualities: gregarious but a loner; sensitive under a crust of insensitivity; unconventional in some ways, conventional in others. He could be caring, yet was not seldom brusque. He could joke about anything, yet be offended when others did so. He gave an impression of being boisterous, yet he said little; he was one of those people it is awkward to talk to on the telephone. In some ways he changed over the years. In his youth he had a lean and hungry look; later he became substantial. His older friends remember him as devoted to discussion. But in later years he was not very open to the ideas of others, and discussion became one-sided: he was willing to give, not to take.

Among human qualities he admired intelligence the most. Becoming a friend of his, one had a feeling of having been admitted to an ordered set, each member ranked according to intelligence; to be lacking in intelligence was a flaw of character. He was certainly himself intelligent, if the word is used in the traditional sense with its emphasis on formal or mathematical ability. Yet it is not clear how much Stig had of the other "intelligences" we hear about today, for example, emotional intelligence, knowledge of self and knowledge of others. Apart from occasional remarks, sometimes very perceptive, he did not like to talk about personal matters.

Like all academics, Stig wanted recognition for his work and, like most, felt that he had not got enough of it. In his case the feeling of frustration may have been justified, for Kanger's work has not had the impact it could have had, had it been better known. Yet the fault was to a certain extent his own. First, he published little; this he saw as a virtue and used to boast that no other Swedish philosophy professor had ever been appointed on so slim a corpus as he had. Second, his publications all appeared in local or at least peripheral venues. Third, his style of writing is off-putting to many readers. Stig loved games, and perhaps he saw writing logic as a game: to give readers as little explanation as possible — but always, in a strict sense, enough — and then challenge them to understand.

It is clear that the years of childhood and adolescence were extremely important in forming the adult Kanger. We know little about this part of his life, but I am certain that it holds the key to understanding this complex man. (One Swedish psychologist — himself a child of missionaries who spent an important part of his childhood in the same missionaries' children's home as Stig and his brother — has written about missionaries that "they should not have children".)

The early years at the University of Stockholm must also have been important. Anders Wedberg (1913–1978), professor of theoretical philosophy at the University of Stockholm from 1949 till his death, was an eminent philosopher; he will be remembered, among other things, for doing history of philosophy in a new way and for being instrumental in bringing formal logic into Swedish philosophy. Wedberg was very gifted but also very critical. There used to be a saying that Wedberg had been able to prove a new theorem: "Almost everything is trivial". Then (the saying went on) Kanger came along and succeeded in strengthening this result, establishing the definitive Wedberg/Kanger Theorem: "Everything is trivial". Wedberg was a perfect example of an analytical philosopher — one good at analysis. For his part, Kanger used to deny that he was one, maintaining that it is not clear what *analytic* philosophy is, if anything, and that at any rate he, Kanger, was a *synthetic* philosopher. One wonders what it was like for the young Kanger — intense and vulnerable, probably then as later given to occasional coarseness — to try to find his way under the refined, patrician, ever critical Wedberg. Kanger admired Wedberg's intellect, perhaps greatly, but his overall attitude

GÖRAN SUNDHOLM

THE PROOF THEORY OF STIG KANGER: A PERSONAL RECOLLECTION.*

I. HOW TO TURN GERHARD GENTZEN ON HIS HEAD: THE SEMANTIC COMPLETENESS OF CUT-FREE SYSTEMS

1. Semantics versus Proof Theory

The term *Proof Theory* shows a certain ambiguity. In the fifties when Stig Kanger carried out his logical work it stood for a cluster of topics pertaining to the syntactic turnstile ⊢, that is, the syntactic counterpart to the semantical notion of (logical) consequence ⊨. On the other hand, and more narrowly, it also stood for investigations of the properties of the syntactic turnstile by means of systematic transformations of derivation trees. Stig Kanger was a proof theorist only in the former sense. For him, model-theoretic semantics, couched in a rich set-theoretic framework, held pride of place, and in this he was very close to the then main European school of logic, namely the Münster School, under the leadership of Heinrich Scholz. There are indeed many questions to be asked with respect to the mere 26 (!!) non-modal pages of *Provability in Logic.*[1] Not the least of these is the question: where did Stig Kanger find his semantics? He admired Alfred Tarski above all other logicians. By the side of *Finnegan's Wake,* Tarski-Mostowski-Robinson, *Undecidable Theories,*[2] and, of course, *Der Wahrheitsbegriff in den formalisierten Sprachen,*[3] would have been with him on the Desert Island. The rare off-print copy of the German (1935) version of Tarski's masterpiece from 1933, formerly in Stockholms Högskolas Humanistiska Bibliotek, now in the University library at Stockholm, bears the mark of careful study, but it does contain the model-theoretic semantics in question only derivatively at pp. 361–62: Tarski's official definition of truth in §3, for the general calculus of classes, is not relativized to a domain of individuals, but quantifies over a universe of everything. The key-concept of model-theoretic semantics, as we now know it, is the three-place relation

$$R(\mathfrak{U}, \varphi, s) =_{def} s \text{ satisfies } \varphi \text{ in } \mathfrak{U}.^4$$

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As far as I know Tarski only gave the explicit definition of R, by a metamathematical recursion over the complexity of the wff φ, for the first time in the classical paper by Tarski and Vaught in 1957. *Provability in Logic* was conceived in 1955, so Kanger did not take the semantics from Tarski and Vaught. If he did not make it himself (which, on the basis of Tarski's remarks in *Der Wahrheitsbegriff*, would certainly have been within his reach), a probable source is the *Mathematische Logik* by Hans Hermes and Heinrich Scholz.[5] Kanger cites the work at the appropriate place in *Provability in Logic* (at p. 16) and it is written in a style that would appeal to him. It is semantically inclined, while syntactically (almost too) precise.[6] Kanger's *Handbok i Logik* shows many similarities with that of Hermes and Scholz.[7]

Anders Wedberg, who held the Chair of Theoretical Philosophy at Stockholm during Kanger's period of study there, was also strongly influenced by Scholz, as is borne out even by a cursory inspection of *Filosofins Historia*, I–III.[8] Indeed, it is no exaggeration to say that, but for Wedberg, only Heinrich Scholz has shown the same appreciation of symbolic logic as a central tool in historical studies, as witnessed by the articles collected in *Mathesis Universalis*.[9] Scholz, however, had no empiricist leanings whatsoever. Wedberg certainly did: in my opinion, he was equally influenced by Moritz Schlick's *Allgemeine Erkenntnislehre*.[10] Scholz, on the other hand, was a classical metaphysician, who began his academic career in the Philosophy of Religion as a favourite pupil of Adolf Harnack. His Platonizing tendencies would not greatly have disturbed Stig Kanger, his fellow Platonist semanticist.[11]

2. *Confluence of Ideas in 1955: Backwards Application of Gentzen Rules*

My Oxford supervisor, the late Robin Gandy, wrote a splendid paper called *The Confluence of Ideas in 1936*, in which he dealt with the origins of recursive function theory and the many different ways of defining the same class of functions by Church, Kleene, Herbrand, Gödel, Hilbert-Bernays, Turing and Post.[12] The early history of contemporary mathematical logic is replete with such confluence of ideas. Post, Lukasiewicz and Wittgenstein provide decision-methods for the propositional calculus. Skolem and Fraenkel emend Zermelo's set theory in similar fashion. Tarski and Herbrand prove the Deduction Theorem. Gödel, Gentzen and Bernays give the intuitionistic Double-negation interpretation in 1932.

Stig Kanger played a central role in two major instances of such confluence. One of these is becoming well-known and concerns the semantics of modal logic.[13] The other concerns the new method for proving completeness of the classical predicate calculus that was independently developed around 1955 by

Ewert Willem Beth,[14] Jaakko Hintikka,[15] Stig Kanger and Kurt Schütte.[16] Beth and Hintikka took the search for a counter-model, where frustration of the search is held to be a proof, as their starting point and worked with, respectively, *semantic tableaux* and *model sets*. [The latter are now also known as 'semi-valuations' (Schütte),[17] '(analytic) consistency properties', and, perhaps even more appropriately, 'Hintikka sets' (Smullyan).] Schütte invented a new book-keeping device in terms of positive and negative parts, but it has never caught on outside his immediate circle. The Beth–Hintikka approach was streamlined by Raymond Smullyan, magician friend of Kanger's, in the 1960's, and codified in his text *First-Order Logic* in terms of 'Consistency properties'.[18] This approach has now become part of standard teaching, above all through the successful Penguin-paperback textbook *Logic* by Wilfrid Hodges.[19]

Of our four inventors only Kanger stayed close to the original Gentzen format.[20] The key observation is the following. Consider the sequent

$$S =_{def} A_1, \ldots, A_n \Rightarrow B_1, \ldots, B_n,$$

or $\Gamma \Rightarrow \Delta$, for short. It holds (logically) when

$$A_1 \,\&\, \ldots \,\&\, A_n \supset B_1 \vee \ldots \vee B_n$$

is (logically) true. However, and this is the main discovery, the sequent S can also be read as a task to be resolved, namely: make every antecedent formula in Γ true and every succedent formula in Δ false. This is what Kanger does. For instance, in order to resolve the task

$$A \supset B, \Gamma \Rightarrow \Delta$$

(that is, the task: make $A \supset B$ and all of Γ true and make all of Δ false), it is necessary either to make all of Γ true and to make A and all of Δ false or to make B and all of Γ true and all of Δ false. Set out in schematic form this becomes:

$$\frac{\Gamma \Rightarrow \Delta, A \qquad B, \Gamma \Rightarrow \Delta}{A \supset, \Gamma \Rightarrow \Delta}$$

This, however, is nothing but an instance of $\supset\Rightarrow$, Gentzen's antecedent introduction-rule for $\supset$. Similar backwards applications of the Gentzen rules yield a systematic search-tree, along whose open branches generate certain semi-valuations — Hintikka sets. The search is frustrated when the systematic decomposition of the sequent-tasks issues in an impossible task. These take the form

(*) $\Phi, C, \Xi \Rightarrow \Theta, C, \Psi,$

where one would have to make the wff C both true and false, which is clearly impossible. From the other perspective, though, the sequent (*) is nothing but an axiom of the Gentzen sequent calculus, so if each branch of the search-tree is thus truncated it is converted into a sequent-calculus proof, when read from top to bottom. An open branch in the search-tree, on the other hand, allows one to read off a semi-valuation which — in the case of first-order logic — can be extended immediately to a total valuation: an atomic wff without a value in the semi-valuation is assigned the value true. We have found the desired counter-model to the original sequent. This procedure is perfectly deterministic: at each stage it can be explicitly laid down which wff to attack and, in the case of a backwards application of the quantifier-rules $\forall\Rightarrow$ and $\exists\Rightarrow$, what new free variable to choose as a witness. In this fashion we get a primitive recursive two-place function T(x, u) such that when S is a sequent and $u = \langle k_1, ..., k_n \rangle$ (with $n \geq 0$), that is, a finite sequence of natural numbers considered as a node in the universal spread, $T(\ulcorner S \urcorner, u)$ gives the (Gödel-number $\ulcorner S' \urcorner$ of the) sequent S′ (if any) that has to be placed at the node u in our derivation. (For nodes v such that no sequent is placed there, $T(\ulcorner S \urcorner, v)$ is trivially put $\equiv 0$.) This information can be squeezed out, in some version or other, from all four approaches to the backwards proof-methods. The treatment becomes particularly smooth when one stays close to the Gentzen format, though, and a beautiful exposition of (essentially) the Kanger treatment can be found in Kleene's *second* text-book *Mathematical Logic* (as Kleene himself came to realise upon completion of his work).[21] Dag Prawitz's contribution to the Schütte-*Festschrift* from 1974 is another useful exposition.[22]

3. *Kanger's Background when Working Backwards*

How did Kanger reach this point? Who, if any, were his precursors? First, and foremost, Gödel and Gentzen. Gödel's original completeness proof can, in retrospect, be seen as working with semi-valuations, that is, minimal counter-models,[23] rather than with the total valuations that are obtained through the Lindenbaum maximalization technique, now very well-known from the Henkin completeness-proof.[24] Gentzen himself, in his dissertation from 1932 (published 1934–35), used the cut-free formalism to prove that intuitionistic propositional logic is decidable, by means of applying the rules backwards in a hypothetical derivation (which, in virtue of the *Hauptsatz*, may be assumed cut-free).[25] Kanger knew Oiva Ketonen's dissertation from 1944,[26] which gives a Gentzen-like refinement of the *Kriterien der Widerlegbarkeit des reinen*

Prädikatkalküls from Hilbert-Bernays[27]. He also knew Erik Stenius' book *Das Interpretationsproblem der formalisierten Zahlenreihe und ihre formale Widerspruchsfreiheit*,[28] which gives a Hilbert-Bernays inspired Herbrand-treatment of the proof-theory of classical predicate logic, and of arithmetic with the omega-rule.

4. How Did the Backwards Method Fare After Kanger?

i) Beth shortly afterwards designed also a version of his semantic tableaux that was complete for an intuitionistic system,[29] but, as was shown by work of Gödel, Kreisel and Dyson, the completeness proof in question was not constructive. After two decades, in the mid-seventies, W. Veldman[31] and H. de Swart[32] at Nijmegen were able to circumvent the Gödel–Kreisel obstacle by considering constructive Beth-models in which $\bot$ was allowed to be true at certain "exploding" nodes. Later refinements were given by Friedman and Dummett.[33]

ii) A decade after the Beth–Hintikka–Kanger–Schütte proof, the method was extended to the then emerging *infinitary logics*. E. G. K. Lopez-Escobar, in particular, gave a Kanger-like completeness-proof for cut-free $L_{\omega_1\omega}$, in his dissertation from 1963, with applications to Craig's Interpolation Theorem and Beth's Theorem.[34] Also Jon Barwise originally developed his version of the theory of admissible sets on a proof-theoretic basis,[35] but Makkai later eliminated the proof theory in favour of an approach in terms of infinitary consistency properties,[36] now conveniently accessible in Keisler's *Model Theory for Infinitary Logic*[37] and Barwise's *Admissible Sets and Structures*.[38]

iii) On a more modest level, it was realised, e. g. by Kent[39] and Lopez-Escobar,[40] that the Shoenfield completeness-theorem for the recursive omega-rule in arithmetic[41] was readily provable using the backwards method of proof, and then it yields even Kalmár-elementary proof-trees.

I have no information as to whether Kanger knew of the work under i)–iii). Lopez-Escobar's thesis was written under the supervision of Dana Scott, a good friend of Kanger's, and it is not improbable that it was known to him.

iv) In the mid-sixties Takeuti's conjecture concerning cut-elimination in second-order predicate calculus[42] was established by Tait,[43] Prawitz(2 ×),[44] and Takahashi.[45] Kanger knew and appreciated Prawitz's extremely elegant *Theoria*-proof: a two-sorted semi-valuation is obtained by running the backwards method. The predicate universes of this semi-valuation need not be closed under definability. The required *total* second-order counter-valuation is obtained by closing the *ramified analytical hierarchy* based on the predicate

DAG PRAWITZ

A NOTE ON KANGER'S WORK ON EFFICIENT PROOF PROCEDURES

Three of Stig Kanger's works belong to proof theory taken in a wide sense: the monograph *Provability in Logic* of 1957, the paper "A simplified proof method for elementary logic" of 1963, and, in between these two, the mimeographed *Handbok i logik* written in 1959. I concur in Göran Sundholm's remark in his paper of the present volume that Kanger's main interest in this connection was not proofs themselves but provability and derivability and in particular the relation of these notions to semantical ones. A case in point is Kanger's variant of Gentzen's calculus of sequents for classical logic, LK, which Kanger develops in *Provability in Logic*. The purpose is there to give a new demonstration of Gödel's completeness result that every valid formula is provable, i.e. has *some* proof, no matter which.

This picture of Kanger's proof theoretical interests is in need of some supplementations and qualifications, however. Kanger also gave a proof of Gentzen's Hauptsatz, a corner-stone in proof theory, which takes up most of Gentzen's classical paper. The theorem is now obtained as an easy corollary of Kanger's completeness result for a cut-free version of LK: If the sequents $\Gamma \Rightarrow \Delta, A$ and $A, \Gamma \Rightarrow \Delta$ are both provable, then in view of the soundness of the calculus they are valid and so is the sequent $\Gamma \Rightarrow \Delta$ (by the semantical validity of the cut rule), and hence by the completeness theorem, the sequent $\Gamma \Rightarrow \Delta$ is provable without use of the cut rule.

Kanger was very fond of his semantical proof of the Hauptsatz and in particular of the ease with which he obtained it. He devotes a section of eight lines to it in his otherwise very condensed monograph. Modestly he remarks that Gentzen's proof is superior to his own since it is finitary. But Kanger did not really care about a result being established in a finitary way. Therefore, his real attitude, as I remember it, was that the Hauptsatz, having been obtained for free, could hardly be a deep theorem.

This indifference to the significance of the Hauptsatz seems to confirm the initial impression that proofs themselves on the object level and their properties were not the kind of things that interested Kanger. But this is not the whole truth. Kanger had an interest in the art of engineering, in how things are made, and he saw that his way of establishing the completeness result for a

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cut-free formalism gave rise to a specific proof procedure. Although proof theory did not please his philosophical interest, model theory being his favourite, more practical questions concerning how to find proofs in an efficient way appealed to him.

Kanger's monograph *Provability in Logic* was his doctoral dissertation, presented in theoretical philosophy in Stockholm in the academic year 1956–57, at which time I was a beginner in philosophy. When my teacher Anders Wedberg, who had also been the teacher of Kanger, was to describe the content of Kanger's thesis to us beginners, he described it as essentially amounting to a new proof procedure for predicate logic, which in principle could be implemented on a computer.[1]

This stimulated me to try to automatize Kanger's procedure, and this in turn led Stig and me to some new insights in that field. I shall here give some glimpses of this early phase of automatic deduction.

1. KANGER'S FIRST PROOF PROCEDURE

The proof procedure that Kanger developed in *Provability in Logic*, also described in Sundholm's paper in this volume, consists essentially in applying the rules of a cut-free version of Gentzen's calculus of sequents backwards until one reaches an axiom at the top of each branch of the resulting tree. What did such a proof procedure amount to?

We may separate the sentential and the quantification part of the procedure. As for the first part, it is instructive to make a comparison. Every beginning student of logic learns after a while that to verify that a formula in sentential logic is a tautology one does not need to go through all the possible truth value assignments to the atomic formulas. Given for instance the formula

$$[A \rightarrow (B \rightarrow C)] \rightarrow [(A \,\&\, B) \rightarrow C]$$

it is sufficient to reason as follows: A falsifying assignment must make the antecedent true and the succedent $(A \,\&\, B) \rightarrow C$ false; to achieve the latter A and B must be assigned truth and C falsity; but to achieve the first, i.e. to make the antecedent $A \rightarrow (B \rightarrow C)$ true, either A must be assigned falsity, which possibility has already been excluded, or $B \rightarrow C$ must get the value truth, requiring in turn that either B is assigned falsity or that C is assigned truth, which two possibilities have also been excluded. Hence there is no falsifying assignment.

This way of reasoning has exactly the same structure as applying the sequent rules as formulated by Kanger backwards: to make a formula true corresponds to putting it in the antecedent of the sequent, i.e. before the arrow, and to make it false to putting it in the succedent of the sequent, i.e. after the arrow; that a formula cannot be both true and false corresponds to the fact that a sequent in which a formula occurs both before and after the arrow is an axiom. To use sequent rules in this way thus seemed to be a very appropriate way of finding proofs in sentential logic corresponding to known short cuts in the handling of truth tables.

As for the quantificational part, backward applications of the quantification rules amounted to the generation of instances of quantified formulas. In the case of an existential formula in the antecedent or a universal formula in the succedent a new constant was introduced to replace the quantified variable (which corresponds semantically to introducing in the counter model sought for the name of an individual that satisfies an instance of the existential formula or falsifies an instance of the universal formula, respectively). In the case of a universal formula in the antecedent or an existential formula in the succedent, instances were instead formed by systematically substituting for the quantified variable all constants that had been introduced in operations of the first kind.

The method was quite straightforward and not very sophisticated. But it was a complete proof search that took advantage of the sub-formula property of cut-free proofs, and the achievement was to formulate it precisely. After Kanger's public defence of his dissertation in the spring of 1957, I sat down in the summer to automatize the procedure. I was then also inspired by Beth's semantical tableaux used in his proof of the completeness result, which parallels Kanger's method in many respects.

At that time there was a computer in Stockholm known as BESK. It was a huge machine occupying several rooms of what had been the premises of the Royal Technical University. Just then it had the record as the fastest computer of the world. But this was a time when there were no programming languages. One had to use the machine code, which I did not know and did not want to learn. Instead I invented a programming language of my own suitable for the particular task in question, and it was translated to the machine code by my father, who had sometimes been using BESK for certain mathematical calculations. The program was run on the machine by Neri Voghera in 1958, and the whole project was presented at the First International Conference on Information Processing, which was arranged in Paris in 1959 by UNESCO. It was published in 1960[2] and was one of the very first automated proof procedures. Some roughly equivalent procedures were implemented on other computers and presented in journals at more or less the same time.

Our program proved simple theorems of logical textbooks, e.g. the one saying that a transitive and irreflexive relation is asymmetric was proved in 12 seconds. But as soon as it came to more advanced theorems, it was hopelessly inadequate, in spite of the record speed of the computer.

2. THE DUMMY METHOD

The reason for the inefficiency of the method was very obvious. After having introduced a few constants because of existential formulas occurring in the antecedent or universal formulas occurring in the succedent, the number of possible substitutions becomes very large when one is to apply the other two quantification rules backwards to universal formulas in the antecedent or existential formulas in the succedent .

As an illustration suppose that we have a sequent of the form

$$\forall x_1 \forall x_3 \forall x_2 \forall x_4 A(x_1,x_2,x_3,x_4) \Rightarrow \forall x_1 \forall x_2 \forall x_3 \forall x_4 A(x_1,x_2,x_3,x_4)$$

where the antecedent and succedent are the same except for a permutation of the quantifiers. A human recognizes the validity of the sequent as soon as she sees that the only difference between the two formulas is the permutation of $\forall x_2 \forall x_3$. However, to prove the sequent by our procedure, one has to try out different ways of breaking down the antecedent formula by substituting the four constants c_i generated by the succedent formula , i.e., after having first obtained the sequent

$$\forall x_1 \forall x_3 \forall x_2 \forall x_4 A(x_1,x_2,x_3,x_4) \Rightarrow A(c_1,c_2,c_3,c_4),$$

one has to test at random substitutions of c_1, c_2, c_3, and c_4 for the four universally quantified variables in the formula in the antecedent. There are thus $4^4 = 256$ possible formulas of the form $A(t_1,t_2,t_3,t_4)$ to generate, and we may assume that only one of them, $A(c_1,c_2,c_3,c_4)$, yields a proof of the original sequent. There was no method for seeking out the relevant substitution instance, and hence one could expect that, on an average, one had to generate a sequent containing 128 substitution instances in the antecedent — quite a long sequent in view of the simplicity of the original sequent. Imagine a similar example but with 10 quantifiers instead of 4 — then there are 10^{10} possible formulas of the form $A(t_1,t_2,...,t_{10})$ to generate, and the right one will be found after 5 000 000 000 steps on an average! Clearly the speed of the computer did not matter much.

At a seminar where my automatization of Kanger's and Beth's proof procedures was presented, I draw attention to this inefficiency of the method and suggested that one should try to find the right substitutions by some kind of calculation. I compared our method to solving equations, say $8x + 37 = 445$, by systematically trying different values for x until the right one was found. Just as we find the roots of equations by performing certain calculations and not by running through possible values 1, 2, 3, ..., we should find the right substitution instance directly by some kind of calculation. At this seminar Kanger suggested as a solution simply to replace the variable by a dummy, which could also be called a meta-variable, and to let it stand while continuing as before until one sees what substitutions are appropriate to make for the dummy. He provokingly pronounced that this dummy-method solved the problem.

I remember that I was slightly irritated by this proposal. Just to replace the variables by something called dummies did not solve the problem; in the essay discussed at the seminar I had also suggested that one should drop quantifiers, replace the variables by meta-variables ranging over the constants to be substituted later, and go on with applying the sentential operations to the formulas — the problem was to "see", as Kanger expressed it, what values should be assigned to the dummies, or how to "put together different proof branches to a proof tree" as I had expressed it. Nevertheless, it did turn out that to replace the variables by dummies or to treat them as meta-variables was a very good strategy, or even *the* right strategy. The crucial idea was to postpone substitutions for the variables in question until there was a better opportunity for making a suitable choice. When attention was focused on this idea, it was not difficult to point out situations in which it was fairly easy to see what the appropriate substitutions should be. Somewhat later it turned out that one could make different selections of such situations.

Kanger described his idea in his *Handbok i logik* and again in the paper "A simplified proof method for elementary logic". This paper is note-worthy also for other features to be mentioned below. The dummy method is described as follows. New constants are introduced by backward applications of quantification rules as before. At applications of the other quantification rules (that do not generate new constants), we substitute not a constant but instead a new dummy d and list at the same time what values the dummies can assume, i.e. we make a note $d/t_1, t_2, .., t_n$, called a substitution list, containing all constants and dummies that occur in the sequent to which the rule is applied. At some stages we stop and "check whether we can choose values for the dummies from the substitution lists in such a way that all top sequents will be directly demonstrable when we replace the dummies by their values". "Directly demonstrable" means here to be either an axiom or derivable from an axiom by inference rules for identity.

Applied to the sequent occurring in the example above, the method works as follows: The formula in the succedent is replaced by $A(c_1,c_2,c_3,c_4)$, where c_1, c_2, c_3, and c_4 are four new constants, while the variables of the formula in the

ANATOLI DEGTYAREV AND ANDREI VORONKOV

KANGER'S CHOICES IN AUTOMATED REASONING

Automated deduction, or automated theorem proving is a branch of science that deals with automatic search for a proof. The contribution of Kanger to automated deduction is well-recognized. His monograph [1957] introduced a calculus **LC**, which was one of the first calculi intended for automated proof-search. His article [1963] was later republished as [Kanger 1983] in the collection of "classical papers on computational logic". Kanger's [1963] (and also [1959]) calculi used some interesting features that have not been noted for a number of years, and the importance of which in the area of automated deduction has been recognized only much later.

Kanger [1963] gives no proofs and uses very succinct presentation. Automated deduction is an area in which very subtle changes in definitions and assertions may lead to inconsistent conclusions. Kanger's [1963] area was theorem proving in sequent calculi with equality and function symbols. Most papers published in this area before 1995 contained serious mistakes, except for Kanger's.

Now, when we are equipped with the impressive amount of techniques developed in this area, we are amazed by the incredible intuition of Kanger that allowed him to choose elegant, interesting (and correct) solutions among many possible choices. This article explains these choices and their place in modern automated deduction.

1. $\models \; \equiv \; \vdash$

The title of this section $\models \equiv \vdash$ is the logo of the Association for Logic Programming: truth is equivalent to provability. The equivalence of validity and provability for classical logic was proved by Gödel [1930] and is known as Gödel's completeness theorem. The notions of truth and validity in logic are formulated as semantical properties, while the notion of provability is defined in a purely syntactical way, so there seems to be a gap between the two notions.

In 1955–1957 several new proofs of Gödel's completeness theorem appeared [Beth 1955, Hintikka 1955, Schütte 1956, Kanger 1957] in which model theory and proof theory were connected in a very natural manner. They

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are based on the idea of searching for countermodels of a given formula F by applying a proof-search procedure to F (i.e. trying to establish $\vdash F$).

Kanger proposed to search for a proof in a *sequent calculus* named **LC** [Kanger 1957]. Cut-free sequent calculi for first-order logic have been introduced by Gentzen [1934]. They turned out to be an important tool for investigating basic proof-theoretic problems [e.g. Gentzen 1936, Girard 1987]. It has also been realized that sequent systems give a convenient tool for designing proof-search algorithms by using the rules of a calculus backwards (i.e. from the conclusion to the premise). To prove a sequent S *"we start from below with S and proceed upwards from level to level in the tree form. At each level the sequent of the next level above are uniquely and effectively determined — if there is such a level. If there is no such level, this fact is effectively determined, so that the process may brought to an end."* [Kanger 1957, page 31]. Consider some choices that arise when one formalizes sequent calculi.

Choice 1 (structure rules) In the original Gentzen's LK a sequent was an expression $\Gamma \to \Delta$, where Γ, Δ are *sequences* of formulas. Since Γ and Δ play the role of a conjunction and a disjunction, respectively, the logical semantics of a sequent is independent of the order of formulas in Γ, Δ. Neither does it depend on duplicate occurrences of formulas in Γ or Δ. Therefore, Gentzen had to introduce several *structure rules* that allow one to interchange and duplicate formulas in Γ, Δ, and also add new formulas:

$$\frac{\Gamma \to \Delta_1, B, A, \Delta_2}{\Gamma \to \Delta_1, A, B, \Delta_2} \qquad \frac{\Gamma_1, B, A, \Gamma_2 \to \Delta}{\Gamma_1, A, B, \Gamma_2 \to \Delta}$$

$$\frac{\Gamma \to \Delta_1, A, A}{\Gamma \to \Delta, A} \qquad \frac{\Gamma, A, A \to \Delta}{\Gamma, A \to \Delta}$$

$$\frac{\Gamma \to \Delta}{\Gamma \to \Delta, A} \qquad \frac{\Gamma \to \Delta}{\Gamma, A \to \Delta}$$

These rules are called *exchange*, *contraction* and *weakening*. The use of these rules introduced unnecessary technical details in proofs of [Gentzen 1934]. In order to avoid complications, other structures than sequences should be adopted. One obvious choice is the use *sets* instead of sequents. This again makes the formalization of sequent calculi quite complex. Suppose that Γ, Δ are sets and consider the following rule of sequent calculi:

$$\frac{\Gamma \rightarrow \Delta \cup \{A\}}{\Gamma \rightarrow \Delta \cup \{A \vee B\}} \ (\rightarrow \vee)$$

Let Γ be empty and consider four different instantiations for Δ: $\{\}$, $\{A\}$, $\{A \vee B\}$, and $\{A, A \vee B\}$. We obtain the following four instances of this rule:

$$\frac{\rightarrow \{A\}}{\rightarrow \{A \vee B\}} \ (\rightarrow \vee) \qquad \frac{\rightarrow \{A\}}{\rightarrow \{A, A \vee B\}} \ (\rightarrow \vee)$$

$$\frac{\rightarrow \{A, A \vee B\}}{\rightarrow \{A \vee B\}} \ (\rightarrow \vee) \qquad \frac{\rightarrow \{A, A \vee B\}}{\rightarrow \{A, A \vee B\}} \ (\rightarrow \vee)$$

The last one is absurd, among all four instances only the first one is enough to preserve completeness. Therefore, if we choose sets, we have to impose several restrictions on the inference rules. If we prohibit A and $A \vee B$ occur in Δ, we may eventually loose completeness. Even if we impose no restrictions we might still be in need of the weakening rule. So what is the right choice for sequent and structure rules in sequent calculi?

Kanger's Choice 1 One distinctive feature of the calculi used in [Kanger 1957, Kanger 1963] is the *full absence of structure rules*. In order to achieve this, sequent are made of *multisets* of formulas and some rules are modified. The use of multisets eliminates the exchange rule. The use of contraction rule is replaced by the explicit duplication of formulas in some (but not all!) rules and changes in some other rules. For example, the $(\rightarrow\exists)$ rule in Kanger's system is

$$\frac{\Gamma \rightarrow \Delta, \exists x\varphi(x), \varphi(t)}{\Gamma \rightarrow \Delta, \exists x\varphi(x)} \ (\rightarrow \exists)$$

(the formula $\exists x\varphi(x)$ is explicitly duplicated), and the rule $(\rightarrow \vee)$ is changed into

$$\frac{\Gamma \rightarrow \Delta, A, B}{\Gamma \rightarrow \Delta, A \vee B} \ (\rightarrow \vee).$$

Finally, to get rid of weakening axioms $\Gamma, A \rightarrow \Delta, A$ are used instead of more traditional $A \rightarrow A$.

Completeness can be proved for virtually any variant of sequent calculi, but even completeness proofs meet small technical problems when it comes to structure rules. The choice made by Kanger to design a system without structure rules at all has now become de facto standard.

Kleene [1952] also described the sequent system G3 with invertible rules, but this property was realized straightforwardly by retaining the principal formula in the premise(s). Later Kleene's G3 was transformed to the system G4 [Kleene 1967][1], which was essentially the system **LC**.

Choice 2 (variants of rules) For some logical connectives, we have a choice among various sequent calculus rules. For example, for the proof disjunction one can use either the following two rules:

$$\frac{\Gamma \to \Delta, A}{\Gamma \to \Delta, A \vee B}\,(\to \vee) \qquad \text{and} \qquad \frac{\Gamma \to \Delta, B}{\Gamma \to \Delta, A \vee B}\,(\to \vee)$$

or just one rule

$$\frac{\Gamma \to \Delta, A, B}{\Gamma \to \Delta, A \vee B}\,(\to \vee)$$

The first choice seems to reflect the semantics of disjunction in a more intuitive way. Nevertheless, in Kanger's system the choice of the second rule is made. Why?

Kanger's Choice 2 The main answer is: all inference rules of Kanger's system are *invertible*. A rule is called invertible if the derivability of the conclusion implies the derivability of the premises. For automatic proof-search invertibility of rules is really a remarkable property. If a sequent $\mathcal{S}$ is unprovable, then *any* derivation tree for $\mathcal{S}$ has a branch containing a countermodel for $\mathcal{S}$. It allowed Kanger to prove completeness *"by means of arguments which are new in some respect and which involve a new turn to the notion of validity"* [Kanger 1957, page 7]. It also allows one to search for a proof in a "don't care" matter: after we have selected a rule to apply, there is no need to undo the selection.

Kleene [1967] notes that the use of his system G4 for proving the completeness theorem *"is quite close to Beth [1955] which gave the present writer the idea for it. In some respect it more resembles Kanger [1957], as the author learned after working it out."*

2. USE OF A SEQUENT CALCULUS AS A DECISION PROCEDURE FOR PREDICATE LOGIC

Kanger was one of the first who used a particular logical calculus as a decision procedure in the backward direction[2]. The point was to guarantee termination of the procedure on target classes of formulas. As examples Kanger considered the class of quantifier-free formulas and the class of $\forall^*\exists^*$ formulas (without functional symbols). Later, Wang [1960] described and implemented a procedure solving this class of formulas, also using backward proof-search in sequent calculi.

The possibility to obtain decision procedures for propositional logics, classical and intuitionistic, using backward proof-search in cut-free Gentzen type calculi was also noted by Kleene [1952]. Later, the use of derivations in machine-oriented calculi to decide some classes of predicate logic has become a generally accepted area of research [Maslov 1964, Kallick 1968, Maslov 1968, Joyner jr 1976, Fermüller, Leitsch, Tammet & Zamov 1993, Leitsch, Fermüller & Tammet 1999].

3. PROOF-SEARCH VIA LOGICAL CALCULI

As soon as the first programs for proving theorems in predicate logic appeared [Prawitz, Prawitz & Voghera 1960, Wang 1960, Gilmore 1960, Davis & Putnam 1960] it has become clear that the main problem consists in instantiating variables in the application of $(\rightarrow \exists)$ and $(\forall \rightarrow))$ rules (also called γ-rules due to [Smullyan 1968, Fitting 1996]).

$$\frac{\Gamma \rightarrow \Delta, \varphi[t/x], \exists x\varphi}{\Gamma \rightarrow \Delta, \exists x\varphi}\ (\rightarrow \exists) \quad \text{and} \quad \frac{\Gamma, \varphi[t/x], \forall x\varphi \rightarrow \Delta}{\Gamma, \forall x\varphi \rightarrow \Delta}\ (\rightarrow \forall)$$

Here sequent are represented by expressions of the form $\Gamma \rightarrow \Delta$, where Γ and Δ are multisets of formulas.

Choice 3 (variable instantiation in γ-rules) How to instantiate variables by terms in γ-rules? The early methods of automated reasoning used the so-called *level saturation*. The set of all variable-free terms was enumerated (usually respecting depending on the term depth) and terms have been substituted one by one in that order. However, it was clear that such a solution is far from the best.

Kanger's Choice 3 The system of Kanger used a new strategy for instantiating variables in the applications of γ-rules. His strategy of instantiating variables

KAJ BØRGE HANSEN

KANGER'S IDEAS ON NON-WELL-FOUNDED SETS: SOME REMARKS

1. INTRODUCTION

1.1 Provability in Logic. Stig Kanger's small book from 1957, *Provability in Logic*, contains eight chapters. The last two chapters are concerned with modal logic. This part has received considerable attention and recognition. Chapters 2–6 treat elementary extensional logic. This part has drawn scantier attention. The present essay contains an exposition and comments on chapters 2–5, with an emphasis on the contributions to set theory and model theory. I take for granted that the reader has access to *Provability in Logic*, either the original edition from 1957 or the reprint in *Collected Papers of Stig Kanger, Vol. I.*

1.2 Content. Kanger develops a calculus LC for one fixed predicate logical language L. His intention seems to be to show that LC is all that is needed for general predicate logic. This forces him to develop new ideas on non-well-founded sets which are of great interest in their own right. These ideas are the main subject of the present essay.

I expose the language L, the calculus LC, the ideas on non-well-founded sets, and the use Kanger makes of them in the model theory for LC.

2. THE CALCULUS LC

2.1 Language. We first indicate the language L.

2.2 Primitive Symbols. The language L is built from the following symbols.

(1) Parentheses.
(2) Propositional constants: $p, q, p_1, q_1, p_2, q_2, \ldots$
(3) Set symbols: For $t = 1, 2, \ldots,$
variables of type t: $x_1^t, x_2^t, \ldots$
constants of type t: $c_1^t, c_2^t, \ldots$
(4) Notation for ordered sets: < >
(5) A two-place predicate: $\in$

G. Holmström-Hintikka, S. Linström and R. Slivinski (eds.), Collected Papers of Stig Kanger with Essays on his Life and Work, Vol. II, 69–86.

(6) Connectives: $\supset$ (material implication), & (conjunction), $\vee$ (disjunction), ~ (negation)
(7) Quantifier symbols: $\forall$ (universal quantification), $\exists$ (existential quantification)
(8) A symbol for Gentzen entailment: $\longrightarrow$

2.3 Formulas. The atomic formulas of L are all expressions of the form

$$(a \in b) \quad \text{or} \quad (<a_1 ... a_n> \in b)$$

where a, b, $a_1,...,a_n$ are set symbols. The set of formulas are obtained in the usual way by closing the set of atomic formulas under the connectives and quantifiers.

2.4 Sequents. Let Γ and Θ be sequences of closed formulas of LC. A *quasi-sequent* is an expression of the form

$$\Gamma \longrightarrow \Theta$$

A quasi-sequent S is a *sequent* if there are infinitely many set constants of each type which do not occur in S. This distinction is relevant for the formulation of the deduction rules *10 and *13 of LC, and we shall not need it in the sequel.

2.5 Intended Interpretation. Kanger considers two interpretations of the atomic formula $(<a_1...a_n> \in b)$. '$\in$' may be interpreted as denoting an arbitrary 2-place relation, or it may be interpreted as representing the membership relation for sets.

2.6 Remark. The language L with the calculus LC is predicate logic without identity. This is important. One of Kanger's completeness theorems for LC cannot be extended to predicate calculus with identity as will be shown in Section 7.

2.7 Remark. All variables and constants in L are typed. This complicates somewhat the semantics of L, the deduction rules of LC, and the constructions in the proofs of the completeness theorems. It is not easy to see any justification for having several types rather than just untyped variables and untyped constants. A typing of symbols may be justified when the domain is a type structure of sets. The set universe Kanger eventually chooses for his model theory contains non-well-founded sets and allows loops like $a \in a$ and $a \in b \in a$. This makes a typing inappropriate.

In the comments later in the essay, I will sometimes make reformulations where types are neglected.

2.8 Remark. The usual way to do general predicate logic is to consider the family $\{L_\alpha\}$ of all predicate logical languages and define a predicate calculus for each L_α. Kanger's intention is clearly to do general predicate logic; but he considers only one fixed language, the language L with one predicate $\in$ and individual constants $c_1, c_2, \ldots$. To motivate this approach, consider, e.g., a language L* which contains a one-place predicate P and a 2-place predicate R and also constants c and d. Let $\mathcal{A}$ be a model for L*. Then

$$(2\text{-}1) \quad \mathcal{A} \models P(c) \quad \Leftrightarrow \quad c^{\mathcal{A}} \in P^{\mathcal{A}}$$
$$(2\text{-}2) \quad \mathcal{A} \models R(c, d) \quad \Leftrightarrow \quad (c^{\mathcal{A}}, d^{\mathcal{A}}) \in R^{\mathcal{A}}$$

We see that atomic formulas in the semantics always are interpreted in terms of the $\in$-relation. This suggests the possibility of doing general predicate logic by having only one predicate, namely $\in$. Individual terms like c, d and predicates like P, R should then be represented by constants intended to denote atoms or sets. This is exactly how Kanger's language L is built up.

The difficulties with such an approach are considerable. The $\in$-relation on the right-hand side of equivalences (2-1) and (2-2) is governed by the axioms of ZF. A logical calculus for L should presumably be incomplete if the '$\in$' of L were interpreted in this way since for completeness, the calculus should have to include at least one of the non-logical ZF axioms. Kanger therefore needs to invent another set universe which contains sets not occurring in Zermelo's cumulative type structure.

It should be pointed out that the considerations stated in the present remark are my own and do not occur in Kanger's work. I nevertheless feel that they must have motivated Kanger in his work. They also make it intelligible that he attaches so great importance to normal models and completeness with respect to normal models (see paragraphs 2.11–2.14 and Section 3 below).

2.9 Semantics. Kanger defines a semantics for L. It consists of a frame together with a valuation. A *frame* for L is an infinite sequence $r = \langle r^1, r^2, \ldots \rangle$ of classes where $r^1 \neq \emptyset$ and $r^t \subseteq r^{t+1}$ for $t = 1, 2, \ldots$.

2.10 Remark. The frame r is the domain of the model. r^t is the class of entities of type at most t. Since $r^t \subseteq r^{t+1}$, the type structure is cumulative.

2.11 Valuations. A primary valuation is a 2-place function V. The first argument is a frame; the second argument is either a propositional constant, the predicate '$\in$', or a set symbol. V satisfies:

(1) V(r, P) = 1 or V(r, P) = 0 if P is a propositional constant. 1 and 0 are the truth-values *true* and *false*, respectively;

(2) V(r, '$\in$') is a class of finite non-unitary ordered sets of elements of r;

(3) V(r, *s*) is an element of r^t if *s* is a set symbol of type t.

A primary valuation is *normal* if it holds for each frame r that any ordered set $<v_1,...,v_n, w>$ of elements of r belongs to V(r,'$\in$') if and only if $<v_1,...,v_n>$ is a member of w, for $n \in \underline{Z}_+$.

The primary valuation gives rise to a secondary valuation T(r, V, S) which, given a frame r and a primary valuation V, assigns a truth-value, 0 or 1, to each formula or sequent S. The extension of V to T is done in the natural way.

2.12 Remark. If we use modern notation and disregard types, the semantics can be reformulated as follows.

A *structure* (sometimes called an *arbitrary structure*) is a sequence

(2-3) $\mathcal{A} = (A, \in^{\mathcal{A}},..., c^{\mathcal{A}},...)$

where $A \neq \emptyset$ is a set, $\in^{\mathcal{A}} \subseteq A^+ \times A$ with $A^+ = \bigcup_{n>0} A^n$, and $c^{\mathcal{A}} \in A$ for each constant c in L. Thus in an arbitrary structure, '$\in$' is interpreted as any relation over $A^+ \times A$.

A *normal structure* is a structure $\mathcal{A}$ such that $\in^{\mathcal{A}}$ is a set theoretical membership relation. Thus if A is a class of atoms and sets, then

(2-4) $\mathcal{A} \models <c_1,...,c_n> \in d \Leftrightarrow <c_1^{\mathcal{A}},...,c_n^{\mathcal{A}}>$ is an element of $d^{\mathcal{A}}$

Note that the definition of a normal structure is vague and ambiguous as is Kanger's concept of a normal primary valuation. This is due to the fact that the exact meaning of "element of" is left open so far. The meaning given to "element of" by the ZF axioms is not adequate for Kanger's purposes. A main task for him is to find a more suitable concept.

2.13 Validity and Logical Truth. Let S be a sequent or sentence of L. S is *valid* if S is true in every structure. S is *logically true* if S is true in every normal structure.

2.14 Remark. Thus validity is defined with reference to structures $\mathcal{A}$ where $\in^{\mathcal{A}}$ is any relation. Logical truth is defined with reference to such structures $\mathcal{A}$ where $\in^{\mathcal{A}}$ is a set theoretical membership relation.

2.15 The Calculus LC. Kanger defines a *sequent calculus LC* for the language L. The rules of the calculus are essentially the usual ones for a sequent calculus, though with some minor changes. The purpose of one of these is to ensure that the proof procedure will be effective. Another one allows the exclusion of the Structural Rule from LC. The purpose of a third change is to cope with the types in the language L.

2.16 Proofs. Let $\Pi = \langle S, T_1, T_2, \ldots \rangle$ be a sequence of sequents. Π is a *quasi-deduction* in LC of S from the class Θ of assumption sequents if the following conditions are satisfied:

(1) Π begins with an occurrence of S;
(2) each component U of Π is either
- an instance of Postulate *1 (the identity axiom), or
- an element of Θ, or
- inferable in one step by a rule of inference from one or two succeeding components of Π.

A *proof* in LC of a sequent S is a finite quasi-deduction of S from the empty class of assumptions. A *theorem* of LC is a sequent which has a proof in LC.

2.17 Remark. (I) A curiosity about the definition is that Kanger defines a deduction as a linear sequence though what he needs and actually uses is the picture of a deduction as a labeled tree.

(II) If S is a provable sequent, then a proof for S is a finite tree with S at the root and with instances of the *Identity Postulate*

(2-5) $\Gamma', B, \Gamma \longrightarrow \Theta', B, \Theta$

labeling the leaves. If S is not provable, then the proof tree for S contains a branch Σ which does not begin with an Identity Postulate and is extended as far backward as the rules of LC allow.

(III) Note that Kanger writes a deduction in the opposite order of the usual one, i.e., he writes the conclusion first and not last. When one works in ordinary formulations of the sequent calculus, one soon discovers that the only reasonable way to construct a deduction is to construct it backward starting