

3

Design of FM/FM Systems

In the design of communication systems, parameters must be determined such that the signal quality is equal to or greater than the design specifications. Generally, in analog systems, the output signal-to-noise ratio is one of the specifications. In FM/FM telemetry systems, the deviation of the carrier with respect to the subcarriers is a variable that will be determined in order to achieve the desired signal quality. Specifically, the preemphasis schedule is developed such that the signal quality in all the channels meets specifications. This chapter is concerned with the design of the preemphasis schedule.

3.1 Learning Objectives

Upon completing this chapter, the reader should understand the following concepts and procedures:

- The function of the modulation parameters in setting the signal-to-noise output;
- The tradeoff between bandwidth and signal-to-noise output;
- A systematic process for designing the preemphasis schedule for FM/FM systems;
- Preemphasis design for operation above threshold;
- Preemphasis design for a specified bandwidth;

- Preemphasis design for a minimum transmission bandwidth or concurrent all-channel dropout;
- Preemphasis design for different signal and subcarrier mod indexes;
- Preemphasis design for different specified output signal-to-noise ratios in the subcarrier channels;
- Setting the receiver IF and video filter bandwidths and discriminator postdetection bandwidths based upon modulation design parameters for the actual operation of the telemetry receiving system;
- Hardware implementation of the preemphasis schedule.

3.2 System Parameters

This section is concerned with what modulation parameters may be used to design the preemphasis schedule. Inspection of (2.27) and (2.28) indicates what parameters—primarily modulation (mod) indexes, or deviation ratios, as they are sometimes called—are available to vary in order to maintain equal quality of the output signals in all the channels. This problem must be addressed since the bandpass filters used to separate the subcarriers must contend with a power spectral density increasing as f^2 . Further, since D_{si} is set, nominally at 5, the bandwidth of the bandpass filters is approximately given by $2f_{dsi}$, which is increasing as f_{si} increases for PBW channels and is constant for CBW channels. In an FM/FM system designed to IRIG specifications, D_{si} is nominally set to 5, and f_{mi} is specified while the carrier IF bandwidth B_c and $[S/N]_c$ are the same for all channels. The only remaining nonfixed variable is D_{ci} , the mod index of the carrier with respect to the i th subcarrier. Since D_{ci} is the ratio f_{dci}/f_{si} and the subcarrier frequency is fixed, the only parameter available to vary in order to maintain equal signal quality is, f_{dci} , the deviation of the carrier by the subcarrier. The design values of the f_{dci} 's are referred to as the preemphasis schedule.

Once the preemphasis schedule has been designed, it is possible and necessary to specify the operational parameters of the receiver such as the IF and video filter bandwidths, postdetection lowpass filter bandwidths, and filter types.

The first step in designing the preemphasis schedule is to solve (2.30) for f_{dci} , which gives

$$f_{dci} = \sqrt{\frac{4}{3}} \frac{f_{si}}{f_{dsi}} \sqrt{f_{mi}^3} \frac{1}{\sqrt{B_c}} \frac{[S/N]_{oi}}{[S/N]_c} \quad (3.1)$$

The difficulty in using this equation is that it is one equation in two unknowns, B_c and f_{dci} . To use (3.1) as is, B_c , the carrier IF bandwidth, must be estimated by a rule of thumb since it is a function of the f_{dci} 's, which are being calculated. A method to circumvent this problem will be discussed in Section 3.3. Early methods developed and used to solve this problem were presented by Downing [1], Nichols and Rauch [2], Law [3], and Rosen [4, 5].

3.3 Design Procedure

A preemphasis schedule design procedure that circumvents the problem of one equation in two unknowns will now be given. The approach is to develop an expression for B_c in terms of f_{dci} and known parameters and substitute that expression into (3.1), converting it into one equation in one unknown. The method to be developed is systematic and amenable to computer programming.

The approach is to define some intermediate variables to facilitate developing one equation in one unknown. Specifically, the equations are arranged such that an expression for B_c in terms of f_{dc1} may be found and substituted into (3.1) and f_{dc1} solved for. Then the other f_{dci} 's are solved for. The process of algebraically manipulating the equations to solve for the actual deviations will be developed next.

In the process, B_c , will be solved for in terms of the norm of the carrier deviations, f_{dn} , and f_{dc1} , to be defined. The first step in the systematic approach is to define all the variables to be used.

Repeating (2.15) gives

$$f_{dn} = \sqrt{f_{dc1}^2 + f_{dc2}^2 + \dots + f_{dcn}^2} \quad (3.2)$$

Let

f_{dc1} = deviation of the carrier by the highest frequency subcarrier;

f_{dc2} = deviation of the carrier by the next highest frequency subcarrier;

⋮

f_{dcn} = deviation of the carrier by the lowest frequency subcarrier.

Let

f_{s1} = highest frequency subcarrier;
 f_{s2} = next highest frequency subcarrier;
 \vdots
 \vdots
 f_{sn} = lowest frequency subcarrier.

Note that because the subcarriers are working from a parabolic noise spectrum, the highest frequency subcarrier will have the maximum noise power to contend with and hence must have the largest f_{dci} to obtain the largest signal-to-noise FM improvement. Normalizing (3.2) with respect to f_{dc1} gives

$$f_{dn} = f_{dc1} \sqrt{1 + \left(\frac{f_{dc2}}{f_{dc1}}\right)^2 + \left(\frac{f_{dc3}}{f_{dc1}}\right)^2 + \dots + \left(\frac{f_{dcn}}{f_{dc1}}\right)^2} \quad (3.3)$$

Define:

$$\begin{aligned}
 A_1 &= 1 \\
 A_2 &= f_{dc2} / f_{dc1} \\
 A_3 &= f_{dc3} / f_{dc1} \\
 &\vdots \\
 &\vdots \\
 &\vdots \\
 A_n &= f_{dcn} / f_{dc1}
 \end{aligned} \quad (3.4)$$

Using the definitions of (3.4) in (3.3) gives

$$f_{dn} = f_{dc1} \sqrt{A_1^2 + A_2^2 + A_3^2 + \dots + A_n^2} \quad (3.5)$$

Defining A_p such that

$$A_p = \sqrt{A_1^2 + A_2^2 + A_3^2 + \dots + A_n^2} \quad (3.6)$$

Then

$$f_{dn} = f_{dc1} A_p \quad (3.7)$$

The A_i 's, and hence A_p , are in terms of the carrier deviations, which are unknown; however, the A_i 's may be expressed in terms of known parameters by formally taking the ratio of the deviations using (3.1), which gives

$$A_i = \frac{f_{dci}}{f_{dc1}} = \frac{\sqrt{4/3} \frac{[S/N]_{oi}}{[S/N]_c} \frac{f_{si}}{f_{dsi}} \sqrt{f_{mi}^3}}{\sqrt{4/3} \frac{[S/N]_{o1}}{[S/N]_c} \frac{f_{s1}}{f_{ds1}} \sqrt{f_{m1}^3}} \quad (3.8)$$

If the output signal-to-noise ratios in all the channels are to be the same, then

$$A_i = \frac{f_{si}}{f_{s1}} \frac{f_{ds1}}{f_{dsi}} \frac{\sqrt{f_{mi}^3}}{\sqrt{f_{m1}^3}} \quad (3.9)$$

Equation (3.9) is important since it allows the A_i 's to be calculated in terms of known parameters. Equation (3.7) gives f_{dn} , the norm of the total carrier deviation in terms of f_{dc1} and A_p , which is known. Although f_{dc1} is not known, the equation with f_{dn} may be used to express B_c in terms of f_{dc1} and substituted into (3.1), reducing this equation into one equation in one unknown. The resulting equation may be used to solve for f_{dc1} . Further, since

$$f_{dci} = f_{dc1} A_i \quad (3.10)$$

the other f_{dci} 's may be solved for. Proceeding and repeating (2.14) gives B_c in terms of f_{dn} such as

$$B_c = 2[f_{dn} + f_{s1}] = 2(f_{dc1} A_p + f_{s1}) \quad (3.11)$$

f_{dc1} must be solved for first. Repeating and rearranging (3.1), then substituting the index 1 for the i th index gives

$$f_{dc1} = \sqrt{4/3} \frac{[S/N]_{oi}}{[S/N]_c} \frac{1}{\sqrt{B_c}} \frac{f_{s1}}{f_{ds1}} \sqrt{f_{m1}^3} \quad (3.12)$$

Multiplying numerator and denominator of (3.12) by $\sqrt{f_{ds1}}$ substituting $D_{s1} = f_{ds1}/f_{m1}$ gives

$$f_{dc1} = \sqrt{4/3} \frac{[S/N]_{oi}}{[S/N]_c} \frac{1}{\sqrt{B_c}} \frac{f_{s1} \sqrt{f_{ds1}}}{\sqrt{D_{s1}^3}} \quad (3.13)$$

Substituting the expression for B_c , (3.11), into (3.13) results in

$$f_{dc1} = \sqrt{4/3} \frac{[S/N]_{oi}}{[S/N]_c} \frac{1}{\sqrt{2(f_{dc1} A_p + f_{s1})}} \frac{f_{s1} \sqrt{f_{ds1}}}{\sqrt{D_{s1}^3}} \quad (3.14)$$

If we let

$$C = \sqrt{4/3} \frac{[S/N]_{oi}}{[S/N]_c}$$

then (3.14) becomes

$$f_{dc1} = C \frac{1}{\sqrt{2(f_{dc1} A_p + f_{s1})}} \frac{f_{s1} \sqrt{f_{ds1}}}{\sqrt{D_{s1}^3}} \quad (3.15)$$

Inspection of (3.15) reveals that f_{dc1} may be solved for without knowing B_c explicitly. That is, if $[S/N]_c$ is specified, the A_i 's are calculated, and since the other parameters are known (the D_{si} 's are nominally set to 5), f_{dc1} can be calculated. Once f_{dc1} is calculated, the remaining f_{dci} 's may be calculated from (3.10). Rearranging (3.15) results in a third order polynomial given by

$$f_{dc1}^3 + \frac{f_{s1}}{A_p} (f_{dc1}^2) - \frac{C^2 f_{s1}^2 f_{ds1}}{2A_p D_{s1}^3} = 0 \quad (3.16)$$

This third-order equation may be used to solve for f_{dc1} explicitly in terms of the known parameters. Once f_{dc1} is solved for, all the other f_{dci} 's may be solved for since the A_i 's are known. That is,

$$f_{dci} = f_{dc1} A_i \quad (3.17)$$

3.4 Design Examples

3.4.1 PBW Channels

Design Example 1

To clarify the use of the FM/FM design process, an FM/FM multiplex design example will be worked for 15 subcarriers of the 7.5% PBW channel. The subcarriers used will be the 1.3 kHz through the 93 kHz.

Parameters:

$$[S/N]_C = 12 \text{ dB (3.98 numerical);}$$

$$D_{si} = f_{dsi} / f_{mi} = 5 \text{ (nominal);}$$

Desired $[S/N]_{oi} = 46 \text{ dB (199 numerical)}$ minimum in all channels.

Step 1

The A_i 's will be calculated. Repeating (3.9) gives

$$A_i = \frac{f_{si}}{f_{s1}} \frac{f_{ds1}}{f_{dsi}} \frac{\sqrt{f_{mi}^3}}{\sqrt{f_{m1}^3}} \quad (3.18)$$

From the IRIG standards:

$$f_{s1} = 93 \text{ kHz;}$$

$$f_{ds1} = 6.975 \text{ kHz;}$$

$$f_{m1} = 1.395 \text{ kHz;}$$

$$f_{s2} = 70 \text{ kHz;}$$

$$f_{ds2} = 5.75 \text{ kHz;}$$

$$f_{m2} = 5.250 \text{ kHz;}$$

$$C = \sqrt{4/3} \frac{[S/N]_{oi}}{[S/N]_C} = 57.87.$$

Specifically, for A_2 ,

$$A_2 = \frac{f_{s2}}{f_{s1}} \frac{f_{ds1}}{f_{ds2}} \frac{\sqrt{f_{mi}^3}}{\sqrt{f_{m1}^3}} = 0.653$$

Using (3.18) to calculate all the A_i 's and noting that $A_1 = 1$ by definition gives the values in column 4 in Table 3.1.

Step 2

Calculate A_p . Using (3.9) to calculate A_p gives

$$A_p = \sqrt{A_1^2 + A_2^2 + A_3^2 + \dots + A_n^2} = 1.31$$

Step 3

Repeating (3.16) gives

$$f_{dc1}^3 + \frac{f_{s1}}{A_p}(f_{dc1}^2) - \frac{C^2 f_{s1}^2 f_{ds1}}{2A_p D_{s1}^3} = 0 \quad (3.19)$$

Solving (3.19), a third-order polynomial, for f_{dc1} and using the parameters for this problem results in

$$f_{dc1} = 66.804 \text{ kHz} \quad (3.20)$$

Table 3.1
Design Example 1: Initial Preemphasis Schedule

f_{si} (kHz)	SCO (kHz) Upper Dev. (f_{ds})	f_{mi} (Hz)	A_i	f_{dci}
93.0	6.975	1,395	1.00	66,804.73
70.0	5.250	1,050	0.653	43,624.44
52.5	3.937	790	0.424	28,379.84
40.0	3.000	600	0.282	18,843.99
30.0	2.250	450	0.183	12,239.53
22.0	1.650	330	0.115	7,686.29
14.5	1.087	220	0.062	4,136.34
10.5	0.787	160	0.038	2,554.39
7.35	0.551	110	0.022	1,484.27
5.4	0.405	81	0.014	934.70
3.9	0.292	59	0.0086	573.69
3.0	0.225	45	0.0058	387.05
2.3	0.172	35	0.0039	259.82
1.7	0.127	25	0.0024	163.48
1.3	0.098	20	0.002	114.0

Step 4

From (3.17), the remaining f_{dci} 's may be calculated, such as

$$f_{dci} = f_{dc1}A_i \quad (3.21)$$

and are shown in column 5 of Table 3.1.

Step 5

Calculate f_{dn} , the norm of the carrier deviations, from (3.7),

$$f_{dn} = f_{dc1}A_p = (66.804)(1.319) = 88.106 \text{ kHz} \quad (3.22)$$

Step 6

Calculate B_c , the carrier IF bandwidth, from (3.11),

$$B_c = 2[f_{dn} + f_{s1}] = 2(88.106 + 93) = 362.2 \text{ kHz} \quad (3.23)$$

Step 7

Calculate BW_v , the bandwidth of the output video filter following the carrier discriminator, in terms of the required bandwidth of the highest frequency subcarrier due to the modulating message, which is the bandwidth, $BW_{bpf s1}$, of the bandpass filter of the highest frequency subcarrier.

$$\begin{aligned} B_v &= f_{s1} + (BW_{bpf s1})/2 \\ &= 93 \text{ kHz} + 2(f_{ds93} + 1,395)/2 \\ &= 93 \text{ kHz} + (6,975 + 1,395) = 101.37 \text{ kHz} \end{aligned}$$

Summary of Results for Design Example 1

The only unspecified parameter available to ensure acceptable and equal signal-to-noise ratios in all the subcarrier channels in the design of an FM/FM multiplex system meeting IRIG specifications is the f_{dci} 's, the deviations of the carrier by the subcarriers. This procedure calculated the necessary deviations for a specified 46-dB signal-to-noise output in 15 PBW channels. Table 3.2 gives the resulting modulation index of the carrier with respect to the subcarrier and the calculated signal-to-noise ratio for the calculated carrier deviations.

Refinements in the Design

Inspection of column 5 in Table 3.1 indicates that the five lowest subcarriers will deviate the carrier less than 1 kHz. Variations of outputs of VCOs,

Table 3.2
 D_{si} and D_{ci} for Design Example 1

f_{si} (kHz)	f_{dci} (kHz)	$(S/N)_{oi}$ (dB)	D_{si}	D_{ci}
93.0	66.80	46	5.000	0.7183
70.0	43.62	46	5.000	0.6232
52.5	28.37	46	4.984	0.5406
40.0	18.84	46	5.000	0.4711
30.0	12.23	46	5.000	0.4080
22.0	7.68	46	5.000	0.3494
14.5	4.13	46	4.943	0.2853
10.5	2.55	46	4.922	0.2433
7.35	1.48	46	5.000	0.2019
5.4	0.934	46	5.000	0.1731
3.9	0.573	46	5.000	0.1471
3.0	0.387	46	5.000	0.1290
2.3	0.259	46	5.000	0.1130
1.7	0.163	46	5.100	0.0962
1.3	0.114	46	5.100	0.088

resistors, transmitters, or intermodulation products in the system may cause larger apparent carrier deviations. Therefore, the deviations of the carrier by the lowest frequency subcarriers should be increased to a larger fixed percentage of f_{dn} . A reasonable increase is to make all the lower deviations equal to 10% of the computed f_{dn} . In this example, it would require increasing the deviations to about 8.8 kHz. However, for the purpose of illustrating the impact of this increase in the lower seven subcarriers, deviations will be increased to 2 kHz.

Increasing the f_{dci} 's changes several parameters. One change is that since $D_{ci} = f_{dci}/f_{si}$, the modulation index of the lower frequency subcarriers with respect to the carrier increases. Observation of (2.27) shows that the signal-to-noise improves as D_{ci} increases, which means that $[S/N]_{oi}$ will exceed the design requirements for the lower frequency channels. This presents an opportunity to increase the information capability of the lower channels. The modulation index of the subcarriers with respect to the message is $D_{si} = f_{dsi}/f_{mi}$ and is nominally set to 5. (Note that f_{mi} is the highest frequency in the message modulating the subcarrier.) Inspection of (2.27) shows that the signal-to-noise also varies directly as D_{si} . Since D_{ci} was increased, D_{si} may be decreased by increasing the allowed maximum frequency of the modulating signal.

Increasing the deviations of the carrier by the lower-frequency subcarriers increases f_{dn} , which makes it necessary to evaluate the increase in the

required carrier IF bandwidth since B_c is a function of f_{dn} . It will be found that there is very little effect on the bandwidth. Design Example 2 will determine the effect on the system parameters of increasing the deviations of the carrier by the lower subcarriers.

Design Example 2

The effect of increasing the deviation of the carrier by the seven lower-frequency subcarriers will be evaluated for the multiplex system of Design Example 1. The deviation on all seven will be increased to 2 kHz.

Step 1

Increase f_{dci} of the seven lower subcarriers. Column 2 in Table 3.3 shows the increase in deviations.

Step 2

Compute the new f_{dn} from (3.2).

$$\begin{aligned} f_{dn} &= \sqrt{f_{dc1}^2 + f_{dc2}^2 + \dots + f_{dcn}^2} \\ &= [(66.8)^2 + (43.6)^2 + (28.4)^2 + (18.8)^2 + (12.2)^2 \\ &\quad + (7.69)^2 + (4.12)^2 + (2.55)^2 + 7(2)^2]^{1/2} \quad (3.24) \\ &= 88.23 \text{ kHz} \end{aligned}$$

Table 3.3
Increased f_{dci} for Lower Subcarriers in Design Example 1

f_{si} (kHz)	f_{dci} (kHz)	$(S/N)_{oi}$ (dB)	D_{si}	D_{ci}
93.0	66.80	46.0	5.000	0.7183
70.0	43.62	46.0	5.000	0.6232
52.5	28.37	46.0	4.984	0.5406
40.0	18.84	46.0	5.000	0.4711
30.0	12.23	46.0	5.000	0.4080
22.0	7.68	46.0	5.000	0.3494
14.5	4.13	46.0	4.943	0.2853
10.5	2.55	46.0	4.922	0.2433
7.35	2.0	48.6	5.000	0.2019
5.4	2.0	52.6	5.000	0.1731
3.9	2.0	56.8	5.000	0.1471
3.0	2.0	60.2	5.000	0.1290
2.3	2.0	63.7	5.000	0.1130
1.7	2.0	67.9	5.100	0.0962
1.3	2.0	71.0	5.100	0.088

Step 3

Compute B_c from (3.11) and the new D_{ci} 's; that is,

$$B_c = 2[f_{dn} + f_{s1}] = 2(88.23 + 93) = 362.5 \text{ kHz} \quad (3.25)$$

and

$$D_{ci} = f_{dci} / f_{si}$$

For example, D_{c15} is given by

$$D_{c15} = 2,000/1,300 = 1.54 \quad (3.26)$$

Step 4

Compute the new $[S/N]_{oi}$ for the seven lowest subcarriers from (2.30).

Repeating (2.31) gives

$$[S/N]_{oi} = \sqrt{3/4} D_{si} D_{ci} \sqrt{\frac{B_c}{f_{mi}}} [S/N]_c \quad (3.27)$$

The new signal-to-noise ratio for the lowest subcarrier is

$$\begin{aligned} [S/N]_{o15} &= (0.866)(3.98)(5)(1.54)[(602)/4.47] = 3,574.2 \text{ (numerical)} \\ [S/N]_{o15} &= 20 \log 35,742 = 71 \text{ dB} \end{aligned} \quad (3.28)$$

Column 3 in Table 3.3 shows the resulting increase in $[S/N]_{oi}$ for the lower-frequency subcarriers. Since this is larger than the design specifications, f_{mi} will be doubled, lowering D_{mi} to approximately 2.5, which decreases $[S/N]_{oi}$.

Step 5

Double f_{mi} of the seven lowest subcarriers, and compute D_{si} and $[S/N]_{oi}$. Note that B_c remains at 362.5 kHz.

The mod index of the 15th subcarrier with respect to the message is

$$D_{s15} = f_{ds15} / f_{m15} = 98/40 = 2.45 \quad (3.29)$$

Using equation (3.27) gives

$$[S/N]_{o15} = \sqrt{3/4} [S/N]_c D_{si} D_{ci} \sqrt{\frac{B_c}{f_{mi}}}$$

$$[S/N]_{o15} = (0.866)(3.98)(2.45)(1.54)[(362,500)/40]^{1/2} = 1,263 \text{ (numerical)}$$

$$[S/N]_{o15} = 20 \log 1,263 = 62 \text{ dB} \quad (3.30)$$

The modified $[S/N]_{oi}$ is shown in column 3 of Table 3.4.

Summary of the Modifications

The deviations of the carrier by the seven lowest subcarriers were increased to 2 kHz, which increased the D_{ci} 's, which increased $[S/N]_{oi}$'s substantially above the design requirement. Consequently, the D_{si} 's were decreased by increasing the maximum frequency of the message, and the new $[S/N]_{oi}$'s were evaluated and found to be above the design requirement in all channels except the 7.35 kHz and the 5.4 kHz. These two channels should have their D_{si} 's increased to possible 4. The modified $B_c = 362.5$ kHz was found to be very close to the original $B_c = 362.2$ kHz.

It was determined that increasing the deviations of the seven lower frequency subcarriers to 2 kHz had very little impact on f_{dn} . In fact, increasing the seven lower subcarrier deviations to 10% of the calculated f_{dn} or to 8.8 kHz has a small effect, as will be shown next.

Table 3.4
Decreased D_{si} for Lower Subcarriers in Design Example 1

f_{si} (kHz)	f_{dci} (kHz)	$(S/N)_{oi}$ (dB)	D_{si}	D_{ci}
93.0	66.80	46.0	5.00	0.72
70.0	43.62	46.0	5.00	0.62
52.5	28.37	46.0	4.98	0.54
40.0	18.84	46.0	5.00	0.47
30.0	12.23	46.0	5.00	0.41
22.0	7.68	46.0	5.00	0.35
14.5	4.13	46.0	4.94	0.28
10.5	2.55	46.0	4.92	0.24
7.35	2.0	40.0	2.51	0.28
5.4	2.0	43.6	2.50	0.37
3.9	2.0	47.7	2.46	0.51
3.0	2.0	51.2	2.50	0.67
2.3	2.0	54.5	2.46	0.87
1.7	2.0	58.9	2.55	1.17
1.3	2.0	62.0	2.45	1.54

From (3.2)

$$\begin{aligned}
 f_{dn} &= \sqrt{f_{dc1}^2 + f_{dc2}^2 + \dots + f_{dcn}^2} \\
 &= [(66.8)^2 + (43.6)^2 + (28.4)^2 + (18.8)^2 + (12.2)^2 \\
 &\quad + (7.69)^2 + (4.12)^2 + (2.55)^2 + 7(8.8)^2]^{1/2} \\
 &= 91.1 \text{ kHz}
 \end{aligned} \tag{3.31}$$

The resulting bandwidth is

$$B_c = 2[f_{dn} + f_{s1}] = 2(91 + 93) = 368.2 \text{ kHz}$$

This small increase in B_c supports the model suggested in Chapter 2 and illustrated in Figure 2.4, although this case has substantially more subcarriers. Further, using only the five highest subcarrier deviations of the carrier gives

$$\begin{aligned}
 f_{dn} &= [f_{dc1}^2 + f_{dc2}^2 + \dots + f_{dc5}^2]^{1/2} \\
 &= [(66.8)^2 + (43.6)^2 + (28.4)^2 + (18.8)^2 + (12.2)^2]^{1/2} \\
 &= 76.7 \text{ kHz}
 \end{aligned}$$

The 76.7 kHz obtained using only the highest five subcarrier deviations compares favorably with the actual 88 kHz using all the deviations for a first-cut look.

Rewriting (3.7) gives

$$f_{dn} = f_{dc1} A_p = 1.3 f_{dc1}$$

This equation indicates that the total RMS carrier deviation or norm of the deviations is about 1.3 times the deviation of the carrier by the highest frequency subcarrier in PBW telemetry systems. This is a rule of thumb used some of the time.

3.4.2 CBW Channels

In CBW channels, f_{dsi} , the deviation of the subcarrier by the i th message, is a constant. The message bandwidth, or equivalently, the maximum frequency, f_{mi} , each message is allowed to contain is the same for all channels. Since f_{mi} and f_{dsi} are constant, then D_{si} is fixed. Inspection of (2.22) shows that

D_{ci} must remain fixed in order for all channels to have the same output signal-to-noise ratio. The parameter used in order to achieve the specified signal-to-noise is f_{dci} and is given by

$$D_{ci} = f_{dci} / f_{si} \quad (3.32)$$

The f_{dci} 's will increase linearly with the f_{si} 's and their value will be determined by the preemphasis design procedure. The design procedure is the same as in the PBW case and it is to compute the A factors, solve for f_{dc1} , and finally solve for the numerical values of the other carrier deviations. A design example will be worked next.

Design Example 3

An example using 14 subcarriers, channels 6A through 19A, will be worked for an FM/FM CBW multiplexed system.

Parameters:

$$[S/N]_c = 12 \text{ dB (3.98 numerical)}$$

$$D_{si} = 5 \text{ nominal}$$

$$\text{Desired } [S/N]_{oi} = 46 \text{ dB (199 numerical)}$$

$$C = \sqrt{3/4} \frac{[S/N]_{oi}}{[S/N]_c} = 57.87$$

From the IRIG standards:

$$f_{s1} = 160 \text{ kHz;}$$

$$f_{ds1} = 2.0 \text{ kHz;}$$

$$f_{m1} = f_{ds1} / D_{s1} = 2,000/5 = 400 \text{ kHz;}$$

$$f_{s2} = 152 \text{ kHz;}$$

$$f_{ds2} = 2.0 \text{ kHz;}$$

$$f_{m2} = f_{ds2} / D_{s2} = 2,000/5 = 400 \text{ kHz.}$$

Step 1

The A_i 's will be calculated using (3.4), which is

$$A_i = \frac{f_{si}}{f_{s1}} \frac{f_{ds1}}{f_{dsi}} \frac{\sqrt{f_{mi}^3}}{\sqrt{f_{m1}^3}} \quad (3.33)$$

Since, in this example, all the D_{si} 's are set at 5, all the f_{dsi} 's are equal, as are the f_{mi} 's and (3.33) becomes

$$A_i = \frac{f_{si}}{f_{s1}} \quad (3.34)$$

Specifically, the expression for A_2 is

$$A_2 = 152/160 = 0.95$$

Using (3.34) the remaining A_i 's are calculated and listed in Table 3.5, column 4.

Step 2

Calculate A_p . Using (3.6) to calculate A_p gives

$$A_p = \sqrt{A_1^2 + A_2^2 + A_3^2 + \dots + A_n^2} = 2.64 \quad (3.35)$$

Step 3

Solve for f_{dc1} using (3.16). Repeating (3.16),

$$f_{dc1}^3 + \frac{f_{s1}}{A_p}(f_{dc1}^2) - \frac{C^2 f_{s1}^2 f_{ds1}}{2A_p D_{s1}^3} = 0$$

Table 3.5
Design Example 3: Preemphasis Schedule

f_{si} (kHz)	f_{dsi} (kHz)	f_{mi} (kHz)	A_i	f_{dci} (kHz)
160	2	400	1.00	48.76
152	2	400	0.95	46.32
144	2	400	0.90	43.88
136	2	400	0.85	41.44
128	2	400	0.80	39.00
120	2	400	0.75	36.57
112	2	400	0.70	34.13
104	2	400	0.65	31.69
96	2	400	0.60	29.25
88	2	400	0.55	26.81
80	2	400	0.50	24.38
72	2	400	0.45	21.94
64	2	400	0.40	19.50
56	2	400	0.35	17.06

Using the parameters of this problem, solving for f_{dc1} gives

$$f_{dc1} = 48,759 \text{ Hz} \quad (3.36)$$

Step 4

Calculate the D_{ci} 's and the remaining f_{dci} 's:

$$D_{c1} = f_{dc1}/f_{s1} = 48,759/160,000 = 0.3047$$

and

$$D_{ci} = 0.3047 \text{ and } D_{si} = 5 \text{ for all channels}$$

Since the D_{ci} 's are fixed, the other f_{dci} 's may be calculated from D_{c1} , that is,

$$f_{dc2} = (D_{c1})(f_{s2}) = (0.3047)(152,000) = 46,321 \text{ Hz}$$

The remaining f_{dci} 's are calculated and tabulated in Table 3.5, column 5.

Step 5

Calculate f_{dn} , the norm of the carrier deviations from (3.11). Using (3.7) and the value of A_p from (3.35) gives

$$f_{dn} = f_{dc1}A_p = (48,759)(2.64) = 128.7 \text{ kHz} \quad (3.37)$$

Step 6

Calculating B_c , the carrier IF bandwidth gives

$$B_c = 2[f_{dn} + f_{s1}] = 2(128.7 + 160) = 577.4 \text{ kHz} \quad (3.38)$$

Step 7

Calculate the output signal-to-noise ratio in each channel. For channel 1, the highest frequency channel, utilizing (2.31) gives

$$[S/N]_{o1} = \sqrt{3/4} D_{s1} D_{c1} [S/N]_c;$$

$$[S/N]_{o1} = (0.866)(5)(0.3047)(760/20)(3.98) = 199 \text{ (numerical);}$$

$$[S/N]_{o1} = 46 \text{ dB.}$$

Column 3 in Table 3.6 gives all the computed signal-to-noise ratios for the channels.

Summary of Results for Design Example 3

The only unspecified parameters available to insure acceptable and equal signal-to-noise ratios in all the subcarrier channels in the design of an FM/FM multiplex system meeting IRIG specifications are the f_{dci} 's, the deviations of the carrier by the subcarriers. This example calculated the necessary deviations for a specified 46-dB signal-to-noise output in 14 CBW channels.

Since the maximum frequency allowed for each channel is constant at 400 Hz, this type of format would be chosen whenever the signals from the various sensors have about the same spectral content. This design example also fixed D_{si} at 5; however, had D_{si} been chosen to be, say 2.5, then the maximum frequency response of each channel would have been 800 Hz. In order to compensate for the decreased D_{si} in the signal-to-noise equation, D_{ci} and hence f_{dci} would have been calculated to be larger, increasing the necessary IF bandwidth.

3.5 Threshold

3.5.1 Signal-to-Noise Ratio for a Bandpass Filter

The FM/FM system may be designed such that the subcarrier signal-to-noise ratios in the bandpass filters are equal to or are larger than the carrier

Table 3.6
 D_{si} and D_{ci} of Design Example 3

f_{si} (kHz)	f_{dci} (kHz)	$(S/N)_{oi}$ (dB)	D_{si}	D_{ci}
160	48.76	46	5	0.3047
152	46.32	46	5	0.3047
144	43.88	46	5	0.3047
136	41.44	46	5	0.3047
128	39.00	46	5	0.3047
120	36.57	46	5	0.3047
112	34.13	46	5	0.3047
104	31.69	46	5	0.3047
96	29.25	46	5	0.3047
88	26.81	46	5	0.3047
80	24.38	46	5	0.3047
72	21.94	46	5	0.3047
64	19.50	46	5	0.3047
56	17.06	46	5	0.3047

IF ratios in the carrier IF so that the threshold will occur later in the subcarrier bandpass filters than in the carrier IF. This is done because data will degrade less for threshold in the carrier IF than for threshold in the subcarrier bandpass filters. This will not give a minimum transmission bandwidth, and often the system is designed for concurrent thresholds.

Equation (2.22) gives the output signal-to-noise ratio for a bandpass filter following an FM discriminator. The output signal-to-noise ratio is also the signal-to-noise ratio in the output bandpass filter. It is desirable to have this greater than 12 dB in order to delay the onset of threshold in the subcarrier bandpass filter until after threshold in the carrier IF.

Multiplying (2.22) by B_c , rearranging, and inserting the subscript i for the i th subcarrier bandpass filter gives

$$\left[\frac{S}{N} \right]_{OBPi} = \frac{1}{2} \frac{f_{dci}^2 B_c}{f_{si}^2 B_i} \left[\frac{A^2}{2\eta B_c} \right] \quad (3.39)$$

The term in the bracket is the carrier-to-noise ratio in the carrier IF. Therefore, (3.39) may be written as

$$\left[\frac{S}{N} \right]_{OBPi} = \frac{1}{2} \frac{f_{dci}^2 B_c}{f_{si}^2 B_i} \left[\frac{S}{N} \right]_c \quad (3.40)$$

For the signal-to-noise ratio of the subcarrier bandpass filter to be greater than 12 dB, the term preceding the bracket must be greater than 1, since the carrier signal-to-noise ratio is 12 dB by design.

3.5.2 Threshold Design Margin

Specifically, after the design of an FM/FM system, since all terms in (3.40) are known and the only design variable is f_{dci} , the term preceding the bracket should be set up as an inequality and checked to see if it is equal to or greater than 1. Proceeding gives

$$\frac{1}{2} \frac{f_{dci}^2 B_c}{f_{si}^2 B_i} \geq 1 \quad (3.41)$$

Solving for f_{dci} gives

$$f_{dci} \geq \sqrt{2} f_{si} \sqrt{B_i / B_c} \quad (3.42)$$

The inequality of (3.42) insures that the signal-to-noise ratio in the i th subcarrier bandpass filter is greater than 12 dB or 3.99 numerical. Taking 10 log of (3.40) gives

$$\left[\frac{S}{N} \right]_{OBPi} \text{ (dB)} = 10 \log \left[\frac{1}{2} \frac{f_{dci}^2}{f_{si}^2} \frac{B_c}{B_i} \right] + \left[\frac{S}{N} \right]_c \text{ (dB)} \quad (3.43)$$

The first term in (3.43) gives the threshold design margin, M_{ii} , for the i th subcarrier bandpass filter.

Design Example 4

Equation (3.43) will be used to determine the design margin for the FM/FM channels of Design Example 2. This example results from the design of 15 FM/FM channels modified for larger f_{dci} 's for the lower frequency channels and with the associated D_{si} 's changed. Noting that f_{dci}/f_{si} is D_{ci} , the threshold margin for the i th subcarrier channel is given by

$$M_{ii} = 10 \log \left[\frac{1}{2} D_{ci}^2 \frac{B_c}{B_i} \right] \quad (3.44)$$

Since the bandpass filter of the i th channel must pass the subcarrier modulated by the message and the mod index is nominally 5, the bandwidth of this filter is given by

$$B_i = 2(f_{dsi} + f_{mi}) \approx 2f_{dsi} \quad (3.45)$$

Using (3.45) and rearranging, (3.44) becomes

$$M_{ii} = 20 \log 0.5 \sqrt{B_c} \frac{D_{ci}}{\sqrt{f_{dsi}}} \quad (3.46)$$

The threshold margin for the highest subcarrier channel is given by

$$M_{i1} = 20 \log 0.5 \sqrt{B_c} \frac{D_{c1}}{\sqrt{f_{ds1}}} \quad (3.47)$$

Using the calculated IF bandwidth, B_c , of 366 kHz and Table 3.4, (3.47) becomes

$$M_{t1} = 20 \log(0.5)(366)^{1/2} \frac{0.718}{(6,975)^{1/2}} = 8.2 \text{ dB} \quad (3.48)$$

Using (3.47) the threshold design margins for the other 14 channels are computed and are listed in Table 3.7.

Design margins calculated for the CBW channels in Design Example 3 are as follows.

The parameters are:

$$B_c = 577 \text{ kHz};$$

$$D_{ci} = 0.3047;$$

$$f_{dsi} = 2 \text{ kHz}.$$

The equation for the design margin is

$$M_{ti} = 20 \log 0.5 \sqrt{B_c} + 20 \log(D_{ci} / \sqrt{f_{dsi}}) \quad (3.49)$$

Since the parameters in this equation in all the channels are the same, the threshold design margin for all channels is

$$M_{ti} = 51.59 - 43.33 = 8.26 \text{ dB}$$

Table 3.7

Threshold Margins for Proportional Bandwidth Channels in Design Example 2

f_{si} (kHz)	M_{ti} (dB)
93	8.3
70	8.3
52.5	8.3
40	8.3
30	8.3
22	8.3
14.5	8.3
10.5	8.3
7.35	11.4
5.4	14.9
3.9	19.1
3.0	22.5
2.3	26.1
1.7	30.0
1.3	33.5

3.6 Changing the Preemphasis Schedule to Utilize Specified IF or Transmission Bandwidth

3.6.1 Bandwidth Utilization

Because the telemetry receivers have only a fixed and specified number of IF bandwidths and because the allocated transmission bandwidth is limited and fixed, it is desirable to design the modulation parameters such that the utilized bandwidth meets specifications and is approximately one of the given IF bandwidths. Once the design process is completed, the following process is used to accomplish that design.

A_i is defined by

$$A_i = \frac{f_{si}}{f_{s1}} \frac{f_{ds1}}{f_{dsi}} \frac{\sqrt{f_{mi}^3}}{\sqrt{f_{m1}^3}} \quad (3.50)$$

also,

$$f_{dci} = f_{dc1} A_i \quad (3.51)$$

and

$$B_c = 2(f_{dn} + f_{sb}) = 2(f_{dc1} A_p + f_{s1}) \quad (3.52)$$

Equation (3.50) shows that the A_i 's are independent of the specified signal-to-noise ratios if they are all equal. Once the A_i 's are calculated for a group of subcarriers, the A_i 's and A_p will not change, even though the specified signal-to-noise may change. (Changing the carrier deviation by the lower frequency subcarriers will change the A_i 's, but not appreciably.) Therefore, for a group of subcarriers whose f_{dci} 's have been determined in the design process but do not utilize the available IF bandwidth, f_{dc1} can be determined such that B_c is the required value. Then the remaining f_{dci} 's may be determined, since each is related to f_{dc1} by A_i .

3.6.2 Design Equations

For example, say A_p and f_{dc1} have been determined for a group of subcarriers and a specified output signal-to-noise ratio, but the calculated IF bandwidth does not use all the allocated bandwidth. Then, (3.52) may be used to solve for f_{dc1} in terms of the desired IF bandwidth. Such as,

$$f_{dc1} = \frac{1}{A_p} \left(\frac{B_c}{2} - f_{s1} \right) \quad (3.53)$$

Then (3.51) may be used to solve for the remaining f_{dci} 's. Specifically,

$$f_{dci}(\text{new}) = f_{dc1}(\text{new}) A_i = f_{dc1}(\text{new}) \frac{f_{dci}(\text{old})}{f_{dc1}(\text{old})} \quad (3.54)$$

$$f_{dci}(\text{new}) = \frac{f_{dc1}(\text{new})}{f_{dc1}(\text{old})} f_{dci}(\text{old}) = P f_{dci}(\text{old})$$

where P is the ratio increase of f_{dc1} .

As (3.54) shows, all the deviations will be increased proportionally by the factor P . Certainly, as the new f_{dci} 's are determined and used, the $[S/N]_o$ ratios will increase. The carrier power must be increased also in order to maintain at least a 12-dB carrier-to-noise ratio in the new bandwidth of the IF bandpass filter.

Design Example 5

An example will be worked here in which the IF bandwidth is expanded to a specified value. The set of proportional bandwidth subcarriers and the design parameters determined in Design Example 1 will be used. The first set of carrier deviations will be used, since increasing the deviations of the lowest frequency subcarriers had little effect on B_c .

Say, it is desirable to utilize a 500-kHz IF or transmission bandwidth. The system parameters of Design Example 1 were:

$$A_p = 1.319;$$

$$B_c(\text{old}) = 362 \text{ Hz};$$

$$f_{dc1} = 66.8 \text{ kHz}.$$

Substituting these values into (3.53) gives

$$f_{dc1} = \frac{1}{1,319} (500/2 - 93) = 119 \text{ kHz}$$

The proportional increase, P , is

$$P = 119/66.8 = 1.78$$

The new f_{dci} 's may be found by multiplying each of the previous carrier deviations by this factor. The resulting deviations are shown in Table 3.8. The f_{dci} 's below 8.8 kHz have been increased to 8.8 kHz for the reasons mentioned previously. Calculating f_{dn} from the sum of the squares of the deviations of the carrier by the subcarriers and calculating B_c from the generalized (3.52), gives $f_{dn} = 158.8$ and $B_c = 502$ kHz. Note that if the 10% rule, which states that all f_{dci} 's should be at least 10% of f_{dn} is implemented, then the 10 lowest subcarriers should have f_{dci} 's = 15.8 kHz in the new preemphasis schedule.

3.7 Designing to a Specified Transmission Bandwidth

3.7.1 Design Process

An example of designing to a specified transmission bandwidth will be worked next. Assume it is desired to design a preemphasis schedule using the five constant bandwidth subcarriers shown in Table 3.9 and using 3 MHz of bandwidth.

Table 3.8
Increased f_{dci} 's for Expanded B_c for Design Example 1

f_{si} (kHz)	f_{dci} (kHz) Initial	f_{dci} (kHz) New
93	66.80	119.0
70	43.60	77.7
52.5	28.37	50.5
40	18.84	33.5
30	12.24	21.78
22	7.68	13.6
14.5	4.13	8.8
10.5	2.55	8.8
7.25	1.48	8.8
5.4	0.93	8.8
3.9	0.57	8.8
3.0	0.38	8.8
2.3	0.25	8.8
1.7	0.16	8.8
1.3	0.11	8.8

Note: $f_{dn} = 158$ kHz; $B_c = 2(f_{dn} + 93) = 2(251) = 502$ kHz.

Table 3.9
Five CBW Channels

Channel	f_{si}	f_{dsi}	f_{mi}
111E	896	32	6.4
95E	768	32	6.4
79E	640	32	6.4
63E	512	32	6.4
47E	384	32	6.4

Note: Desired bandwidth = 3 MHz.

Design Example 6

The first step is to calculate the A_i 's. The equation for A_i is given by

$$A_i = \frac{f_{si}}{f_{s1}} \frac{f_{ds1}}{f_{dsi}} \frac{\sqrt{f_{mi}^3}}{\sqrt{f_{m1}^3}} \quad (3.55)$$

For the CBW channels:

f_{dsi} = constant for all five channels;

f_{mi} = constant for all five channels if D_{si} is the same in all the channels.

Then

$$A_i = \frac{f_{si}}{f_{s1}} \quad (3.56)$$

and

$$A_1 = 1;$$

$$A_2 = 768/896 = 0.85.$$

The equation for A_p is given by

$$A_p = \sqrt{A_1^2 + A_2^2 + A_3^2 + \dots + A_n^2} \quad (3.57)$$

$$A_p = [1 + (0.85)^2 + (0.714)^2 + \dots + (0.428)^2]^{1/2} = 1.66$$

Now f_{dc1} may be calculated,

$$f_{dc1} = \frac{1}{A_p} \left(\frac{B_c}{2} - f_{s1} \right) = 1/1.66(3/2 - 896 \text{ kHz}) = 363 \text{ kHz} \quad (3.58)$$

The other f_{dci} 's are given by

$$f_{dci} = A_i f_{dc1}$$

Then

$$f_{dc2} = A_2 f_{dc1} = (0.857)(363) \text{ kHz} = 311.8 \text{ kHz}$$

The signal-to-noise out must be checked and is given by

$$[S/N]_{o1} = \sqrt{3/4} D_{si} D_{ci} \frac{\sqrt{B_c}}{\sqrt{f_{mi}}} [S/N]_C \quad (3.59)$$

Numerically evaluating (3.59) gives

$$[S/N]_{o1} = 43.6 \text{ dB}$$

The specified and the computed parameters are shown in Table 3.10.

If the five lowest frequency subcarriers in the E group, 79E, 63E, 47E, 31E, and 15E had been chosen with all the same required parameters such as $B_c = 3 \text{ MHz}$, then $[S/N]_{oi} = 50 \text{ dB}$. All things being equal, it would be

Table 3.10
Parameters for Design Example 6

Channel	f_{si} (kHz)	f_{dsi} (kHz)	f_{mi} (kHz)	A_j	f_{dci} (kHz)	D_{ci}	$[S/N]_{oi}$ dB
111E	896	32	6.4	1.0	362	0.4	43.6
95E	768	32	6.4	0.857	311	0.4	43.6
79E	640	32	6.4	0.714	259	0.4	43.6
63E	512	32	6.4	0.571	207	0.4	43.6
47E	384	32	6.4	0.428	155	0.4	43.6

desirable to use the lower-frequency subcarriers in this group, since they use less bandwidth while handling the same signal frequency; hence, for a specified bandwidth, the output signal-to-noise will be larger (see Problem 3.42).

3.8 Designing the Preemphasis Schedule for Different Values for the D_{si} 's

3.8.1 Design Equations and Procedures

All the design procedures above have assumed that the D_{si} 's would all be the same. Typically, all the D_{si} 's were assumed to be 5. However, in those design procedures, all the D_{si} 's could have been set to any value between 1 and 5 as long as they were all the same. A design procedure will now be given for developing the preemphasis schedule for different values of the D_{si} 's. Repeating (3.55) gives

$$A_i = \frac{f_{si} f_{ds1} (f_{mi})^{3/2}}{f_{s1} f_{dsi} (f_{m1})^{3/2}} \quad (3.60)$$

This can be written as

$$A_i = \frac{f_{si} f_{ds1} f_{mi} \sqrt{f_{mi}}}{f_{s1} f_{dsi} f_{m1} \sqrt{f_{m1}}} \quad (3.61)$$

$$A_i = \frac{f_{si} D_{s1} \sqrt{f_{mi}}}{f_{s1} D_{si} \sqrt{f_{m1}}}$$

Once the group of subcarriers have been chosen, (3.61) allows the A_i 's to be computed based upon the assigned value of the D_{si} 's rather than some fixed value. This procedure will design the preemphasis schedule based upon the specified output signal-to-noise ratio with the required bandwidth falling where it will. Once the A_i 's are calculated, the design procedure is the same as the first design procedure above. Any necessary modifications to the required bandwidth can be achieved as shown in the procedure in Section 3.6.

The design procedure steps are as follows:

1. Select the group of subcarriers compatible with the sensors.
2. Specify the required signal-to-noise in the output channels.
3. Specify the D_{si} 's for the individual subcarrier channels based upon the maximum f_{mi} 's the channels will be required to handle.
4. Calculate the A_i 's.
5. Calculate A_p .
6. Solve for f_{dc1} using the cubic (3.16).
7. Calculate the remaining f_{dci} 's using (3.17).
8. Calculate the required bandwidth, B_c .

This completes the design of the preemphasis schedule. A design example will be worked next.

Design Example 7

A design example will be worked for specified different D_{si} 's. It will be assumed that the sensor requirement will be satisfied by the 5 sensors of Design Example 6. The required output signal-to-noise ratio is specified at 43.3 dB; thus, the required bandwidth cannot be specified and will be a function of the computed carrier deviations by the subcarriers. The specifications and frequency requirements for the subcarriers are given in Table 3.11. Note that the lowest frequency channel, 47E, must handle a signal frequency of 32 kHz and will therefore have a mod index, D_{si} , of 1. Calculating the A_i 's based upon (3.61) gives the values listed in column 6, Table 3.11.

Calculating A_p as the square root of the sum of the squares of the A_i 's gives

$$A_p = 5.053$$

Table 3.11
Parameters for Design Example 7

Channel	f_{si}	f_{dsi}	f_{mi}	D_{si}	A_i	f_{dci}	D_{ci}
111E	896	32	6.4	5	1	286	0.32
95E	768	32	6.4	5	0.86	245	0.32
79E	640	32	6.4	5	0.71	204	0.32
63E	512	32	6.4	5	0.57	163	0.32
47E	384	32	32	1	4.79	1,370	3.57

Frequencies are given in kilohertz.

Solving the cubic (3.16) for f_{dc1} and (3.17) for the other f_{dci} 's results in the following carrier deviations given in column 7 of Table 3.11. Solving for f_{dn} as the square root of the sum of the squares of the f_{dci} 's gives

$$f_{dn} = 1.4 \text{ MHz}$$

The resulting D_{ci} 's are listed in column 8 of Table 3.11. The required IF bandwidth is given by

$$B_c = 2(f_{dc1}A_p + f_{s1}) = 4.69 \text{ MHz}$$

This is in contrast to the required bandwidth of 3 MHz for Design Example 5 for the same required output signal-to-noise ratio of 43.3 dB. The greater required bandwidth is the result of decreasing the mod index of the signal with respect to the subcarrier and making it up by increasing D_{c5} in order to keep an output signal-to-noise of 43.3 dB.

In essence, bandwidth has been exchanged for output signal quality. This will always occur in FM/FM as the D_{si} 's are reduced in order to handle higher signal frequencies and the D_{ci} 's are increased in order to maintain the output signal-to-noise ratios.

3.8.2 Summary of the Results of Design Example 7

The required transmission bandwidth is a function of which subcarrier is assigned to handle the higher signal frequency. Listed below in Table 3.12 are the required bandwidths, B_c , where four of the five subcarriers are assigned 6.4-kHz message bandwidths, while the fifth is given the 32-kHz message. Only the subcarrier modulated by the 32-kHz message will be listed. For CBW subcarriers, it appears that assigning the highest frequency message to the lowest frequency subcarrier will require the lowest overall transmission bandwidth.

Table 3.12
 B_c for Different Subcarriers Assigned the High-Frequency Message

Channel	111E	95E	79E	63E	47E
B_c	7.06	6.43	5.87	5.27	4.69

3.9 Designing the Preemphasis Schedule for the Minimum Transmission Bandwidth with Equal D_{si} 's (Concurrent All-Channel Dropout)

For a given set of subcarriers, this minimum transmission or IF bandwidth occurs when threshold happens simultaneously in both the carrier and subcarrier bandpass filters. This is referred to as concurrent all-channel dropout. The design procedure can be used to design the preemphasis schedule for a minimum transmission or IF bandwidth, B_c , resulting in concurrent all-channel dropout. Reducing the design transmission bandwidth further will endanger the data, since the receiver and subcarrier bandpass filters will be operating below threshold. All other parameters being fixed, reducing the required transmission bandwidth is achieved by reducing the required channel output signal-to-noise ratio, which will reduce the design f_{dci} 's and hence B_c . The threshold design margin, M_{ti} , will also be reduced. The minimum bandwidth achievable without threshold occurring will occur when the threshold design margin equals 0 dB. Alternatively, the minimum bandwidth will occur if the preemphasis schedule is designed such that threshold occurs simultaneously in all the bandpass filters.

3.9.1 Design Equations and Procedures

The mathematical equation for determining threshold in the subcarrier bandpass filters is (3.39), which is

$$[S/N]_{OBPi} = \left[\frac{1}{2} \frac{f_{dci}^2}{f_{si}^2} \frac{B_c}{B_i} \right] \left[\frac{A^2}{2\eta B_c} \right] \quad (3.62)$$

The term in the second bracket is the carrier-to-noise ratio in the carrier IF and by link design is set at a minimum of 12 dB, where threshold occurs. If the first term is made larger than 1, then the signal-to-noise in the subcarrier bandpass filter will be above 12 dB and the onset of threshold will occur later in this filter than in the carrier IF, which is desirable if there is transmission bandwidth available. However, if the system is being designed for minimum bandwidth utilization, it is necessary for the two thresholds to occur at the same time.

To insure the threshold occurred later in the subcarrier filter, the first term in (3.62) was set up as the inequality

$$\frac{1}{2} \frac{f_{dci}^2}{f_{si}^2} \frac{B_c}{B_i} \geq 1 \quad (3.63)$$

In Section 3.5, after the preemphasis design procedure was completed, this inequality was checked to determine the threshold design margin. On the other hand, if the equal sign is used in (3.68), the threshold design margin is 0 dB and the thresholds will occur simultaneously.

The procedure for designing the preemphasis schedule for a minimum bandwidth is developed next. The process is started by equating the terms in (3.63) to 1. Rearranging and noting that $B_i = 2(f_{dsi} + f_{mi})$ for large or small mod indexes gives

$$f_{dci}^2 B_c = 4f_{si}^2 (f_{dsi} + f_{mi}) \quad (3.64)$$

The problem in using (3.64) as the starting point in a design procedure after the i th index has been converted to the 1 index is that it is one equation in two unknowns, whereas (3.63) was used after the preemphasis schedule was designed, and the parameters were known and used for checking the inequalities. The way around this dilemma is to note the relationship in (3.11) between f_{dc1} and B_c :

$$B_c = 2[f_{dn} + f_{s1}] = 2[f_{dc1}A_p + f_{s1}] \quad (3.65)$$

The design procedure is the same as the procedure used in designing for a specified output signal-to-noise ratio in that once a group of subcarriers is selected, the A_i 's and A_p are calculated. Equation (3.65) is substituted into (3.64) to convert the equation into one equation in one unknown with the i th index replaced by the highest frequency index of 1. Rearranging and normalizing gives

$$f_{dc1}^3 + \frac{f_{s1}}{A_p} f_{dc1}^2 - 2 \frac{f_{s1}^2 (f_{ds1} + f_{m1})}{A_p} = 0 \quad (3.66)$$

This is a cubic equation, which must be solved for f_{dc1} . Using the A_i 's determined early, the other f_{dci} 's may be calculated, giving the preemphasis schedule which insures that all thresholds will occur simultaneously and, hence, will use the minimum transmission bandwidth. This cubic equation is for a design procedure for minimum bandwidth utilization, whereas the

earlier cubic equation was for a specified output signal-to-noise ratio and, in fact, had a signal-to-noise factor as a coefficient.

The design procedure for minimum bandwidth and concurrent all-channel dropout is as follows:

1. Select the group of subcarriers compatible with the sensors.
2. Specify the D_{si} 's for the individual subcarrier channels.
3. Calculate the A_i 's.
4. Calculate A_p .
5. Solve for f_{dc1} using the cubic (3.66).
6. Calculate the remaining f_{dci} 's using (3.17).
7. Calculate the signal-to-noise in the output channels.
8. Calculate the required bandwidth, B_c , from (3.65).

Design Example 8

The eight steps in Section 3.9.1 will be used to design for concurrent thresholds and, hence, a minimum transmission bandwidth.

1. Step 1 will be carried out by selecting the five constant bandwidth channels of Design Example 6 to demonstrate the minimum bandwidth design procedure.
2. All the D_{si} 's will be set to 5.
3. The A_i 's are the same as in Design Example 6.
4. From Design Example 6, $A_p = 1.66$.
5. Substituting the specific values of these parameters into the cubic (3.66) and solving gives

$$f_{dc1} = 221 \text{ kHz.}$$

6. Using the relationship $f_{dci} = A_i f_{dc1}$, gives the following values in kilohertz in Table 3.13.
7. The output signal-to-noise in all the channels is computed to be

$$[S/N]_{oi} = 38.53 \text{ dB.}$$

8. The required bandwidth, B_c and f_{dn} , is

$$f_{dn} = 367 \text{ kHz;}$$

$$B_c = 2.5 \text{ MHz.}$$

Table 3.13
Parameters for Design Example 8

Channel	f_{si} (kHz)	f_{dsi} (kHz)	f_{mi} (kHz)	A_i	f_{dci} (kHz)
111E	896	32	6.4	1.0	221
95E	768	32	6.4	0.857	189
79E	640	32	6.4	0.714	158
63E	512	32	6.4	0.571	126
47E	384	32	6.4	0.482	95

All the D_{ci} 's = 0.246.

B_c cannot be reduced further without having the receiver operate in threshold in the subcarrier bandpass filters if the carrier IF is operating at 12 dB.

In designing for all-channel dropout, regardless of the set of subcarriers chosen, whether using CBW or PBW subcarriers, the signal-to-noise output will be 38.5 dB if the D_{si} 's equal 5 and the carrier-to-noise ratio is 12 dB. This can be shown by setting B_i equal to $2(f_{dsi} + f_{mi})$ in (3.64) and solving for B_c which gives

$$B_c = \frac{4(f_{dsi} + f_{mi})}{D_{ci}^2} \quad (3.67)$$

Substituting (3.67) into (2.29) results in

$$[S/N]_{oi} = \frac{3}{4} D_{si}^2 D_{ci}^2 \frac{4(f_{dsi} + f_{mi})}{D_{ci}^2} \frac{1}{f_{mi}} [S/N]_c \quad (3.68)$$

$$[S/N]_{oi} = 3D_{si}^2(D_{si} + 1) [S/N]_c$$

Setting $D_{si} = 5$ and taking 10 log of the result will give 38.5 dB. Equation (3.68) shows that whenever the design margin is set equal to 0 dB for the concurrent threshold design procedure, the output signal-to-noise is a function of only D_{si} and the input carrier-to-noise ratio. For example, setting $D_{si} = 1$ gives a $[S/N]_c = 16.77$ dB, which is an unsatisfactory result. *Prudence must be exercised whenever an all-channel dropout design is initiated with the D_{si} 's less than 5.*

3.10 Summary of Design Examples 6 and 8

To put the results of these two examples into context and show the results of accepting a lower output signal-to-noise ratio for a smaller transmission bandwidth, the results of a preemphasis design example for a specified output signal-to-noise ratio, 46 dB, will also be given. The required bandwidth and design threshold margin resulting from this design will be given first:

1. $[S/N]_{oi} = 46$ dB;
 $B_c = 3.3$ MHz;
 $M_{ti} = 8.3$ dB.
2. $[S/N]_{oi} = 43.3$ dB;
 $B_c = 3.0$ MHz;
 $M_{ti} = 5.6$ dB.
3. $[S/N]_{oi} = 38.7$ dB;
 $B_c = 2.4$ MHz;
 $M_{ti} = 0$ dB.

From an observation of 1, 2, and 3, it can be seen that the higher the required output signal-to-noise ratio, the larger the transmission bandwidth. An implicit assumption in these examples is that the transmitted power may be increased whenever necessary in order to keep the carrier-to-noise ratio at 12 dB as the IF bandwidth is increased.

3.11 Designing the Preemphasis Schedule for All-Channel Dropout and Unequal D_{si} 's

In designing the preemphasis schedule for all-channel dropout, but with unequal D_{si} 's, (3.8) must be used before the signal-to-noise ratios are divided out because this design will create unequal output signal-to-noise ratios. Under these conditions, (3.8) for A_i becomes

$$A_i = \frac{f_{dci}}{f_{dc1}} = \frac{\sqrt{4/3} \frac{[S/N]_{oi}}{[S/N]_c} \frac{f_{si}}{f_{dsi}} \sqrt{f_{mi}^3}}{\sqrt{4/3} \frac{[S/N]_{o1}}{[S/N]_c} \frac{f_{s1}}{f_{ds1}} \sqrt{f_{m1}^3}} \quad (3.69)$$

Rearranging (2.31) gives

$$\frac{[S/N]_{oi}}{[S/N]_C} = \sqrt{3/4} D_{si} D_{ci} \sqrt{\frac{B_c}{f_{mi}}} \quad (3.70)$$

Substituting the value of B_c from (3.67) into (3.70) gives

$$\frac{[S/N]_{oi}}{[S/N]_C} = \sqrt{3} D_{si} \sqrt{1 + D_{si}} \quad (3.71)$$

Solving for the signal-to-noise ratios for the first channel in the same manner gives

$$\frac{[S/N]_{oi}}{[S/N]_C} = \sqrt{3} D_{s1} \sqrt{1 + D_{s1}} \quad (3.72)$$

Substituting (3.71) and (3.72) into (3.69) results in

$$A_i = \frac{D_{si}}{D_{s1}} \sqrt{\frac{1 + D_{si}}{1 + D_{s1}}} \frac{f_{si}}{f_{s1}} \frac{f_{ds1}}{f_{dsi}} \frac{\sqrt{f_{mi}^3}}{\sqrt{f_{m1}^3}} \quad (3.73)$$

The A_i 's can be computed, then A_p solved for f_{dc1} can be solved for from the cubic (3.66). The other f_{dci} 's may be solved for using the computed values of the A_i 's.

Design Example 9

An example will be worked for an all-channel dropout or concurrent threshold design with different specified D_{si} 's. The first three subcarriers of Design Example 8 will be used, given that we want channel 111E to handle a 32-kHz signal and the other two channels to handle 6.4-kHz signals. This information is summarized in Table 3.14.

The first step is to compute the A_i 's from (3.73). These are listed in column 6 of Table 3.14. Computing A_p gives

$$A_p = 1.32$$

Solving (3.66) for f_{dc1} gives

$$f_{dc1} = 284 \text{ kHz}$$

Table 3.14
Parameters for Design Example 9

Channel	f_{si}	f_{dsi}	f_{mi}	D_{si}	A_j	f_{dci}	D_{ci}	M_{ti}	S/N_o
111E	896	32	32	1	1.0	284	0.32	0	19.7
95E	768	32	6.4	5	0.66	189	0.24	0	38.5
79E	640	32	6.4	5	0.55	157	0.24	0	38.5

Note: $B_c = 2.5$ MHz.

Because $f_{dci} = A_i f_{dc1}$, these deviations may be solved for and are listed in column 7 of Table 3.14, and the D_{ci} 's may then be computed. The threshold margin in each channel is 0 dB. Notice that the output signal-to-noise ratios in the channels vary, and the ratio is probably unacceptably low in the 111E channel. This is a result of requiring D_{s1} to be 1. Again it should be noted that requiring small D_{si} 's in channels for the all-channel dropout design will result in small output signal-to-noise ratios.

3.12 Designing the Preemphasis Schedule for Different Specified Signal-to-Noise Ratios in the Channels

The equations and the process for designing the preemphasis schedule for different specified signal-to-noise ratios will be given. Since different signal-to-noise ratios are to be specified, (3.69) must be used to calculate the A_i 's. Rearranging gives

$$A_i = \frac{[S/N]_{oi} f_{si} f_{ds1} \sqrt{f_{mi}^3}}{[S/N]_c f_{s1} f_{dsi} \sqrt{f_{m1}^3}} \quad (3.74)$$

Once the A_i 's are calculated using (3.74), which has the i th channel specified signal-to-noise ratio as a factor, all the steps given in Section 3.2 are appropriate. These steps will be repeated in this section in working an example.

Design Example 10

Design a preemphasis schedule for the three CBW subcarriers of Design Example 9 with the same D_{si} 's all equal to 5, but with different specified signal-to-noise ratios. The steps are as follows:

1. Select the set of subcarriers to be used.
2. Specify the different $[S/N]_{oi}$'s.
3. Calculate the A_i 's using (3.74).
4. Calculate A_p .
5. Solve (3.16) for f_{dc1} .
6. Complete the preemphasis schedule by solving for the remaining f_{dci} 's using $f_{dci} = f_{dc1} A_i$.
7. Compute B_c .

The specified $[S/N]_{oi}$'s are given in column 4 of Table 3.15. The computed A_i 's are given in column 6 and the preemphasis schedule in column 7. The required B_c is 3.6 MHz.

The design was also done for the same subcarrier channels, but with all the $[S/N]_{oi}$'s specified at 40 dB. The resulting f_{dci} 's and D_{ci} 's are given in columns 9 and 10 respectively. The required B_c is 2.56 MHz. Note that requiring channel 111E to have an output signal-to-noise of 49 dB adds an additional 1 MHz of required bandwidth.

Note that the additional bandwidth requires additional transmitted power in order to maintain an acceptable carrier-to-noise ratio in the IF. This example was not worked with fixed transmitter power.

If the three lowest frequencies subcarriers in the E -channels are used, and the D_{si} 's = 5 and $[S/N]_{oi} = 40$ dB, the required bandwidth will be less. For this case, $B_c = 1.17$ MHz. All things being equal, and since the lowest frequency subcarriers will handle the same signal frequencies as the higher subcarriers, the lowest frequency subcarriers would be the subcarriers of choice.

3.13 Hardware Implementation of the Preemphasis Schedule

Once the deviation schedule of the carrier with respect to the subcarriers is determined, the next step is to implement the schedule in hardware. An S-band transmitter might have a deviation sensitivity such as 500 kHz/V RMS.

A typical maximum voltage output of the subcarrier modulator, referred to as the subcarrier oscillator (SCO), would be 0.325V RMS operating into 10K ohms. Off-the-shelf SCOs with output amplitude control can have the output signal voltage set without affecting the SCO's output impedance. This special feature allows the user to precisely set the outputs of each SCO for the desired system preemphasis. With the amplitude pin open-circuited, the signal output amplitude is a maximum, and with the amplitude pin connected to ground, the signal output amplitude is zero. The desired signal output amplitude is obtained by connecting a selected resistor between the amplitude pin and ground. The resistor size may be obtained from Figure 3.1.

Design Example 11

Say we want to implement the preemphasis schedule given in Table 3.8 for the new f_{dci} 's. The maximum output voltage from the SCO will cause the following deviation

Table 3.15
Parameters for Design Example 10

Channel	f_{si}	f_{mi}	S/N_o	D_{si}	A_j	f_{dci}	D_{ci}	f_{dci}	D_{ci}
111E	896	6.4	49dB	5	1.0	615	0.68	259	0.29
95E	768	6.4	40dB	5	0.85	527	0.68	222	0.29
79E	640	6.4	40dB	5	0.71	439	0.68	185	0.29

Note: $B_c = 3.6$ MHz (different $[S/N]_{oi}$'s). $B_c = 2.56$ MHz (all $[S/N]_{oi}$'s = 40 dB).

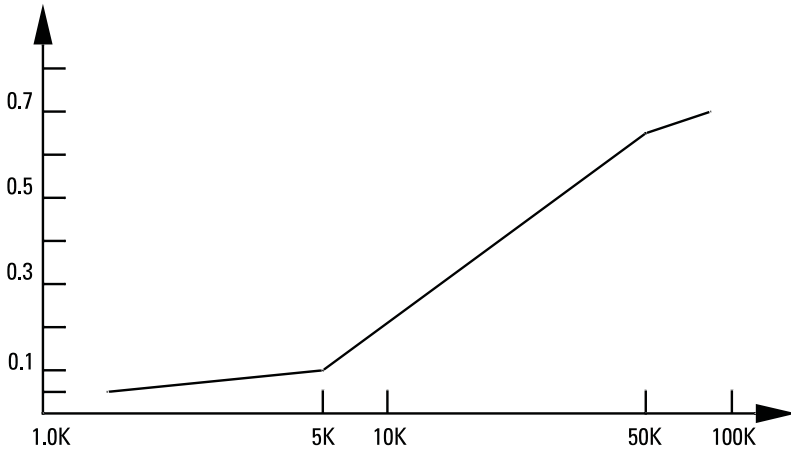


Figure 3.1 Relative amplitude of output for values of amplitude resistor.

$$f_{dc\max} = (500 \text{ kHz}) \left(\frac{0.325}{1} \right) = 162.5 \text{ kHz}$$

Now say we want to set the deviation of the 93-kHz subcarrier at 119 kHz.

Therefore, the output voltage should be set to

$$v_{o93} = \left(\frac{119}{162.5} \right) (0.325) = (0.73)(0.325) = 0.238$$

The ratio of the desired voltage to the max is

$$R_{93} = \frac{0.238}{0.325} = 0.73$$

From the graph in Figure 3.1, it can be seen that the 0.73 ratio requires about a 100-k resistor to be connected between the amplitude pin and the ground. Inspection of the equations shows that the ratio to be determined in order to go to the graph is the ratio of the desired deviation to the maximum obtainable with the maximum output voltage for the particular SCO. The process is repeated until all resistor sizes are determined.

Specifically, for the 8.8-kHz deviation of the lower frequency subcarriers, say the 1.3-kHz subcarrier, the ratio is given by

$$R(N)_{1.3} = \frac{8.8}{162.5} = 0.054$$

Then, from the graph of Figure 3.1, the resistor size would be approximately $R_{1.3} = 1.2\text{K}$. From Figure 3.1 it can be seen that this is about the smallest resistor size that can be read accurately from the figure.

3.14 Summary of Design Procedures

The design of the preemphasis schedule for various options was studied. Systematic design procedures were developed for the various options. Design procedures were developed for three main options: (1) specified output signal-to-noise ratios, (2) a specified transmission bandwidth, and (3) concurrent-all-channel dropout. There were a number of modifications to these three options, such as different D_{st} 's for the channels and different specified output signal-to-noise ratios.

The transmission bandwidth used was based upon the root mean square of the carrier deviations by the subcarriers. This is neither the 99% bandwidth nor the -60-dB bandwidth discussed in the *Telemetry Standards* document IRIG 106-00 [6]. However, extensive investigations by [4] indicate that there is less than 1% of the sideband power outside this bandwidth. It seems to be a good estimate of the 99% bandwidth, but this has not been verified in an IRIG document.

The equation for the FM/FM channel output signal-to-noise ratio used is the one from the document IRIG 106-00. This equation does not include the degradation due to bandedge effects, nonidealized filters, intermodulation products, or harmonics from one channel falling in another. For this type of analysis, the reader is referred to [3, 7].

3.15 IRIG B_{IF} Specifications

Some receiver bandwidths available at the ranges are shown in Table 3.16. Statements from IRIG 106-00 concerning channel spacing for transmission are as follows.

Narrowband telemetry channel spacing is in increments of 1 MHz, beginning with the frequencies 1,435.5; 2,200.5; and 2,310.5 MHz. These numbers will be used as the base from which all frequency assignments are to be made. Medium bandwidth channels, utilizing 1 to 3 MHz and wideband channels using 3 to 10 MHz are permitted and will be centered on the center frequency of the narrowband channels.

Table 3.16
IRIG IF Bandwidths at the Ranges

B_{IF}
300 kHz
500
750
1.0 MHz
1.5
2.4
3.3
4.0
6.0
10.0

Preliminary Problems

Preliminary Problem 3.1

Using the three CBW channels, 111E, 95E, and 79E, design the preemphasis schedule with specifications as follows:

$$[S/N]_c = 12 \text{ dB (3.98 numerical);}$$

$$D_{si} = f_{dsi}/f_{mi} = 5 \text{ (nominal);}$$

$$[S/N]_{oi} = 40.$$

Answer: $A_p = 1.49$, $f_{dc1} = 259 \text{ kHz}$, $f_{dn} = 388 \text{ kHz}$, and $B_c = 2.57 \text{ MHz}$.

Preliminary Problem 3.2

For Preliminary Problem 3.1, compute D_{ci} for all the channels.

Answer: $D_{ci} = 0.289$ in all channels.

Preliminary Problem 3.3

Compute M_{ti} in decibels for all the channels for Preliminary Problem 3.1.

Answer: 1.46 dB in all channels.

Preliminary Problem 3.4

Using the five PBW channels, 15, 16, 17, 18, and 19, design the preemphasis schedule with specifications as follows:

$$[S/N]_c = 12 \text{ dB (3.98 numerical)};$$

$$D_{si} = f_{dsi}/f_{mi} = 5 \text{ (nominal)};$$

$$[S/N]_{oi} = 43.5.$$

Answer: $A_p = 1.31$, $f_{dc1} = 66.8 \text{ kHz}$, $f_{dn} = 87.6 \text{ kHz}$, $B_c = 361 \text{ kHz}$.

Preliminary Problem 3.5

Using the five CBW channels, 111E, 95E, 79E, 63E, and 47E, design the preemphasis schedule with specifications as follows:

$$[S/N]_c = 12 \text{ dB (3.98 numerical)};$$

$$D_{si} = f_{dsi}/f_{mi} = 5 \text{ (nominal)};$$

$$[S/N]_{oi} = 49.$$

Answer: $A_p = 1.66$, $f_{dc1} = 602 \text{ kHz}$, $f_{dn} = 999 \text{ kHz}$, $B_c = 3.7 \text{ MHz}$.

Preliminary Problem 3.6

Using the five lowest frequency subcarrier CBW E-channels, 79E, 63E, 47E, 31E, and 15E, design the preemphasis schedule with specifications as follows:

$$[S/N]_c = 12 \text{ dB (3.98 numerical)};$$

$$D_{si} = f_{dsi}/f_{mi} = 5 \text{ (nominal)};$$

$$[S/N]_{oi} = 49.$$

Compare with Preliminary Problem 3.5.

Answer: $A_p = 1.48$, $f_{dci} = 502, 402, 302, 201, 100 \text{ kHz}$, $f_{dn} = 745 \text{ kHz}$, and $B_c = 2.7 \text{ MHz}$.

Regular Design Problems

Problem 3.1

Using the 15 PBW 7.5% channels, 5 through 19, and these specifications: $[S/N]_c = 12 \text{ dB (3.98 numerical)}$ and $D_{si} = f_{dsi}/f_{mi} = 5 \text{ (nominal)}$, (a) design the system for $[S/N]_{oi} = 49 \text{ dB (281 numerical)}$ minimum in all channels and complete Tables 3.1 and 3.2, and (b) compute f_{dn} and B_c .

Problem 3.2

Using the 15 PBW 7.5% channels, 5 through 19, and these specifications: $[S/N]_c = 12$ dB (3.98 numerical) and $D_{si} = f_{dsi}/f_{mi} = 5$ (nominal), (a) design the system for $[S/N]_{oi} = 43$ dB (141 numerical) minimum in all channels and complete Tables 3.1 and 3.2, and (b) compute f_{dn} and B_c .

Problem 3.3

Using the 15 PBW 7.5% channels, 5 through 19, and these specifications: $[S/N]_c = 12$ dB (3.98 numerical) and $D_{si} = f_{dsi}/f_{mi} = 5$ (nominal), (a) design the system for $[S/N]_{oi} = 40$ dB (100 numerical) minimum in all channels and complete Tables 3.1 and 3.2, and (b) compute f_{dn} and B_c .

Problem 3.4

Using the 15 PBW 7.5% channels, 5 through 19, and these specifications: $[S/N]_c = 12$ dB (3.98 numerical) and $D_{si} = f_{dsi}/f_{mi} = 5$ (nominal), (a) design the system for $[S/N]_{oi} = 38$ dB (79 numerical) minimum in all channels and complete Tables 3.1 and 3.2, and (b) compute f_{dn} and B_c .

Problem 3.5

Using the 15 PBW 7.5% channels, 5 through 19, and these specifications: $[S/N]_c = 12$ dB (3.98 numerical) and $D_{si} = f_{dsi}/f_{mi} = 5$ (nominal), (a) design the system for $[S/N]_{oi} = 36$ dB (63 numerical) minimum in all channels and complete Tables 3.1 and 3.2, and (b) compute f_{dn} and B_c .

Problem 3.6

For Problem 3.1, increase the deviation of the carrier by all subcarriers to be at least 10% of f_{dn} and (a) compute the new $[S/N]_{oi}$ for the channels with the changed deviation of the carrier by the subcarriers; then (b) decrease the D_{si} to 2.5 on the channels with the increased deviation and compute the new $[S/N]_{oi}$.

Problem 3.7

Compute the new B_c , for Problem 3.6.

Problem 3.8

(a) For Problem 3.1 and Problem 3.6(b), double the frequency of the message modulating the subcarriers such that the new $D_{si} = 2.5$ on all channels and

calculate the new $[S/N]_{oi}$; then (b) determine a new preemphasis schedule for Problem 3.1 by doing the complete design starting with a required $D_{si} = 2.5$. Compute the new B_c .

Problem 3.9

For Problems 3.1 through 3.5, check the design parameters to determine if the inequality of (3.43) is satisfied.

Problem 3.10

Determine the threshold margin for all channels for Problems 3.1, 3.2, 3.3, 3.4, and 3.5.

Problem 3.11

Using the 14 CBW channels, 6A through 19A, and these specifications: $[S/N]_c = 12$ dB (3.98 numerical) and $D_{mi} = f_{dsi}/f_{mi} = 5$ (nominal), (a) design the system for $[S/N]_{oi} = 49$ dB (281 numerical) minimum in all channels and complete Tables 3.1 and 3.2, and (b) compute f_{dn} and B_c .

Problem 3.12

Using the 14 CBW channels, 6A through 19A, and these specifications $[S/N]_c = 12$ dB (3.98 numerical) and $D_{mi} = f_{dsi}/f_{mi} = 5$ (nominal), (a) design the system for $[S/N]_{oi} = 43$ dB (141 numerical) minimum in all channels and complete Tables 3.1 and 3.2, and (b) compute f_{dn} and B_c .

Problem 3.13

Using the 14 CBW channels, 6A through 19A and these specifications: $[S/N]_c = 12$ dB (3.98 numerical) and $D_{mi} = f_{dsi}/f_{mi} = 5$ (nominal), (a) design the system for $[S/N]_{oi} = 40$ dB (100 numerical) minimum in all channels and complete Tables 3.1 and 3.2, and (b) compute f_{dn} and B_c .

Problem 3.14

Using the 14 CBW channels, 6A through 19A, and these specifications: $[S/N]_c = 12$ dB (3.98 numerical) and $D_{mi} = f_{dsi}/f_{mi} = 5$ (nominal), (a) design the system for $[S/N]_{oi} = 38$ dB (79 numerical) minimum in all channels and complete Tables 3.1 and 3.2, and (b) compute f_{dn} and B_c .

Problem 3.15

Using the 14 CBW channels, 6A through 19A and these specifications: $[S/N]_c = 12$ dB (3.98 numerical) and $D_{mi} = f_{dci}/f_{mi} = 5$ (nominal), (a) design the system for $[S/N]_{oi} = 36$ dB (63 numerical) minimum in all channels and complete Tables 3.1 and 3.2, and (b) compute f_{dn} and B_c .

Problem 3.16

Using Figure 3.1, determine the size of the resistors on the amplitude pin of the SCOs necessary to implement the deviations of the carrier by the subcarriers as indicated in Table 3.5. Assume the sensitivity of the transmitter is $k = 500$ kHz/V RMS and that the maximum voltage out from the VCOs is 0.325 RMS.

Problem 3.17

Using Figure 3.1, determine the size of the resistors on the amplitude pins of the SCOs necessary to implement the preemphasis schedule of Problem 3.11. Assume the sensitivity of the transmitter is $k = 500$ kHz/V RMS and that the maximum voltage out from the VCOs is 0.325 RMS.

Problem 3.18

Determine the threshold design margins for Problems 3.11, 3.12, 3.13, 3.14, and 3.15.

Problem 3.19

For Problem 3.11, design the system such that $B_c = 500$ kHz; then (a) list the new f_{dci} 's and (b) compute the new $[S/N]_{oi}$'s.

Problem 3.20

Design a preemphasis schedule for eight of the 15% channels using alternate channels starting with channel A, given the design specifications: $[S/N]_c = 12$ dB and $[S/N]_{oi} = 46$ dB. Start with a nominal $D_{si} = 5$. (a) Find f_{dn} and B_c and (b) redesign the preemphasis package such that B_c is designed to the nearest integer MHz bandwidth from part (b).

Problem 3.21

(a) Design a preemphasis schedule for three of the 30% channels using AA, FF, and KK. $[S/N]_c = 12$ dB, $[S/N]_{oi} = 46$ dB, and $D_{si} = 5$. (b) Find f_{dn} and B_c . (c) Redesign the preemphasis schedule such that B_c is designed to the nearest integer MHz bandwidth from part (b).

Problem 3.22

(a) Design a preemphasis schedule for the 11 CBW B channels. $[S/N]_c = 12$ dB, $[S/N]_{oi} = 46$ dB, and $D_{si} = 5$. (b) Find f_{dn} and B_c . (c) Design the preemphasis schedule such that B_c is designed to the nearest integer MHz.

Problem 3.23

Design a preemphasis schedule for six of the CBW B channels using 13B through 23B. $[S/N]_c = 12$ dB, $[S/N]_{oi} = 46$ dB, and $D_{si} = 5$. Find f_{dn} and B_c .

Problem 3.24

Design a preemphasis schedule for 15 of the 7.5% PBW channels 5 through 19, and 6 of the B CBW channels using 13B through 23B. $[S/N]_c = 12$ dB, $[S/N]_{oi} = 46$ dB, and $D_{si} = 5$. (a) Find f_{dn} and B_c and compare the B_c of this schedule with that of Problem 3.23 before the 15 PBW channels were added.

Problem 3.25

For Problem 3.24, modify the preemphasis schedule of the lower-frequency subcarriers such that all deviations are at least 10% of f_{dn} . Compute the new f_{dn} , and new B_c . Compute the new $[S/N]_{oi}$ for the subcarriers with the changed f_{dcs} .

Problem 3.26

For Problem 3.25, change the D_{si} of the modified subcarriers to 2.5 and calculate the new $[S/N]_{oi}$'s.

Problem 3.27

For Problem 3.26, calculate all the threshold margins.

Problem 3.28

Show that the all-channel dropout design procedure will give a $[S/N]_{oi} = 38.5$ dB by solving (3.69) for B_c and substituting the resulting equation into (2.27). Set $B_i = 2(f_{dsi} + f_{mi})$ and $D_{si} = 5$ and $[S/N]_c = 12$ dB.

Problem 3.29

Design a preemphasis schedule for the five channels of Preliminary Problem 3.5, except let $[S/N]_{oi} = 43.5$ and $D_{s1} = 1$. Let all the other D_{si} 's = 5. Answer: $A_p = 1$, $f_{dc1} = 2.6$ MHz, and $B_c = 7.06$ MHz.

Problem 3.30

Repeat Problem 3.29, except set $D_{s2} = 1$. Let all the other D_{si} 's = 5. Answer: $A_p = 9.6$, $f_{dc1} = 240$ kHz, $f_{dc2} = 2.3$ MHz, and $B_c = 6.4$ MHz.

Problem 3.31

Repeat Problem 3.29, except set $D_{s3} = 1$. Let all the other D_{si} 's = 5. Answer: $A_p = 8.1$, $f_{dc1} = 0.251$ MHz, $f_{dc2} = 0.21$ MHz, $f_{dc3} = 2$ MHz, and $B_c = 5.8$ MHz.

Problem 3.32

Repeat Problem 3.29, except set $D_{s5} = 1$. Let all the other D_{si} 's = 5. Answer: $A_p = 5.0$, $f_{dc1} = 0.287$ MHz, $f_{dc2} = 0.25$ MHz, $f_{dc3} = 0.2$ MHz, $f_{dc5} = 1.37$ MHz, and $B_c = 4.7$ MHz.

Problem 3.33

Repeat Problem 3.29, except set $D_{s5} = 2$. Let all the other D_{si} 's = 5. Answer: $A_p = 2.33$, $f_{dc1} = 338$ kHz, $f_{dc2} = 0.290$ MHz, $f_{dc5} = 0.57$ MHz, and $B_c = 3.37$ MHz.

Problem 3.34

Repeat Problem 3.33, except set $D_{s1} = 2$. Let all the other D_{si} 's = 5. Answer: $A_p = 1$, $f_{dc1} = 1.18$ MHz, $f_{dc2} = 0.25$ MHz, $f_{dc5} = 0.12$ MHz, and $B_c = 4.29$ MHz.

Problem 3.35

Using the three CBW channels, 111E, 95E, and 79E, design the preemphasis schedule for all-channel dropout, that is, set $M_{ti} = 1$ or 0 dB. This will also give the minimum transmission bandwidth. Specifications are as follows: $[S/N]_c = 12$ dB (3.98 numerical) and $D_{si} = f_{dsi}/f_{mi} = 5$ (nominal).

Answer: $A_p = 1.49$, $f_{dc1} = 0.224$ MHz, $f_{dn} = 335$ kHz, $B_c = 2.46$ MHz, and $D_{ci} = 0.25$.

Problem 3.36

For Problem 3.35, compute $[S/N]_{oi}$ for all the channels.

Answer: $[S/N]_{oi} = 38.5$ dB.

Problem 3.37

Repeat Problem 3.35, except set $D_{si} = 1$ in all the channels.

Answer: $A_p = 1.49$, $f_{dc1} = 0.279$ MHz, $f_{dn} = 418$ kHz, $B_c = 2.6$ MHz, and $D_{ci} = 0.31$.

Problem 3.38

For Problem 3.37, compute $[S/N]_{oi}$ for the channels.

Answer: $[S/N]_{oi} = 19.7$ dB. Note that requiring the channels to handle 32 kHz of message bandwidth, effectively setting $D_{si} = 1$, gives an unsatisfactory $[S/N]_{oi}$ when designing for all-channel dropout.

Problem 3.39

For the three CBW channels 111E, 95E, and 79E, and required $[S/N]_{oi}$ ratios equal to 49 dB, 40 dB, and 40 dB, respectively, and with these specifications: $[S/N]_c = 12$ dB (3.98 numerical) and $D_{si} = f_{dsi}/f_{mi} = 5$, (a) design the preemphasis schedule, (b) determine B_c , (c) find D_{ci} , and (d) compute M_{ti} .

Answer: $A_p = 1.5$, $f_{dc1} = 0.614$ MHz, $f_{dc2} = 0.527$ MHz, $f_{dc3} = 0.439$ MHz, $B_c = 3.6$ MHz, $D_{ci} = 0.686$, and $M_{ti} = 10.46$ dB.

Problem 3.40

In Design Example 10, $B_c = 2.56$ MHz was given for the three subcarriers when a specified $[S/N]_{oi} = 40$ was used. Show this to be true. Also find the f_{dci} 's.

Answer: f_{dci} 's = 0.289 MHz.

Problem 3.41

At the end of Design Example 10, it was pointed out that if the three lowest frequency subcarriers in the E-channels were used, $B_c = 1.17$ would be required. Show this to be true. Also find the f_{dci} 's.

Answer: 164, 109, and 54.7 kHz.

Problem 3.42

Work Design Example 6 using the five lowest frequency subcarriers in the E group and show $[S/N]_{oi} = 50.8$. Explain. Find D_{ci} .

Answer: 0.9.

Problem 3.43

Using the five CBW channels, 111E, 95E, 79E, 63E, and 47E, design the preemphasis schedule with specifications as follows:

$$[S/N]_c = 12 \text{ dB (3.98 numerical);}$$

$$D_{si} = f_{dsi}/f_{mi} = 5 \text{ (nominal) in the 3 highest frequency channels;}$$

$$D_{si} = 1 \text{ in the other two;}$$

$$[S/N]_{oi} = 46 \text{ dB.}$$

Answer: $A_p = 8.12$. $f_{dci} = 315$; 270; 225; 2,000; and 1,500 kHz. $f_{dn} = 2.5$ MHz. $B_c = 6.9$ MHz.

Problem 3.44

Observing the new nomenclature and using the five CBW F-channels, 3,840F; 3,584F; 3,328F; 3,584F; and 3,840F, design the preemphasis schedule with specifications as follows:

$$[S/N]_c = 12 \text{ dB (3.98 numerical);}$$

$$D_{si} = f_{dsi}/f_{mi} = 2.5 \text{ (nominal);}$$

$$[S/N]_{oi} = 46 \text{ dB.}$$

Answer: $A_p = 1.95$. $f_{dci} = 3,175$; 2,963; 2,752; 2,540; and 2,328 kHz. $f_{dn} = 6.190$ MHz. $B_c = 20$ MHz.

Problem 3.45

Using the channels and specified parameters in Problem 3.44, except $[S/N]_{oi} = 46$ dB, design the preemphasis schedule for a specified $B_c = 12,000$ kHz.

Answer: $A_p = 1.95$. $f_{dci} = 1,108; 1,034; 960; 886; \text{ and } 813$ kHz. $D_{ci} = 0.287$. $[S/N]_{oi} = 34.6$ dB.

Problem 3.46

Using the channels and specified parameters of Problem 3.44, except $[S/N]_{oi} = 46$ dB, design the preemphasis schedule for all channel dropout or minimum transmission bandwidth.

Answer: $A_p = 1.95$. $f_{dci} = 711, 663, 616, 568, \text{ and } 521$ kHz. $f_{dn} = 1.38$ MHz. $D_{ci} = 0.185$. $[S/N]_{oi} = 30.2$ dB.

Problem 3.47

Observing the new nomenclature and using the lowest five constant bandwidth F-channels, 256F; 512F; 768F; 1,024F; and 1,280F, design the preemphasis schedule with specifications as follows:

$$[S/N]_c = 12 \text{ dB (3.98 numerical);}$$

$$D_{si} = f_{dsi}/f_{mi} = 2.5 \text{ (nominal);}$$

$$[S/N]_{oi} = 46 \text{ dB.}$$

Problem 3.48

Using the channels and specified parameters of Problem 3.47, except $[S/N]_{oi} = 46$ dB, design the preemphasis schedule for a specified $B_c = 12,000$ kHz.

Problem 3.49

Using the channels and specified parameters of Problem 3.47, except $[S/N]_{oi} = 46$ dB, design the preemphasis schedule for all-channel dropout or minimum transmission bandwidth.

Problem 3.50

Observing the new nomenclature and using the five CBW G-channels, 3,584G; 3,072G; 2,560G; 2,048G; and 1,536G, design the preemphasis schedule with specifications as follows:

$$[S/N]_c = 12 \text{ dB (3.98 numerical)};$$

$$D_{si} = f_{dci}/f_{mi} = 5 \text{ (nominal)};$$

$$[S/N]_{oi} = 46 \text{ dB.}$$

Answer: $A_p = 1.66$. $f_{dci} = 1,821; 1,565; 1,304; 1,043$; and 282 kHz. $f_{dn} = 3.02$ MHz, $D_{ci} = 0.5$. $B_c = 13.2$ MHz.

Problem 3.51

Using the channels and specified parameters of Problem 3.50, except $[S/N]_{oi} = 46$ dB, design the preemphasis schedule for a specified $B_c = 12,000$ kHz.

Answer: $A_p = 1.66$. $f_{dci} = 1,459; 1,254; 1,045; 836$; and 627 kHz. $D_{ci} = 0.408$. $[S/N]_{oi} = 43.6$ dB.

Problem 3.52

Using the channels and specified parameters of Problem 3.50, except $[S/N]_{oi} = 46$ dB, design the preemphasis schedule for all-channel dropout or minimum transmission bandwidth.

Answer: $A_p = 1.66$. $f_{dci} = 882, 758, 632, 505$, and 379 kHz. $f_{dn} = 1.47$ MHz. $D_{ci} = 0.247$. $B_c = 10$ MHz. $[S/N]_{oi} = 38.5$ dB.

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