Introduction

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When the English mathematician Henry Briggs learned in 1616 of the invention of logarithms by John Napier, he determined to travel the four hundred miles north to Edinburgh to meet the discoverer and talk to him in person. The meeting of Briggs and Napier is one of the great tales in the history of mathematics. According to William Lily, who had it from Napier's friend John Marr, it happened when Napier had given up hope of seeing his long-awaited southern guest:

> It happened one day as John Marr and Lord Napier were speaking of Mr. Briggs "Ah John", saith Marchiston, "Mr. Briggs will not come." At the very instant one knocks at the gate. John Marr hastened down, and it proved Mr Briggs, to his great contentment. He brings Mr. Briggs to my Lord's chamber, where almost one quarter of an hour was spent each beholding the other with admiration, before one spoke: at last Mr. Briggs began: "My lord, I have undertaken this long Journey purposely to see your Person, and to know by what Engine of Wit or Ingenuity you came first to think of this most excellent Help unto Astronomy, *viz.*, the *Logarithms*; but, my Lord, being by you found out, I wonder nobody else found it out before, when now known it is so easy." He was nobly entertained by Lord Napier, and every summer after this, during Lord Napier's being alive, this venerable man, Mr. Briggs, went to Scotland to visit him.

The many layers of significance of this story make it an invaluable resource for mathematics teachers at all levels. For younger pupils, the idea of two grown men sitting looking at each other in silence for fifteen minutes on first meeting is sufficiently strange to provoke mirth and a vivid sense of how important mathematical ideas were to them. Pupils need no knowledge of logarithms to recognise from this that mathematics is something which has been invented by people at particular stages of history, not something which has always been there. Questions arise for young pupils too about the practicalities of life in old times, about travelling long distances as well as how before the days of photographs and television people generally had no accurate idea of what each other looked like unless they met in the flesh. Some may notice that in ancient times people were sometimes called by their name ('Napier') and sometimes by where they lived ('Marchiston'), as Napier lived in a castle called Merchiston Castle. It is in elementary and middle school, too, that teachers can introduce pupils to another of Napier's inventions, his 'rods' or 'bones' for speeding up multiplication. These lay bare the structure of multiplying in the decimal place-value numeral system (Hindu-Arabic numbers, as we call them) in a way which deepens student understanding and memorisation of the process.

Older pupils who are beginning to learn about logarithms are reinforced in understanding their importance, through reflecting on the lengths to which Briggs went in wanting to meet and admire their discoverer. Or were logarithms invented, not discovered? Teachers can explain how arduous calculations could be before logarithms, and tell pupils of Kepler's remark that thanks to Napier the astronomer's life-span had been doubled. This invention is a microcosm of the activity of mathematicians down the ages: the *point* of mathematics is to make things happen more easily and to save people trouble. (This revelation will be quite surprising to some pupils!—or at least to their parents with unhappy memories of their school mathematics lessons.) The possible benefits of the story work on a number of levels. Once students know, for example, how happy the astronomer was when multiplication of two ten-digits numbers reduced to a simple addition, they will never have a problem in remembering which is the correct rule: $\log ab = \log a + \log b$, not $\log(a+b) = \log a \times \log b$.

Senior students will begin to recognise just how significant logarithms are: that a device for easing the activity of calculating turns out to be one of the most influential and far-reaching of ideas in all of mathematics, a function of immense power and reach which pulls together ideas from different areas of mathematics. This illuminates another general truth about the amazing power of mathematics, the way different parts of it reinforce each other. Here, it is little short of miraculous how ideas from ancient Greece (curves from slicing cones, called conic sections), from early seventeenth century Scotland, and from later in the seventeenth century (a general method for finding the areas bounded by curves) all come together to generate a complex of mathematics of great power, and the student who is trained to understand and share in these ideas is immensely empowered as a result.

Trainee teachers reflecting on the story can absorb all these resonances and also notice what the story of Briggs's meeting with Napier tells us about the psychology of learning mathematics: it is every pupil's experience that once some difficult idea has been learned it seems so natural that you cannot understand why you did not understand it before! The concept of an "Engine of Wit or Ingenuity" is a very deep one. The apparent tension in this phrase between mechanical and psychological images is characteristic of the seventeenth century, prefiguring perhaps the 'mechanical philosophy' promoted by René Descartes and others a few decades later.

There are lessons for those designing mathematics education syllabuses too. The curriculum designer will appreciate that an apparently straightforward observation made by several mathematicians from Archimedes onwards, that multiplying numbers can correspond to adding powers of another number, or more simply that geometrical and arithmetical series can run in parallel, took many centuries to be recognised as a key perception to build upon for calculational purposes. The curricular implications may be (put in a rather general way) that what seems simple after the event can pose difficulties for students until they are prepared for new ways of looking at things.

This one short tale from four centuries ago can in this way be seen to lay the grounding for a number of valuable interactions between teacher and student in the mathematics classroom over several school years. A teacher able to support, encourage and lead students in this way through their school career is a better teacher: better prepared, better resourced, more empowered. History, we might say, is an Engine of Mathematical Wit. This story, and the pedagogical reflections which it generates, are to this extent a microcosm of what we hope the present book will achieve.

The background to this study

Does history of mathematics have a role in mathematics education? This book has been made by people who believe that the answer is positive, that the history of mathematics can play a valuable role in mathematical teaching and learning. It is the report of a study instigated by the International Commission on Mathematical Instruction (ICMI). We describe later how the study was carried out, but first sketch the problem setting of the study, the general background of concerns from several quarters which have led to a flourishing of work in this area in recent decades.

Mathematicians, historians and educators in many countries have long thought about whether mathematics education can be improved through incorporating the history of mathematics in some way. This arises from the recognition that mathematics education does not always meet its aims for all pupils, and that so long as some students emerge from their education with less understanding of mathematics than might be useful for them, or indeed with an actual fear or phobia about mathematics, then it is worth exploring possible avenues for improving the process. Nor have they only thought about the possibility of using history; many teachers in classrooms across the world have tried out various pedagogic possibilities. It soon emerges that there is a wide range of views and experiences of how history of mathematics can help. Some educators believe that mathematics is intrinsically historical: so learning the subject must involve its history, just as studying art involves learning about art history. Others see a number of ways in which history can aid the teacher's, and thus the learner's, task, from the apparently banal (such as giving more information about the names students may meet—which, by the way, are often wrong attributions in any case, as in the cases of Pascal's triangle and L'Hôpital's rule, not to speak of Pythagoras' theorem) to a deeper way of teaching mathematics in a historical vein.

It is not only teachers who are concerned with perceived failings in school and college mathematics. Parents, employers and politicians all vie repeatedly in urging attention to the system's ability to deliver enough students passing mathematics examinations. Whatever the truth behind such fears and concerns, resolving them is evidently a political matter, and thus adoption of the contribution offered by this Study, to improve mathematics education through the provision and use of historical resources, is a political choice to be made or influenced at any or all of the several layers of decision-making in complex modern societies.

The ICMI Study

ICMI, the International Commission on Mathematical Instruction, was established in 1908 at the International Congress of Mathematicians held in Rome, its first chair being Felix Klein. After an interruption of activity between the two World Wars, it was reconstituted in 1952 as a commission of the International Mathematical Union

(IMU). The IMU itself was formed at the 1920 International Congress of Mathematicians, held in Strasbourg. The history of these international bodies is thus closely linked with twentieth century internationalisation of mathematical activity, in particular with the efforts of mathematicians to re-energise international co-operation after major wars, as part of the healing and reconciliation process and in a spirit of optimism about building a better future for everyone. In 1972, at the second International Congress on Mathematical Education in Exeter, UK, the idea was developed of an International Study Group on the Relations between History and Pedagogy of Mathematics, which was formally affiliated to ICMI at the 1976 International Congress (ICME-3) at Karlsruhe, Germany. HPM has continued ever since to explore and advise on these relations through the activities of its members, who are mathematics educators, teachers and historians across the world., who are mathematics educators, teachers and historians across the world.

Since the mid 1980s ICMI has engaged in promoting a series of studies on essential topics and key issues in mathematics education, to provide an up-to-date presentation and analysis of the state of the art in that area. The tenth ICMI Study, whose report is presented in the present volume, was conceived in the early 1990s in order to tease out the different aspects of the relations between history and pedagogy of mathematics, in recognition of how the endeavours of how the *HPM* Study Group had encouraged and reflected a climate of greater international interest in the value of history of mathematics for mathematics educators, teachers and learners. Concerns throughout the international mathematics education community began to focus on such issues as the many different ways in which history of mathematics might be useful, on scientific studies of its effectiveness as a classroom resource, and on the political process of spreading awareness of these benefits through curriculum objectives and design. It was judged that an ICMI Study would be a good way of bringing discussions of these issues together and broadcasting the results, with benefits, it is to be hoped, to mathematics instruction world-wide.

ICMI Studies typically fall into three parts: a widely distributed *Discussion Document* to identify the key issues and themes of the study; a *Study Conference* where the issues are discussed in greater depth; and a *Study Volume* bringing together the work of the Study so as to make a permanent contribution to the field. The current study has followed this pattern.

The *Discussion Document* was drawn up by the two people invited by ICMI to co-chair the Study, John Fauvel (Open University, UK; HPM chair 1992-1996) and Jan van Maanen (University of Groningen, Netherlands; HPM chair 1996-2000), with the assistance of the leading scholars who formed the International Programme Committee: Abraham Arcavi (Israel), Evelyne Barbin (France), Jean-Luc Dorier (France), Florence Fasanelli (US, HPM Chair 1998-1992), Alejandro Garciadiego (Mexico), Ewa Lakoma (Poland), Mogens Niss (Denmark) and Man-Keung Siu (Hong Kong). The Discussion Document was widely published, in for example the *ICMI Bulletin* **42** (June 1997), 9-16, and was translated into several other languages including French, Greek and Italian. From the responses and from other contacts, some eighty scholars were invited to a Study Conference in the spring of 1998, an invitation which in the event between sixty and seventy were able to accept.

The *Study Conference* took place in the south of France, at the splendid country retreat of the French Mathematical Society, CIRM Luminy (near Marseille), from 20 to 25 April 1998. Local organisation was in the hands of Jean-Luc Dorier (University of Grenoble). The scholars attending were from a variety of backgrounds: mathematics educators, teachers, mathematicians, historians of mathematics, educational administrators and others. This rich mix of skills and experiences enabled many fruitful dialogues and contributions to the developing study.

The means by which the Study was advanced, through the mechanism of the Conference, is worth description and comment. Most participants in the Conference had submitted papers, either freshly written or recent position papers, for the others to read and discuss, and several studies were made available by scholars not able to attend the meeting. These, together with whatever personal qualities and experiences each participant was bringing to the Conference, formed the basis for the work. Apart from a number of plenary and special sessions, the bulk of the Conference's work was done through eleven working groups, corresponding, in the event, to the eleven chapters of the Study Volume. Each participant belonged to two groups, one meeting in the mornings and one in the afternoons. Each group was led by a convenor, responsible for co-ordinating the group's activities and playing a major part in the editorial activity leading to the eventual chapters of the book. Each group's work continued for several months after the Conference, with almost everyone participating fully in writing, critical reading, bibliographical and other editorial activities.

This way of group working for a sustained period towards the production of a book chapter was a fresh experience to many participants, since the pattern of individual responsibility for separate papers is a more common feature of such meetings and book productions. In this instance the participants proved remarkably adept at using the new structures to come up with valuable contributions to the development of the field, all the more valuable for their being the results of consensual discussions and hard-written contributions, which have been edited and designed into the present Study Book.

Authorship of contributions

As just explained, this ICMI Study adopted a style of collective group work in which international teams worked together on the various issues, each led by a convenor, whose reports form the basis of the chapters in this book. We have experienced this as a very useful and productive way of working for the teachers, educators and researchers involved, who were able to share insights, experiences and ideas, and develop strategies together for future progress in the field. It follows from the working style that it is not quite as straightforward as usual to attribute responsibility and authorship to particular sections of text. As will be seen, each chapter is credited to a team, listed in alphabetical order, headed by the name of the chapter co-ordinator. Within the chapters, sometimes names may appear as responsible for subsections and sometimes not. In the construction of the book some sections retained individual responsibility (while commented on and modified by the help of the rest of the group), and others were by the end of the process a genuinely