The Phases of Quantum Chromodynamics

This book discusses the physical phases of quantum chromodynamics (QCD) in ordinary as well as in extreme environments of high temperatures and high baryon number. A major theme of this book is the idea that, to understand the dynamics of QCD in ordinary circumstances, one needs to master them in extreme environments. QCD is thought to be characterized in ordinary circumstances by quark confinement through the formation of flux tubes and chiral-symmetry breaking. These properties are believed to be lost in environments of extreme conditions and new phases, the quark–gluon plasma and color superconductivity, are thought to exist.

The book is aimed at graduate students and researchers entering the fields of lattice-gauge theory, heavy-ion collisions, nuclear theory, and high-energy phenomenology, as well as astrophysicists interested in the phases of nuclear matter and their impact on our current ideas of the interiors of dense stars. It is suitable for use as a textbook on lattice-gauge theory, effective Lagrangians, and field-theoretical modeling for nonperturbative phenomena in QCD.

JOHN KOGUT obtained his Ph.D. from Stanford University in 1971. He is a Fellow in the American Physical Society and has held Guggenheim (1987–8) and Sloan Foundation (1976–8) Fellowships. He is co-author of over 200 original papers in elementary-particle, high-energy, and condensed-matter physics, and is the author of one previous book. He is currently Professor of Physics at the University of Illinois, Urbana-Champaign.

MIKHAIL STEPANOV obtained his Ph.D. from the University of Oxford in 1994. He was a Postdoctoral Research Associate at the University of Illinois, Urbana-Champaign from 1994 to 1997 and then spent two years at the State University of New York at Stony Brook. He is at present Associate Professor at the University of Illinois at Chicago and RHIC Fellow at the RIKEN-BNL Center. He has published over 50 original papers on particle and nuclear physics, lattice quantum-field theory, matter under extreme conditions, and heavy-ion collisions. His research is supported by a DOE Outstanding Junior Investigator Award, as well as by an Alfred P. Sloan Foundation Fellowship.
CAMBRIDGE MONOGRAPHS ON
PARTICLE PHYSICS
NUCLEAR PHYSICS AND COSMOLOGY
21

General Editors: T. Ericson, P. V. Landshoff

1. K. Winter (ed.): Neutrino Physics
2. J. F. Donoghue, E. Golowich and B. R. Holstein: Dynamics of the Standard Model
3. E. Leader and E. Predazzi: An Introduction to Gauge Theories and Modern Particle Physics, Volume 1: Electroweak Interactions, the ‘New Particles’ and the Parton Model
4. E. Leader and E. Predazzi: An Introduction to Gauge Theories and Modern Particle Physics, Volume 2: CP Violation, QCD and Hard Processes
5. C. Grupen: Particle Detectors
6. H. Grosse and A. Martin: Particle Physics and the Schrödinger Equation
7. B. Andersson: The Lund Model
9. I. I. Bigi and A. I. Sanda: CP Violation
10. A. V. Manohar and M. B. Wise: Heavy Quark Physics
12. D. Green: The Physics of Particle Detectors
15. E. Leader: Spin in Particle Physics
16. J. D. Walecka: Electron Scattering for Nuclear and Nucleon Structure
17. S. Narison: QCD as a Theory of Hadrons
18. J. F. Letessier and J. Rafelski: Hadrons and Quark–Gluon Plasma
19. A. Donnachie, H. G. Dosch, P. V. Landshoff and O. Nachtmann: Pomeron Physics and QCD
20. A. Hofmann: The Physics of Synchrotron Radiation
21. J. B. Kogut and M. A. Stephanov: The Phases of Quantum Chromodynamics
The Phases of Quantum Chromodynamics:
From Confinement to Extreme Environments

JOHN B. KOGUT
University of Illinois, Urbana-Champaign

MIKHAIL A. STEPHANOV
University of Illinois, Chicago
Contents

1 Introduction 1
2 Background in spin systems and critical phenomena 10
  2.1 Notation and definitions and critical indices 10
  2.2 Correlation-length scaling and universality classes 14
  2.3 Properties of the Ising model 17
  2.4 The Kosterlitz–Thouless model 19
  2.5 Coulomb gas, duality maps, and the phases of the planar model 24
  2.6 Asymptotic freedom in two-dimensional spin systems 30
  2.7 Instantons in two-dimensional spin systems 36
  2.8 Computer experiments and simulation methods 37
  2.9 The transfer matrix in field theory and statistical physics 41

3 Gauge fields on a four-dimensional euclidean lattice 53
  3.1 Lattice formulation, local gauge invariance, and the continuum action 53
  3.2 Confinement and the strong-coupling limit 57
  3.3 Confinement mechanisms in two and four dimensions: vortex and monopole condensation 63

4 Fermions and nonperturbative dynamics in QCD 74
  4.1 Asymptotic freedom and the continuum limit 74
  4.2 Axial symmetries and the vacuum of QCD 76
  4.3 Two-dimensional fermionic models of confinement, axial symmetries, and $\theta$ vacua 77
  4.4 Instantons and the scales of QCD 84
Contents

5 Lattice fermions and chiral symmetry 93
  5.1 Free fermions on the lattice in one and two dimensions 93
  5.2 Fermions and bosons on Euclidean lattices 101
  5.3 Staggered Euclidean fermions 104
  5.4 Block derivatives and axial symmetries 107
  5.5 Staggered fermions and remnants of chiral symmetry 109
  5.6 Exact chiral symmetry on the lattice 111
  5.7 Chiral-symmetry breaking on the lattice 117
  5.8 Simulating dynamical fermions in lattice-gauge theory 126
  5.9 The microcanonical ensemble and molecular dynamics 127
  5.10 Langevin and hybrid algorithms 132

6 The Hamiltonian version of lattice-gauge theory 136
  6.1 Continuous time and discrete space 136
  6.2 Quark confinement in Hamiltonian lattice-gauge theory and thin strings 145
  6.3 Relativistic thin strings, delocalization, and Casimir forces 146
  6.4 Roughening and the restoration of spatial symmetries 150

7 Phase transitions in lattice-gauge theory at high temperatures 158
  7.1 Finite-temperature transitions at strong coupling 158
  7.2 Simulations at nonzero temperature 162
  7.3 Pure gauge-field simulations at nonzero temperature 165
  7.4 Restoration of chiral symmetry and high temperature 170
  7.5 Hadronic screening lengths 172
  7.6 Thermal dilepton rates and experimental signatures for the quark–gluon plasma 174
  7.7 A tour of the three-flavor QCD phase diagram 176

8 Physics of QCD at high temperatures and chemical potentials 182
  8.1 The thermodynamic background 182
  8.2 Hadron phenomenology and simple models of the transition to the quark–gluon plasma 202
  8.3 A tour of the $T-\mu$ phase diagram 207
  8.4 The quark–gluon plasma and the energy scales of QCD 223
  8.5 The extreme environment at a relativistic heavy-ion collider 226

9 Large chemical potentials and color superconductivity 236
  9.1 Color superconductivity and color–flavor locking 236
  9.2 Calculating the gap at asymptotically large $\mu$ 239
Contents

9.3 Lowest excitations of the CFL phase 260
9.4 Comments and some further developments 278

10 Effective Lagrangians and models of QCD at nonzero chemical potential 280
10.1 QCD at finite $\mu$ and the sign problem 280
10.2 The random-matrix model of QCD 282
10.3 Two-color QCD and effective Lagrangians 295
10.4 QCD at nonzero isospin chemical potential 307
10.5 Pion propagation near and below $T_c$ 316

11 Lattice-gauge theory at nonzero chemical potential 324
11.1 Propagators and formulating the chemical potential on a Euclidean lattice 324
11.2 Naive fermions at finite density 326
11.3 The three-dimensional four-Fermi model at nonzero $T$ and $\mu$ 330
11.4 Four-flavor SU(2) lattice-gauge theory at nonzero $\mu$ and $T$ 335
11.5 High-density QCD and static quarks 345
11.6 The Glasgow algorithm 347
11.7 The Fodor–Katz method for high $T$, low $\mu$ 349
11.8 QCD at complex chemical potential 350

12 Epilogue 355

References 357

Index 363