KOENO GRAVEMEIJER, RICHARD LEHRER, BERT VAN OERS AND LIEVEN VERSCHAFFEL

INTRODUCTION AND OVERVIEW

Abstract. This chapter describes the structure of the book, and provides a general overview of the various sections.

1. INTRODUCTION

The growing appreciation of constructivism and socio-cultural theory within the mathematics education community has created the need to find an alternative for the now questionable use of tactile and visual models as embodiments of mathematical concepts. This book explores the option of building on symbolizing, modeling and tool use as personally meaningful activities of students, in the context of social practices. The metaphors of 'transmission of knowledge' with help of symbols that function as 'carriers of meaning' are replaced by the image of students constructing their own ways of symbolizing as part of their mathematical activity. The corresponding instructional approach builds on the idea that symbolizing and meaning co-evolve in a reflexive process. A first key element, therefore, is to capitalize on students' own informal self-construed ways of symbolizing. In this respect, interest is in symbol use in the context of play, the use of idiosyncratic ways of symbolizing, the process of exchanging initial ways of symbolizing for new ones, and the criteria students use when evaluating and improving informal ways of symbolizing. A second issue concerns ways in which dialectical processes of symbolizing and meaning development can be promoted and guided in the context of instruction. Here interest is in issues of instructional design, analysis of classroom practices, and discourse. Third, interest is in a deeper understanding of the underlying psychological processes, a re-conceptualization of the notion of transfer, and the matter of educational goals and assessment.

These three areas of interest can also be characterized by the amount in which the researchers zoom in into or zoom out from the activity of individual students in school. The position with respect to this variation in perspective forms the basis for the organization of this book. The first section of the book takes the closest look at the activity of students in school, by focussing at the spontaneous, informal, design and use of symbols and models by the students in semi-experimental instructional practices in school. The second section of the book takes more distance, while addressing issues of instructional design. Section three steps even further back from actual instructional practice in school, by shifting the attention to consequences of

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instruction and educational goals. Each section is complemented with a short introduction by one of the editors and a discussion by an expert in the field.

These three sections are preceded by a preamble. The purpose of this preamble is to place the book in a historical context. It describes how the role of symbols and models has become a central topic of attention in the mathematics education community. Characteristic for this (renewed) interest is the concern for the way students actually use tools and symbols, and for what tools and symbols signify for them. This represents a shift away from approaches in which symbols are discussed in agentless terms. The preamble sketches how this current interest is elaborated within the two now dominant theoretical perspectives, constructivism and socio-cultural theory. It further describes what semiotics has to offer in the context of the abandonment of a representational view. All in all, the preamble tries to situate the studies presented in this book.

2. SECTION 1, EMERGENT MODELING

The overall objective of the first section of the book is to pool ideas and investigations that might shed some light onto how processes of modeling and symbolization emerge from (problem solving) activities of children. The basic idea is that forms of symbolization (in schemes, diagrams, models or even verbal terms) emerge in the context of activities that require the availability of such symbolic tools, and that the functional requirements of these activities stimulate the improvement of the children's way of symbolizing. The chapters in this section address questions like: what are the developmental paths of the symbolizing activity that may lay the basis for later mathematizing? What kind of psychological processes may occur when children are getting involved in mathematical practices that prompt activities of tool formation, or tool improvement? What processes are involved in ascribing meaning to mathematical models and representations in educational settings?

In the first chapter of this section, Bert van Oers addresses the problem of the development of mathematical symbol use in the context of young children's play and everyday talk. Starting out from a Vygotskian perspective, he assumes that the development of the meaning of everyday mathematics-like terms gradually acquire a more definite mathematical meaning as a result of the children's participation in mathematical activities in school or everyday practices, and of the provoked reflection on the interrelationships between signs and meanings (the so called 'semiotic activity'). In order to examine some of the assumptions involved the author reports a number of observational and interview studies with primary school children.

Next, Richard Lehrer and Carrie Pritchard describe a design experiment within which children of 8 and 9 years of age symbolize the familiar large-scale space of their school's playground. They elaborate how the students mathematize their ways of symbolizing by generating re-descriptions, which eventually evolve into polar coordinates. This development relied upon the emergence of conceptions of scale,

origin, and the appropriation of coordinates to describe position and direction. They further report about long-term effects, the involvement of parents, and related professional development

Luciano Meira discusses the use and production of mathematical notations, in connection with children's continuous process of making and transforming the meanings of word problems and physical devices. The author summarizes a set of his own studies on children's representational activity, focusing on the microgenesis of tables of values and algebraic expressions on paper. As such, the studies aim at describing the process of how children build symbolic representations in the context of their problem solving activities.

Andrea diSessa reports on a study that aims at revealing the criteria students use in determining the quality of different representations. In prior work it was observed that, in the course of designing representations, students employed an iterative process of innovating, critiquing, selecting, refining, and combining representations. Prior work also catalogued a rich set of ideas for the design of representation. The present question is: What kind of resources do students use to evaluate and thus improve representations?

Norma Presmeg closes the section with a discussion of the four chapters on emergent modeling.

3. SECTION 2, INSTRUCTIONAL-DESIGN PERSPECTIVES ON MODELING

The chapters in the second section represent the shift in the way models are viewed in instructional design. The conventional focus on didactical models that embody the formal mathematics that is to be taught at least implicitly a viewpoint in which the properties of symbols are analyzed independently of their use. Within the alternative perspectives presented here, the ways that symbols are used and the meanings they come to have are seen to be mutually constitutive. An instructional design heuristic that takes this dialectic relation into account is the RME notion of emergent models that forms a common tread in the four chapters.

In the first chapter of this section, Koeno Gravemeijer and Michelle Stephan take the design of an instructional sequence, which deals with flexible mental computation strategies for addition and subtraction up to one hundred, as an instance for elaborating on the role of 'emergent models' as an RME design heuristic. It is explicated, how the label 'emergent' refers both to the character of the process by which models emerge within RME, and to the process by which these models support the emergence of formal mathematical ways of knowing.

In his chapter, Paul Cobb looks at symbolizing and mathematical learning from a social constructivist perspective that is motivated by an interest in instructional design. The central theme is that of a concern for the way students actually use tools and symbols. The point of departure is in analyses treat people's activity with symbols as an integral aspect of their mathematical reasoning rather than as external aids to it. Against this background an analysis is presented of the mathematical practices established during a seventh-grade classroom teaching experiment that focused on statistical data analysis, that is based on RME theory. This analysis is

supplemented with a description of the taken-as-shared ways in which two computer-based analysis tools were used in the classroom.

Next, Patrick Thompson discusses ways in which conceptual analyses of mathematical ideas from a radical constructivist perspective might complement RME's attention to emergent models, symbolization, and participation in classroom practices. The discussion draws on examples from research in quantitative reasoning, in which radical constructivism serves as a background theory. A theory of quantitative reasoning enables one to describe mathematical understandings one hopes students will have, and the way in which students might express their understandings in action or communication. Within a quantitative-reasoning perspective, the focus is not so much on ways of influencing students' activity, but more on things students might re-perceive, and things about which a teacher might hold fruitful discussions. The central claim of the chapter is that quantitative reasoning and realistic mathematics education provide complementary foci in both design of instruction and evaluation of it.

Christoph Selter closes this section with a discussion of the three chapters on instructional-design perspectives on modeling.

4. SECTION 3, MODELS, SITUATED PRACTICES AND GENERALIZATION

In the third section, mathematical modeling is considered in connection to situated practices, on the one hand, and generalization and transfer, on the other hand. Key issues addressed in this section are the nature of mathematical models and generalizations, and the circumstances in which they are used. Recent theorizing argues that this is a complex matter that cannot be fully understood on the basis of symbolic tools or problem solving abilities alone. A deeper grasp of human symbolizing capacity also requires reflection on the evolution of brain functions and the language faculty. Hence, the issue of 'models, situated practices and generalization' is here not only addressed from a microgenetic perspective, but also from a sociogenetic and phylogenetic point of view. Furthermore, implications for how we understand the symbolic aspects of mathematics and how they may be learned and transferred will be drawn.

Ricardo Nemirovsky opens this section by hooking on to the ongoing discussion in the domain of psychology of (mathematics) education about the consequences of the 'situated cognition' paradigm for the issue of transfer of learning. He first elaborates on the nature of generalizing. He then shifts to the topic of transfer of learning. The central question here is: How does one experience become part of another? A review of the many studies and debates on the notion of transfer of learning developed during the twentieth century, delineates dominant themes and concerns of the transfer literature. The paper articulates the claims through an in-depth analysis of an interview with an 11-year old girl working with problems involving the graphical representation of motion.

Lieven Verschaffel, Brian Greer, and Erik De Corte present a review of research on modeling of mathematical word/world problems by elementary-school students.

They report on studies that show that after several years of traditional mathematics instruction children have developed a tendency to ignore their common-sense knowledge and realistic considerations about the problem context. Further, two related but different lines of follow-up studies are presented. The first line of research investigated the effects of different forms of scaffolds, the second one looked at the effectiveness of attempts to increase the authenticity of the testing setting. The second line of research aimed at changing students' perceptions of word problem solving by taking a radical modeling perspective.

James Kaput and David Shaffer look at the development of human representational competence from an evolutionary point of view. They base themselves on work done by the evolutionary psychologist Merlin Donald, who argues that human cognition has developed across evolutionary time through a series of four distinct stages. They began with episodic (ape-like) memory and passed through mimetic (physical-action-based), mythic (spoken), and theoretical (written) transformations. The authors argue that we are entering a fifth stage of cognitive development leading to a virtual culture, which will replace the writing-based theoretic culture and which will support and be supported by a new hybrid mind, just as each of the predecessor stages subsumed its prior stage.

David Carraher and Analucia Schlieman close this section with a discussion of the three chapters on models, situated practices and generalization.