INTRODUCTION

"Today we really learnt something!" Mary exclaimed after she, together with Adam, had concentrated for almost two hours on setting up a spreadsheet. Something significant seems to have happened for Mary, something that should be considered when theorising about the learning of mathematics. In this study we are going to meet with Mary and Adam and many other students in the mathematics classroom. The main purpose of this meeting is to gather empirical resources to gain a better understanding of the role of communication in learning mathematics.

The initial idea that guides our investigations can be condensed in the following hypothesis: *The qualities of communication in the classroom influence the qualities of learning mathematics.* This is not a very original statement and certainly very general. If the statement is to be provided with meaning it is important to clarify at least the two expressions: 'qualities of communication' and 'qualities of learning mathematics'. In this introduction, as well as during the rest of this book, we are going to struggle with clarifying in what sense communication and learning can be connected, and how to conceptualise this connection.

QUALITIES OF COMMUNICATION

In many different contexts, both inside and outside school, special attention is paid to communication. Thus, companies organise workshops and courses on communication in order to improve the way they operate (see, for instance, Isaacs, 1999a; Kristiansen and Bloch-Poulsen, 2000). The improvement of communication is expected not only to have an influence on the atmosphere of the workplace, but also on the way the company operates in terms of business, as expressed in figures and budgets. Communication becomes related to the idea of the 'learning organisation'.

Qualities of communication can be expressed in terms of interpersonal relationships. Learning is rooted in the act of communicating itself, not just in the information conveyed from one party to another. Thus, communication takes on a deeper meaning. In *Freedom to Learn*, first published in 1969, Carl Rogers (1994) considers interpersonal relationships as the crucial point in the facilitation of learning. Learning is personal, but it takes place in the social contexts of interpersonal relationships. Accordingly, the facilitation of learning depends on the quality of contact in the interpersonal relationship that emerges from the communication between the participants. In other words, the context in which people communicate affects what is learned by both parties.

This brings forward the idea that some 'qualities of communication' could be clarified in terms of *dialogue*. The word 'dialogue' has many everyday descriptive references but the important factor common to all is that they involve at least two parties. For instance, it is possible to talk about the dialogue between East and West and about the breakdown of the dialogue between Palestine and Israel. Such references to dialogue are not strictly part of our concern. In philosophical contexts the notion of dialogue occurs in many places. Plato presented his ideas as dialogues; in 1632 Galileo Galilei wrote *The Dialogue Concerning the two Chief World Systems* (which brought him close to the Inquisition), and Imre Lakatos (1976) presented his investigation of the logic of mathematical discovery in the form of a dialogue taking place in an imaginary classroom. Such uses of 'dialogue' refer first of all to analytical forms and presentations of inquiries and of 'getting to know'. As soon as we enter the field of 'getting to know', dialogue becomes relevant to epistemology. However, although our concept of dialogue is also related to epistemology in this way, it will diverge from the traditional philosophical use of the term by being related to 'real' dialogues and not to in-principle dialogues. We use the word 'dialogue' for a conversation with certain *qualities*, and the specification of 'dialogue' is one of the tasks awaiting for us as part of this study.

In talking of qualities related to conversation, we recognise that the notion of quality may have a double meaning. On the one hand, quality may refer to properties of a certain entity. Thus, we can talk (almost in Aristotelian terms) about the quality of a cup as being different from the quality of a glass. In this sense quality refers to descriptive aspects of an entity. However, quality may also contain a normative element. Thus, we can talk about one glass being of a better quality that another glass. Maintaining the distinction between descriptive and normative references to quality is not simple. For instance, we may prefer the quality of a glass to the quality of a cup when drinking wine. In a similar way, we may prefer a dialogue when we think of certain forms of learning, bearing in mind that dialogue refers to certain properties of an interaction.

Paulo Freire (1972) emphasises the importance of interpersonal relationships in terms of dialogue. To Freire dialogue is not just any conversation. Dialogue is fundamental for the freedom to learn. The notion of dialogue is integral to concepts like 'empowerment' and 'emancipation', and from this perspective Freire makes a connection between the quality of what is happening between people and the possibility of pursuing political actions. He defines dialogue as a meeting between people in order to 'name the world', which means talking about events and the possibility of changing these. In this way dialogue is seen as existential. Dialogue cannot exist without love (respect) for the world and for other people, and it cannot exist in relations of dominance (Freire, 1972, 77f.). Further, taking part in a dialogue presupposes some kind of humility. You cannot enter a dialogic relationship being self-sufficient. The participants have to believe in each other and to be open-minded towards each other in order to create an equal and faithful relationship. As the dialogue is directed by the hope of change, it cannot exist without the engagement of the partners in critical thinking (Freire, 1972, 80f.). To Freire the co-operation of the participants is a central parameter of dialogic communication. In co-operation the participants throw light on the world that surrounds them and the problems that connect and challenge them. Freire points out the importance of co-operation between action and reflection (Freire, 1972, 75f.). Hand and head have to go together. Acting without reflecting would end up in pure activism and reflection without action would result in verbalism. However, in a dialogue, reflection and action can enrich each other. According to Freire, the educational dialogue is supposed to examine the universe of the people – its thematic universe – which announces emancipation through education. Freire's program was originally aimed at illiterate people, and it has to be remembered that only in May 1985 did illiterate people in Brazil get the right to vote.[1] To Freire, dialogue clearly refers to a form of interaction with many specific qualities.

In classical philosophy, dialogue first of all refers to a presentation (and confrontation) of two or more different (and contradictory) points of view, with the aim of identifying a conclusion that can be agreed on.

[1] Freire makes the revolutionary leaders responsible for the communication remaining dialogic. They should not invade the perspectives of the people and inform or instruct them, nor should they just adapt to the expectations of the people. They should learn about the people's world together with the people – by naming the world (Freire, 1972, 161). The relationship between the revolutionary leaders and the people, as suggested by Freire, can be interpreted in terms of the relationship between teacher and students – an asymmetrical relationship.

Freire and Rogers, however, also viewed dialogue as encompassing interpersonal relationships, where listening and accepting on the part of the participants is fundamental. Dialogue is not just a mode of analysis, but also a mode of interaction. In the following clarification of the notion of dialogue we shall maintain this combination of epistemic and relational aspects of dialogue.

Rogers and Freire have much in common although they work from different historical positions. This is perhaps not surprising as they both relate to the German philosopher Martin Buber (1957), who emphasises the relationship, 'the interhuman', in the dialogue as a certain way of meeting the other with unconditional acceptance. Rogers calls his approach to learning 'person-centered' as opposed to the 'traditional mode', and he describes the two approaches as opposite poles of a continuum (Rogers, 1994, 209f.). He argues that the person-centered mode prepares the students for democracy, whereas the traditional mode socialises the students to obey power and control. In the traditional mode, he argues, "the teacher is the possessor of knowledge and power," and "rule by authority is accepted policy in the classroom". Students are expected to be recipients of knowledge, and examinations are used to measure their receptivity. Rogers emphasises that "trust is at a minimum," and "democratic values are ignored and scorned in practice". In the person-centred mode, he argues, the environment is trustful and the responsibility for the learning processes is shared. "The facilitator provides learning resources," and "the students develop their program of learning alone and in co-operation with others". The main principle is learning how to learn, and self-discipline and self-evaluation guarantee a continuing process of learning. This growth-promoting climate not only facilitates learning processes but also stimulates the students' responsibility and other competencies for democratic citizenship: "I have slowly come to realize that it is in its politics that a person-centred approach to learning is most threatening. The teacher or administrator who considers using such an approach must face up to the fearful aspects of sharing of power and control. Who knows whether students or teachers can be trusted, whether a process can be trusted? One can only take the risk, and risk is frightening." (Rogers, 1994, 214)

Freire contrasts his dialogic approach with 'banking education', where the teacher makes an investment, and where the students are considered boxes and are supposed to preserve what is invested. To both Rogers and Freire, dialogue represents certain forms of interaction fundamental to processes of learning, which, in Freire's terms, can ensure empowerment, and which in Rogers' terms can ensure person-centered learning and students' responsibility. In this sense they find that qualities of communication can turn into qualities of learning, referring to both descriptive and normative elements. When we talk about qualities of communication and qualities of learning, we also have in mind both descriptive and normative elements. We want to locate certain aspects of communication which may support certain aspects of learning, and at the same time it becomes important to support these aspects of learning.

Many studies of communication concentrate on classrooms that are situated in the school mathematics tradition. Here, we refer to a tradition where the textbook plays a predominant role, where the teacher explains the new mathematical topics, where students solve exercises within the subject, and where correction of solutions and mistakes characterise the overall structure of a lesson. We have observed classrooms from a school mathematics tradition where there is a nice atmosphere, and where the teacher-student communication appears friendly. So, by the school mathematics tradition we do not simply refer to the non-attractive features of the mathematics classroom, where a never-smiling teacher dominates the students. However, within the school mathematics tradition we can locate characteristic patterns of communication which have certain qualities, but we are not tempted to refer to these patterns as dialogue.

The form of communication depends on the context of communication, and, like many others, we find that the school mathematics tradition frames the communication between students and teacher in a particular way. In the first chapter of this study we will summarise a few of our observations and analyses of this phenomenon, but in the rest of the book we primarily undertake our investigations in classrooms outside the school mathematics tradition. We are interested in situations where the students become involved in more complex and also unpredictable processes of inquiry. This opens a new space for communication, where new qualities can emerge.

In many cases the mathematics classroom has undergone radical changes. Thematic approaches and project work challenge tradition in such a way that the distinction between learning mathematics and learning something else is not always sharply maintained.

With the exception of Chapters 1 and 2, we describe projects where the planning of the subject matter was a shared process between the teachers and us. Then, when it came to the classroom practice, the teachers were in charge. One reason for this division of labour is simply that the teacher's professionalism in real-life classrooms is much higher than ours. We discussed the interpretations of the observations with the teachers, and we have included their suggestions for possible interpretations. In some cases, we also interviewed the students about

CHAPTER 1

COMMUNICATION IN THE MATHEMATICS CLASSROOM

The purpose of teaching mathematics is to point out mistakes and correct them! This seems to be a common understanding of mathematics education among many students.[12] We have even seen examples of preschool children expressing the same view in a role play about teaching mathematics.[13] One child was playing the role of the teacher the rest were 'students'. One 'student' was supposed to do an exercise on the blackboard and wrote some serious-looking symbols in a long row. Afterwards, the 'teacher' erased a couple of those symbols and wrote some others while accusing the 'student' of being mistaken. Thus, even before having any school experiences of their own and without having an understanding of what symbols might mean, the children showed an understanding of mistakes and of the correction of mistakes as being a central parameter in mathematics education.

One reason why the notion of 'mistakes' seems so important in mathematics education can be related to the search for 'truth' in mathematics. A main task of a philosophy of mathematics has been to give an adequate explanation of 'truth'. Absolutism in epistemology is associated with the idea that the individual has the possibility to acquire absolute truth. This idea connects with the Euclidean ideal in epistemology. Relativism, though, maintains that truth is always located by someone in a certain context at a certain time. Thus, truth cannot be grasped in absolute terms. With mathematics in mind, relativism has been put forward by both radical and social constructivism.[14]

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[12] Alrø and Lindenskov (1994). This chapter is a rewriting of Alrø and Skovsmose (1996a, 1998).

[13] See Fosse (1996).

[14] See, for instance, Glasersfeld (1995) and Ernest (1998a).

Somehow the philosophic discussion of mathematical truths, becomes reflected in a discussion of mistakes in the mathematics classroom.[15] Like the concept of 'truth', the concept of 'mistake' has two extremes – one absolutist and one relativistic. The absolutist interpretation apparently has a sound basis. For instance, to think that 12 multiplied by 13 equals 155 seems a simple mistake. But the situation looks somewhat different if we come to the applications of mathematics. If we measure one side of a play ground to be (about) 12 m and the other side to be (about) 13 m, its area may well be 155 m^2 – the ground looks rectangular. Relativism may have a bearing when the application of mathematics is considered. Nevertheless, it often seems possible to make absolute mistakes when applications of mathematics are presented in mathematics textbooks. (We will return to this point in the section 'From exercises to landscapes of investigation' in Chapter 2.)

In the first section of this chapter we discuss mistakes and correcting of mistakes on the basis of classroom observations.[16] We suggest the notion of bureaucratic absolutism to characterise the type of learning environment, where mistakes are handled in absolute terms. Mistakes are simply *mistakes* and have to be elimininated. This learning environment corresponds very well with the communication pattern: Guess What the Teacher Thinks.[17] Further, referring to a non-bureaucratic classroom, we introduce the notion of perspective in order to describe student understandings and pre-understandings as resources for learning. Here the

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[15] Normally the (theoretical) discussion of mistakes in the mathematics classroom has concentrated on the mistakes of the students. We could as well look at teacher mistakes, teacher ways of interpreting own mistakes, student ways of interpreting teacher mistakes, teacher ways of hiding mistakes, etc. The study of mistakes can take a variety of directions. Nevertheless, we shall follow the mainstream and concentrate on student mistakes, and teacher ways of interpreting and correcting these.

[16] The observed mathematics lessons we refer to in Chapter 1 and Chapter 2 were part of the normal teaching programme. Many analyses of traditional mathematics classrooms have from different theoretical perspectives pointed to the fact that communication plays an important role for the dynamics of the classroom. We are especially inspired by the microethnografic approach of the German group of symbolic interactionists and their studies of routines, relationships and patterns of communication that can be found in the traditional mathematics classroom, e.g. Bauersfeld (1980, 1988, 1995); Krummheuer (1983, 1995, 2000b) and Voigt (1984, 1985, 1989).

Other important contributions to this field of analysis are Cestari (1997); Cobb and Bauersfeld (eds.) (1995); Jungwirth, H. (1991); Lemke (1990); Pimm (1987); Sfard (2000) and Steinbring (1998, 2000). For a discussion of the culture of the mathematics classroom, see also Brown (2001); Lerman (ed.) (1994); Nickson (1992); Seeger, Voigt and Waschescio (eds.) (1998) and Wood (1994).

[17] The term 'Guess What the Teacher Thinks' is used by Young (1992, 106f.).

students' guessing can be understood as their zooming-in on the classroom agenda. Finally, we discuss learning in terms of action, including the crucial notion of intention.

MISTAKES AND CORRECTIONS

As 'truth' is a key term in the philosophy of mathematics, so are 'mistakes' a key to grasp an implicit philosophy prevailing in many mathematics classrooms. Correction of mistakes opens a backdoor to the classroom philosophy of mathematics.

Philosophical absolutism maintains that some absolute truth can be obtained by the individual. Classroom absolutism comes about when (students') mistakes are treated as absolute: 'This is wrong!' 'You have to correct these calculations!' Thus, classroom absolutism seems to maintain that mistakes are absolute and can be eliminated by the teacher. Our point is not, however, that no mistakes in the mathematics classroom should be stated as real mistakes. We do not want to maintain an absolute relativism. But it seems like absolutism in the philosophy of mathematics automatically leads to absolutism in pedagogy that justifies certain forms of classroom interaction.

We can conceive of different types of mistakes found in mathematics education. In what follows we talk about 'mistakes' in the broadest way to include 'real' mistakes, other sorts of (mis)conceptions, as well as simply alternative conceptions. The mistake could concern the output of some algorithm: 'This calculation is wrong!' The mistake could concern the used algorithm: 'You should not add these numbers but do a subtraction!' The mistake could concern the sequence in which things are done: 'When drawing a graph you first have to calculate some values of the function!' The mistake could have to do with the way the text is interpreted: 'No, when the exercise is formulated like this, you first have to find the value of x!' Or it could have to do with the organisation of the tasks for the students: 'No, no, those exercises are for tomorrow!'

Although the content of these mistakes is quite different, the corrections can be expressed in the same absolute terms. The basic assumption is that the aim of a correction is to correct a mistake. The phenomenon that all sorts of mistakes are treated as absolute, i.e. as real mistakes, we refer to as classroom absolutism.

CHAPTER 2

INQUIRY CO-OPERATION

What counts as traditional mathematics education will naturally vary during time, and also from country to country. Thus, it is difficult to provide any general characteristic of 'tradition'. We shall, however, suggest that the school mathematics tradition is characterised by certain ways of organising the classroom. For instance, a mathematics lesson can be divided into two parts: First, the teacher presents some mathematical ideas and techniques. This presentation is normally closely related to the presentation in the given textbook. Secondly the students work with selected exercises. These exercises can be solved by using the just presented techniques. The solutions are checked by the teacher. An essential part of the students' homework is to solve exercises from the textbook. The time spent on teacher presentation and on students doing exercises can naturally vary. Other elements can be included as for instance students' presentations of selected topics and solutions.[38]

In the school mathematics tradition the patterns of teacher-students communication can also become a routine, and much research has tried to identify the communicative patterns that dominate this tradition. We are interested in possible causes for such communicative patterns, as for instance the quizzing pattern of communication we described in Chapter 1, and here we shall pay attention to one particular aspect of the school mathematics tradition, the *exercise paradigm*. This paradigm has a deep influence on mathematics education, concerning the organisation of the individual lessons, the patterns of communication between teacher and students, as well as the social role that mathematics may play in society, for instance operating as a gatekeeper (the mathematical exercises fit nicely into processes of exams and tests). Normally, exercises in mathematics are formulated by an authority from outside the classroom. It is neither the teacher nor the students who have formulated the

[38] See Blomhøj (1995) for a similar characteristic of traditional mathematics education.

exercises. They are set by an author of a textbook. This means that the justification of the relevance of the exercises is not part of the mathematics lesson itself. Most often, the mathematical texts and exercises represent a 'given' for the classroom practice, including the classroom communication.

The exercise paradigm has been challenged in many ways: by problem solving, problem posing, thematic approach, project work, etc. To put it more generally, the exercise paradigm can be contrasted by investigative approaches.[39] We see the activities of solving exercises as being much more restrictive for the students than being involved in investigations. We want to elaborate on learning as action and not as a forced activity, and this makes us pay special attention to students being part of an investigative approach. In order to create possibilities for making investigations, it is important to consider possibilities outside the exercise paradigm. 'Openness from the start', illustrated by the project 'How much does a newspaper fill?' shows what it could mean to leave the well known frame of the exercise paradigm.

In this chapter we try to characterise more generally challenges to the exercise paradigm in terms of *landscapes of investigation*. We will discuss what it would mean to enter such a landscape. And by discussing an episode from the project 'What does the Danish flag look like?' we try to clarify the notion of *inquiry co-operation* as a particular form of student-teacher interaction when exploring a landscape of investigation. This co-operation we will specify into an *Inquiry-Co-operation Model* (IC-Model) that designates a significant pattern of communication. Such a pattern cannot easily be identified within a classroom practice located in the exercise paradigm.

FROM EXERCISES TO LANDSCAPES OF INVESTIGATION

Let us look at an example of an exercise in mathematics education: Shopkeeper A sells dates for 85p per kilogram. B sells them at 1.2 kg for

[39] An investigative approach can take many forms. One example is project work, as exemplified for primary and secondary school education in Nielsen, Patronis and Skovsmose (1999); Skovsmose (1994) and for university studies in Vithal, Christiansen and Skovsmose (1995). See also Cobb and Yackel (1998).

£1. (a) Which shop is cheaper? (b) What is the difference between the prices charged by the two shopkeepers for 15 kg of dates?[40]

Clearly we are dealing with dates, shops and prices. But most likely the person who constructed this exercise neither made any empirical investigation of how dates are sold, nor interviewed anyone to find out under what circumstances it would be relevant to buy 15 kg of dates. The situation is artificial. The exercise is located in a semi-reality. Solving exercises with reference to a semi-reality is an elaborated competence in mathematics education, based on a well specified (although implicit) agreement between teacher and students.[41]

Some of the principles in the agreement are the following: The semi-reality is fully described by the text of the exercise. No other information concerning the semi-reality is relevant in order to solve the exercise, and accordingly not relevant at all. The whole purpose of presenting the exercise is to solve the exercise. Asking any other questions about the specific nature of the semi-reality is similar to any form of disturbance of the mathematics lesson. A semi-reality is a world without sense impressions (to ask for the taste of the dates is out of the question), only the measured quantities are relevant. Furthermore, all the quantitative information is exact, as the semi-reality is defined completely in terms of these measures. For instance, the question whether it is OK to negotiate the prices or to buy somewhat less that 15 kg of dates is non-existing. The exactness of the measurements combined with the assumption that the semi-world is fully described by the provided information makes it possible to maintain the one-and-only-one-answer-is-correct assumption. The metaphysics of the semi-reality makes sure that this assumption gets a validity, not only when references are made exclusively to numbers and geometric figures, but also when references are made to 'shops', 'dates', 'kilograms', 'prices', 'distances', etc.[42]

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[40] The example is taken from Dowling (1998), where he describes the 'the myth of references'. The following presentation and discussion of landscapes of investigation is based on Skovsmose (2000b, 2000c, 2001a, 2001b). The notion of 'virtual reality' referring to the world set by the mathematical exercises has been used by Christiansen (1994, 1997).

[41] See Brousseau (1997) and Christiansen (1995) for a discussion of 'the didactical contract'.

[42] If it is not realised that the way mathematics fits the semi-reality has nothing to do with the relationship between mathematics and reality, then the ideology of certainty finds a place for growing. For a discussion of the ideology of certainty, see Borba and Skovsmose (1997).

CHAPTER 3

FURTHER DEVELOPMENT OF THE INQUIRY CO-OPERATION MODEL

The IC-Model has been developed with reference to a particular example of communication between a teacher and a group of students. The following key notions describe elements of inquiry: *getting in contact, locating, identifying, advocating, thinking aloud, reformulating, challenging and evaluating.* In this chapter we will reconsider these elements by looking at students' mutual inquiry co-operation.

We analyse group work that contains several new inquiring elements that seem to relate to the notions of the IC-Model. In the final section we will summarise these elements in order to develop the IC-Model to become not only a model of teacher-student communication, but a general model of inquiry co-operation in teaching and learning mathematics that aims at concretising inquiry as a communicative practice. This also includes a discussion of observed communicative patterns that seem to be an obstacle to inquiry co-operation.

'BATMAN & CO.'

We are in a 10th grade mathematics class. This week the students have two lessons in mathematics from 10 to 12 o'clock every day. In this particular school it is possible to set up a different schedule every week, so the students are used to special arrangements. We look at a course of inquiry that takes place in a classroom environment of group work where the teacher plays the role of a consultant. There are no given exercises, but the teacher introduces the students to a landscape of investigation with clearly defined vantage points that allow the students to raise mathematical questions and to solve mathematical problems. This means that they should be able to obtain an idea of what they could be doing in this landscape. Naturally, this does not prevent them from having to face difficulties. The presented landscape includes references to a semi-reality, but it also includes real-life references.

The class is going to imagine to be the Danish division of an American factory, 'Run for Your Life', that makes sports articles. Every day they get new information and orders that they have to consider. The first day the introduction is: "For the coming promotion campaign we need a large amount of balls. We have bought some black and white leather – 25 m^2 of each colour..." and a big shopping trolley with many sorts of balls is placed in the 'factory hall'. Cardboard, scissors and glue are also available. Some students begin to examine the well-known black and white handballs and soccer balls. They consist of 12 pentagons and 20 hexagons each. How can the factory begin a production? Later they get this fax: "Our sports centre has burnt down. We have rented a bubble hall of 25×40 meters. We need grounds for handball, basketball, badminton and volleyball. Please help us!" There is a lot of serious (and non-serious) work in the classroom. The students define their own tasks, they produce a lot, they calculate a lot.

A particular job is requested from 'Batman & Co'. This company needs bats for table tennis. The price must be no more than 89 Danish Kroner, and the Danish division of 'Run For Your Life' has no bats of that price in stock. However, a Swedish supplier is able to sell the bats at 70 Swedish Kroner. Naturally, the students also have to consider insurance and freight charges that are estimated to be 1.5 %. They are informed that the exchange rate between Danish and Swedish Kroner is 82.14; another source says 81.29. Duty is 8% and the profit is expected to be 25%. Finally, the VAT (Value Added Tax) in Denmark is 25%. As mentioned, 'Batman & Co' only wants to pay 89 Danish Kroner per bat. How to handle this situation? We will see how two students cope with this and how the teacher tries to facilitate their progress in work.

Mary and Adam from one of the groups get a computer and find a spreadsheet to solve the problem. They struggle hard and concentrate on this work during the two-hour lesson without any break.[59] A couple of times they are interrupted and challenged by the teacher. The 'factory hall' is filled with humming and shouting voices of the other 'workers',

[59] This is especially remarkable in the case of Adam, who is considered a problem child by many teachers. He has not done much during the first days of the project, but before today's lesson the math teacher, with whom he obviously has a respectful relationship, has kindly asked him to pull himself together and show his capability. The teacher has confidence in Adam, and his idea was to challenge him by bringing the computer into the classroom. See also Alrø, Skovsmose and Skånstrøm (2000).

but Mary and Adam do not allow themselves to be disturbed, not even when other group members try to interfere in what they are doing.[60] On this day the whole class is going on an excursion, and the bus will leave a few minutes after the lesson. But Mary and Adam keep working, and they do not stop when the teacher ends the lesson (in this school there is no bell ringing). They remain all alone in the room working at the spreadsheet.[61]

After some time they realise that they have to stop in order to join the others: Mary: "Well, should we give this up?" Adam: "Yes, no, we'll save it, won't we?" Mary: "Yes, it's actually very interesting, we have been quite clever, don't you think?" Leaving the classroom, Mary blushes when she addresses the teacher: *"Today we really learned something!"*

Prices in Danish Kroner

Mary and Adam have not been in a group together before, but they seem enthusiastic about what they are going to do. They start trying to set up a spreadsheet with the information from the teacher's introduction.[62] They start with the cost price of one bat, *C1*, which is 70 Swedish Kroner. Then they add insurance and transport which is 1.5 % of the cost price. They construct the formula $C2 = C1 + 0.015C1$. Then follows the transaction into Danish Kroner. Mary clears her throat:

Mary: OK, then there's the rate of exchange if you are to work out what it is in Danish Kroner, right?

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[60] Actually the surrounding voices disturb the tape recorder, so that it is difficult to hear what Mary and Adam say. That is one of the reasons for many incomprehensible utterances [ic] in the transcript.

[61] We present and analyse the whole course, but in what follows some sequences of the transcript are omitted.

[62] Mary and Adam are going to construct the following sequence of formulae (in principle):

C1 (the original price)
$C2 = C1 + 0.015C1$ (insurance and freight is added)
$C3 = 0.8129C2$ (transaction into Danish Kroner)
$C4 = C3 + 0.08C3$ (duty is added)
$C5 = C4 + 0.25C4$ (profit is added)
$C6 = C5 + 0.25C5$ (VAT is added)

Our numeration is a bit simplified compared to Mary and Adam's. The conversation is strongly indexically anchored which means that the situational context including the computer gives meaning to the large number of features like deixis, pointing, facial expressions etc. This meaning can (easily) be understood by the students in the situation, but it needs translation or explanation when presented in another context.