# Contents

## Part III Appendices

# List of Figures

# List of Symbols

| | |
|---|---|
| $a, b, c, \alpha, \beta, \mu, ...$ | Scalars over $\mathbb{R}$ or $\mathbb{C}$ |
| $\mathbf{A}, \mathbf{B}, \mathbf{X}, \mathbf{Y}...$ | Matrices; Sect. A.2 |
| $\mathbf{A} \leq \mathbf{B}$ | Partial ordering; Sect. A.2 |
| $\mathbf{A}^{-1}$ | Matrix inverse; Sect. A.3 |
| $\mathbf{A}^T$ | Transpose matrix; Definition A.3 |
| $\mathrm{A}_K(\mathbf{X})$ | Eq. (1.8) |
| $\mathrm{A} \times \mathrm{B}$ | Cartesian product |
| $\mathsf{A}$ | Sect. 5.2.1 |
| $\mathbf{A} \circ \mathbf{B}$ | Hadamard product; Sect. A.2 |
| | |
| $\mathrm{B}_K$ | Sect. 1.6.2 |
| $\overline{\mathrm{B}}_K$ | Sect. 1.6.2 |
| $\mathrm{B}$ | Sects. 4.3 and 5.2.1 |
| | |
| $\mathbb{C}$ | Sect. A.1 |
| $\mathrm{cl}(\mathrm{A})$ | Closure |
| $\mathrm{C}$ | Eq. (5.11) |
| $\tilde{\mathrm{C}}$ | Eq. (5.15) |
| | |
| $\mathrm{diag}(\mathbf{u})$ | Diagonal matrix; Sect. A.2 |
| $\det(\mathbf{A})$ | Matrix determinant; Sect. A.3 |
| $\delta_l$ | The Kronecker delta |
| | |
| $\mathbf{e}_i$ | Sect. A.1 |
| $\mathrm{E}_K(\mathbf{X})$ | Sect. 1.6.2 |
| $\mathrm{E}_K^+(\mathbf{X})$ | Sect. 1.6.2 |
| $\boldsymbol{\eta}(\mathbf{p})$ | Eq. (6.30) |
| | |
| $\mathrm{F}$ | Eq. (1.53) and Eq. (2.5) |
| $\partial\mathrm{F}$ | Eq. (1.55) |
| $\mathrm{F}^c$ | Eq. (1.60) |

| | |
|---|---|
| $\mathrm{F}(P_{\mathrm{t}})$ | Eq. (2.9) |
| $\mathrm{F}(P_1,\dots,P_K)$ | Eq. (2.11) |
| $\mathrm{F}_k(\alpha)$ | Eq. (2.12) |
| $\mathrm{F}(P_{\mathrm{t}};P_1,\dots,P_K)$ | Eq. (2.13) |
| $\partial\mathrm{F}(P_{\mathrm{t}})$ | Definition 2.15 |
| $\partial\mathrm{F}(P_1,\dots,P_K)$ | Definition 2.15 |
| $\mathrm{F}^c(P_{\mathrm{t}})$ | Sect. 2.4 |
| $\mathrm{F}_\gamma$ | Eq. (5.32) |
| $\mathrm{F}_\gamma(\mathrm{P})$ | Eq. (5.31) |
| $\partial\mathrm{F}_\gamma(\mathrm{P})$ | Eq. (5.34) |
| $f'(x), x\in\mathbb{R}$ | The first derivative; Sect. B.1 |
| $f''(x), x\in\mathbb{R}$ | The second derivative; Sect. B.1 |
| $F(\mathbf{p})$ | Eq. (6.2) |
| $F_e(\mathbf{s})$ | Eq. (6.12) |
| | |
| $\boldsymbol{\Gamma}(\boldsymbol{\omega})$ | Eqs. (1.48) and (5.12) |
| $g_k(\mathbf{p})$ | (6.21) |
| | |
| $h_k(\mathbf{s})$ | Eq. (6.14) |
| | |
| $\mathbf{I}$ | Identity matrix; Sect. A.2 |
| $I_k(\mathbf{p})$ | Eq. (6.5) |
| | |
| $\mathsf{K}$ | Sect. 4.1 |
| $\mathsf{K}(n)$ | Sect. 4.1 |
| | |
| $\lambda_p(\boldsymbol{\omega})$ | Sect. 1.3.1 |
| $\mathrm{LC}_K(\Omega)$ | Definition 1.34 |
| $\mathrm{lc}(\Omega)$ | Sect. 2.3 |
| $\mathsf{L}$ | Sect. 4.1 |
| | |
| $\mathbb{N}$ | Natural numbers |
| $\mathbb{N}_0$ | Nonnegative integers |
| $\mathsf{N}$ | Sect. 4.1 |
| $\mathrm{N}_K=\mathbb{R}_+^{K\times K}$ | Definition A.18 |
| $\mathrm{N}_K^+$ | Sect. 1.6.2 |
| $\mathrm{N}_K(\Omega)$ | Definition 1.32 |
| $\mathrm{N}_{K,\mathbf{r}}(\Omega)$ | Eq. (2.8) |
| $\Vert\mathbf{u}\Vert$ | Vector norms; Sect. A.1 |
| $\Vert\mathbf{X}\Vert$ | Matrix norms; Sect. A.2 |
| $\nabla_k f(\mathbf{x})$ | Definition B.9 |
| $\nabla f(\mathbf{x})$ | Eq. (B.3) |
| $\nabla^2 f(\mathbf{x})$ | Definition B.12 |

| | |
|---|---|
| $\mathbf{1}$ | Sect. A.1 |
| $\Omega \subset \mathbb{R}^K$ | Eq. (1.45) |
| | |
| $\Pi_K$ | Sect. 1.1 |
| $\Pi_K^+$ | Sect. 1.1 |
| $\mathrm{P}_K = \mathbb{R}_{++}^{K\times K}$ | Definition A.18 |
| $\mathrm{P}_K(\Omega)$ | Sect. |
| $\boldsymbol{\omega}$ | Sect. 1.3.1 |
| $\mathbf{p}(\mathbf{X})$ | Eq. (1.2) |
| $\mathbf{p}(\boldsymbol{\omega})$ | Eq. (2.4) |
| $\varPhi$ | Eq. (4.7) |
| $\varPsi$ | Definition 5.5 |
| $\varPsi_e$ | Definition 5.5 |
| $\psi$ | Eq. (5.36) |
| $\psi_e$ | Definition 6.1 |
| $\varPi_{\mathrm{S}}$ | Eq. (B.19) |
| $\mathrm{P} \subset \mathbb{R}_+^K$ | Eq. (4.6) |
| $\mathrm{P}_n$ | Eq. (4.6) |
| $\mathrm{P}_+$ | Eq. (6.10) |
| | |
| $\mathbf{q}(\mathbf{X})$ | Eq. (1.2) |
| $\mathrm{Q} \subset \mathbb{R}$ | Sect. 1.3.1 |
| | |
| $\mathbb{R}$ | Real numbers |
| $\mathbb{R}_+ \subset \mathbb{R}$ | Sect. A.4 |
| $\mathbb{R}_{++} \subset \mathbb{R}_+$ | Sect. A.4 |
| $\mathbb{R}^K$ | Sect. A.1 |
| $\mathbb{R}^{K\times K}$ | Sect. A.2 |
| $\mathbb{R}_{++}^K(\Omega)$ | Sect. 2.1 |
| | |
| $\sigma(\mathrm{A})$ | Matrix spectrum; Definition A.7 |
| $\rho(\mathbf{X})$ | Spectral radius; Definition A.7 |
| $\mathrm{S}_K$ | Sect. 1.2 |
| $\mathrm{S}_K(\mathbf{X})$ | Eq. (1.3) |
| $\mathrm{SIR}_k(\mathbf{p})$ | Eq. (4.2) |
| $\mathrm{S} \subset \mathbb{R}^K$ | Eq. (6.11) |
| $\mathsf{S}$ | Sect. 4.1 |
| | |
| $\mathrm{trace}(\mathbf{X})$ | Matrix trace; Sect. A.2 |
| $\boldsymbol{\theta}(\mathbf{p})$ | Eq. (6.30) |
| | |
| $\mathbf{u} \le \mathbf{v}$ | Partial ordering; Sect. A.1 |
| | |
| $\mathbf{p}, \mathbf{q}, \mathbf{s}, \mathbf{u}, \mathbf{v}, \mathbf{z}, \ldots$ | Vectors; Sect. A.1 |

| | |
|---|---|
| $\mathbf{V}$ | Eq. (4.4) |
| $\mathrm{W}_K(\mathbf{X})$ | Eq. (1.14) |
| $\mathrm{X}_K \subset \mathrm{N}_K$ | Definition A.21 |
| $\mathrm{X}_K(\Omega)$ | Definition 1.32 |
| $\mathrm{X}_{K,\mathbf{\Gamma}}(\Omega)$ | Eq. (1.49) |
| $\mathrm{X}^s_{K,\mathbf{\Gamma}}(\Omega)$ | Sect. 1.4.1 |
| $\mathrm{X}^p_{K,\mathbf{\Gamma}}(\Omega)$ | Sect. 1.4.2 |
| $\mathrm{X}^0_{K,\mathbf{\Gamma}}(\Omega)$ | Sect. 1.5 |
| $\mathbf{0}$ | Zero vector; Sect. A.1 |