
Contents

List of Symbols	XIX
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Part I Theory

1	On the Perron Root of Irreducible Matrices	3
1.1	Some Basic Definitions	3
1.2	Some Bounds on the Perron Root and Their Applications ..	4
1.2.1	Concavity of the Perron Root on Some Subsets of Irreducible Matrices	11
1.2.2	Kullback–Leibler Divergence Characterization	14
1.2.3	Some Extended Perron Root Characterizations	15
1.2.4	Collatz–Wielandt-Type Characterization of the Perron Root	18
1.3	Convexity of the Perron Root	22
1.3.1	Some Definitions	22
1.3.2	Sufficient Conditions	24
1.3.3	Convexity of the Feasibility Set	26
1.3.4	Necessary Conditions	28
1.4	Special Classes of Matrices	30
1.4.1	Symmetric Matrices	31
1.4.2	Symmetric Positive Semidefinite Matrices	32
1.5	The Perron Root Under the Linear Mapping	34
1.5.1	Some Bounds	35
1.5.2	Disproof of the Conjecture	38
1.6	Some Remarks on Arbitrary Nonnegative Matrices	41
1.6.1	Log-Convexity of the Spectral Radius	42
1.6.2	Characterization of the Spectral Radius	43
1.6.3	Collatz–Wielandt-Type Characterization of the Spectral Radius	46
1.7	Bibliographical Notes	47

2	On the Positive Solution to a Linear System with Nonnegative Coefficients	51
2.1	Basic Concepts and Definitions	51
2.2	Feasibility Sets	53
2.3	Convexity Results	56
2.3.1	Log-Convexity of the Positive Solution	56
2.3.2	Convexity of the Feasibility Set	59
2.3.3	Strict Log-Convexity	60
2.3.4	Strict Convexity of the Feasibility Sets	65
2.4	The Linear Case	66

Part II Applications and Algorithms

3	Introduction	71
4	Network Model	75
4.1	Basic Definitions	75
4.2	Medium Access Control	76
4.3	Wireless Communication Channel	79
4.3.1	Signal-to-Interference Ratio	81
4.3.2	Power Constraints	83
4.3.3	Data Rate Model	84
4.3.4	Two Examples	85
5	Resource Allocation Problem in Communications Networks	91
5.1	End-to-End Rate Control in Wired Networks	91
5.1.1	Fairness Criteria	92
5.1.2	Algorithms	95
5.2	Problem Formulation for Wireless Networks	97
5.2.1	Joint Power Control and Link Scheduling	98
5.2.2	Feasible Rate Region	101
5.2.3	End-to-End Window-Based Rate Control for Wireless Networks	103
5.2.4	MAC Layer Fair Rate Control for Wireless Networks	105
5.2.5	Utility-Based Power Control	107
5.3	Interpretation in the QoS Domain	112
5.4	Remarks on Joint Power Control and Link Scheduling	115
5.4.1	Optimal Joint Power Control and Link Scheduling	115
5.4.2	High SIR Regime	118
5.4.3	Low SIR Regime	119
5.4.4	Wireless Links with Self-Interference	122
5.5	Remarks on the Efficiency–Fairness Trade Off	123
5.5.1	Efficiency of the Max-Min Fair Power Allocation	125

5.5.2	Axiom-Based Interference Model	128
6	Power Control Algorithm	129
6.1	Some Basic Definitions	130
6.2	Convex Statement of the Problem	131
6.3	Strong Convexity Conditions	133
6.4	Gradient Projection Algorithm	137
6.4.1	Global Convergence	138
6.4.2	Rate of Convergence	140
6.4.3	Diagonal Scaling	142
6.4.4	Projection on a Closed Convex Set	142
6.5	Distributed Implementation	143
6.5.1	Local and Global Parts of the Gradient Vector	143
6.5.2	Adjoint Network	145
6.5.3	Distributed Handshake Protocol	148
6.5.4	Noisy Measurements	150

Part III Appendices

A	Some Concepts and Results from Matrix Analysis	155
A.1	Vectors and Vector Norms	155
A.2	Matrices and Matrix Norms	157
A.3	Square Matrices and Eigenvalues	158
A.3.1	Spectral Radius and Neumann Series	159
A.3.2	Orthogonal, Symmetric and Positive Semidefinite Matrices	160
A.4	Perron–Frobenius Theory	161
A.4.1	Perron–Frobenius Theorem for Irreducible Matrices	162
A.4.2	Perron–Frobenius Theorem for Primitive Matrices	165
A.4.3	Some Remarks on Reducible Matrices	166
A.4.4	The Existence of a Positive Solution \mathbf{p} to $(\alpha\mathbf{I} - \mathbf{X})\mathbf{p} = \mathbf{b}$	168
B	Some Concepts and Results from Convex Analysis	171
B.1	Sets and Functions	171
B.2	Convex Sets and Functions	175
B.2.1	Strong Convexity	176
B.3	Log-Convex Functions	177
B.3.1	Inverse Functions of Monotonic Log-Convex Functions	179
B.4	Convergence of Gradient Projection Algorithms	180
	References	185

List of Figures

1.1	The feasibility set F for some $\mathbf{X} \in X_{K,\Gamma}^p(\Omega)$ with $\gamma(x) = x, x > 0, K = 2$ and $\Omega = \mathbb{Q}^2$	34
2.1	Illustration of Example 2.3: The feasibility set $F(P_t; P_1, P_2)$ with $\mathbf{X}(\boldsymbol{\omega}) \equiv 0, \gamma(x) = e^x - 1, x > 0$, and $\mathbf{u}(\boldsymbol{\omega}) = (e^{\omega_1} - 1, e^{\omega_2} - 1)$. The constraints P_1, P_2 and P_t are chosen to satisfy $0 < P_1, P_2 < P_t$ and $P_t < P_1 + P_2$	55
2.2	The l^1 -norm $\ \mathbf{p}(\boldsymbol{\omega}(\mu))\ _1$ as a function of $\mu \in [0, 1]$ for some fixed $\hat{\boldsymbol{\omega}}, \check{\boldsymbol{\omega}} \in \mathbb{Q}^K$ chosen such that $\ \mathbf{p}(\hat{\boldsymbol{\omega}})\ _1$ and $\ \mathbf{p}(\check{\boldsymbol{\omega}})\ _1$ are independent of the choice of γ	65
2.3	$F(P_1, P_2)$ is equal to the intersection of $F_1(P_1)$ and $F_2(P_2)$. Thus, $F^c(P_1, P_2)$ is equal to the union of $F_1^c(P_1)$ and $F_2^c(P_2)$, each of which is a convex set if $\gamma(x) = x, x > 0$. However, the union of these sets is not convex in general.	67
4.1	There are five nodes represented by $\mathbf{N} = \{1, 2, 3, 4, 5\}$ and 10 wireless links: $(1, 2), (2, 1), (2, 3), (3, 2), (2, 4), (4, 2), (3, 5), (5, 3), (4, 5), (5, 4)$. The wireless links are not numbered in the figure. Two flows entering the network at nodes 1, 3 and destined for node 4 establish 6 logical links $\mathbf{K} = \{1, 2, 3, 4, 5, 6\}$. For instance, logical links (or MAC layer flows) originating at node 2 are 2 and 3 so that we have $\mathbf{K}(2) = \{2, 3\}$. These links share wireless link $(2, 4)$. The flow rates are ν_1 and ν_2 . Packets of flow 2 take two different routes to their destination that is node 4.	77

- 5.1 Three flows compete for access to two links [58, 1]. Whereas flows 1 and 2 are one-link flows going through links 1 and 2, respectively, flow 3 uses both links. The links have fixed capacities C_1 and C_2 , respectively. Clearly, the maximum total throughput is $C_1 + C_2$ and, in the maximum, the longer flow must be shut off ($\nu_3 = 0$) so that the one-link flows can be allocated rates of $\nu_1 = C_1$ and $\nu_2 = C_2$. In contrast, if $C_1 \leq C_2$, the max-min fair allocation is $\nu_1 = C_1/2, \nu_2 = C_2 - C_1/2$ and $\nu_3 = C_1/2$. Thus, the total throughput is $C_2 + C_1/2$ which is strictly smaller than $C_1 + C_2$. Note that if $C_1 = C_2$, then all source rates are equal under the max-min fair solution. 93
- 5.2 Assuming $\Phi^{-1}(x) = e^x - 1, x \in \mathbb{R}$, the figure compares the modified utilities $U(x) = \Psi(\Phi^{-1}(x)), x > 0$, with the traditional ones $U(x) = \Psi(x), x > 0$, for $\Psi(x) = \log(x), \Psi(x) - 1/x, x > 0$, and $\Psi(x) = \log x/(1+x), x > 0$ 110
- 5.3 The feasible rate region for two mutually orthogonal links subject to a sum power constraint. The region is a strictly convex set so that link scheduling between arbitrary points on the boundary of the feasible rate region is suboptimal. 117
- 5.4 The feasible SIR region for two users under total power constraint P_t and individual power constraints on each link $P_1 < P_t$ and $P_2 < P_t$. If there were no individual power constraints, a MAC policy involving a time sharing protocol between the points E and F , corresponding to power vectors $(0, P_t)$ and $(P_t, 0)$, respectively, would be optimal. In contrast, when in addition individual power constraints are imposed, a time sharing protocol between A and D (that correspond to power vectors $(0, P_2)$ and $(P_1, 0)$, respectively) is suboptimal. In this case, it is better to schedule either between A and B or between B and C or between C and D depending on the target signal-to-interference ratios. 121
- 6.1 In the primal network, the received signal samples at E1 and E2 are $y_1 = h_{1,1}X_1 + h_{1,2}X_2$ and $y_2 = h_{2,2}X_2 + h_{2,1}X_1$, respectively, where X_1, X_2 are zero-mean independent information-bearing symbols with $E[|X_1|^2] = p_1, E[|X_2|^2] = p_2$. In the adjoint network, E1 and E2 transmits $X_1/h_{1,1}$ and $X_2/h_{2,2}$, respectively, so that the received signal samples are $\tilde{y}_1 = X_1 + h_{2,1}/h_{2,2}X_2$ and $\tilde{y}_2 = X_2 + h_{1,2}/h_{1,1}X_1$ 147

List of Symbols

$a, b, c, \alpha, \beta, \mu, \dots$	Scalars over \mathbb{R} or \mathbb{C}
$\mathbf{A}, \mathbf{B}, \mathbf{X}, \mathbf{Y} \dots$	Matrices; Sect. A.2
$\mathbf{A} \leq \mathbf{B}$	Partial ordering; Sect. A.2
\mathbf{A}^{-1}	Matrix inverse; Sect. A.3
\mathbf{A}^T	Transpose matrix; Definition A.3
$\mathbf{A}_K(\mathbf{X})$	Eq. (1.8)
$\mathbf{A} \times \mathbf{B}$	Cartesian product
\mathbf{A}	Sect. 5.2.1
$\mathbf{A} \circ \mathbf{B}$	Hadamard product; Sect. A.2
\mathbf{B}_K	Sect. 1.6.2
$\overline{\mathbf{B}}_K$	Sect. 1.6.2
\mathbf{B}	Sects. 4.3 and 5.2.1
\mathbb{C}	Sect. A.1
$\text{cl}(\mathbf{A})$	Closure
\mathbf{C}	Eq. (5.11)
$\tilde{\mathbf{C}}$	Eq. (5.15)
$\text{diag}(\mathbf{u})$	Diagonal matrix; Sect. A.2
$\det(\mathbf{A})$	Matrix determinant; Sect. A.3
δ_l	The Kronecker delta
\mathbf{e}_i	Sect. A.1
$\mathbf{E}_K(\mathbf{X})$	Sect. 1.6.2
$\mathbf{E}_K^+(\mathbf{X})$	Sect. 1.6.2
$\boldsymbol{\eta}(\mathbf{p})$	Eq. (6.30)
\mathbf{F}	Eq. (1.53) and Eq. (2.5)
$\partial \mathbf{F}$	Eq. (1.55)
\mathbf{F}^c	Eq. (1.60)

$F(P_t)$	Eq. (2.9)
$F(P_1, \dots, P_K)$	Eq. (2.11)
$F_k(\alpha)$	Eq. (2.12)
$F(P_t; P_1, \dots, P_K)$	Eq. (2.13)
$\partial F(P_t)$	Definition 2.15
$\partial F(P_1, \dots, P_K)$	Definition 2.15
$F^c(P_t)$	Sect. 2.4
F_γ	Eq. (5.32)
$F_\gamma(\mathbf{P})$	Eq. (5.31)
$\partial F_\gamma(\mathbf{P})$	Eq. (5.34)
$f'(x), x \in \mathbb{R}$	The first derivative; Sect. B.1
$f''(x), x \in \mathbb{R}$	The second derivative; Sect. B.1
$F(\mathbf{p})$	Eq. (6.2)
$F_e(\mathbf{s})$	Eq. (6.12)
$\Gamma(\boldsymbol{\omega})$	Eqs. (1.48) and (5.12)
$g_k(\mathbf{p})$	(6.21)
$h_k(\mathbf{s})$	Eq. (6.14)
\mathbf{I}	Identity matrix; Sect. A.2
$I_k(\mathbf{p})$	Eq. (6.5)
\mathbf{K}	Sect. 4.1
$\mathbf{K}(n)$	Sect. 4.1
$\lambda_p(\boldsymbol{\omega})$	Sect. 1.3.1
$\text{LC}_K(\Omega)$	Definition 1.34
$\text{lc}(\Omega)$	Sect. 2.3
\mathbf{L}	Sect. 4.1
\mathbb{N}	Natural numbers
\mathbb{N}_0	Nonnegative integers
\mathbf{N}	Sect. 4.1
$\mathbf{N}_K = \mathbb{R}_+^{K \times K}$	Definition A.18
\mathbf{N}_K^+	Sect. 1.6.2
$\mathbf{N}_K(\Omega)$	Definition 1.32
$\mathbf{N}_{K,\Gamma}(\Omega)$	Eq. (2.8)
$\ \mathbf{u}\ $	Vector norms; Sect. A.1
$\ \mathbf{X}\ $	Matrix norms; Sect. A.2
$\nabla_k f(\mathbf{x})$	Definition B.9
$\nabla f(\mathbf{x})$	Eq. (B.3)
$\nabla^2 f(\mathbf{x})$	Definition B.12

$\mathbf{1}$	Sect. A.1
$\Omega \subset \mathbb{R}^K$	Eq. (1.45)
Π_K	Sect. 1.1
Π_K^+	Sect. 1.1
$P_K = \mathbb{R}_{++}^{K \times K}$	Definition A.18
$P_K(\Omega)$	Sect.
ω	Sect. 1.3.1
$\mathbf{p}(\mathbf{X})$	Eq. (1.2)
$\mathbf{p}(\omega)$	Eq. (2.4)
Φ	Eq. (4.7)
Ψ	Definition 5.5
Ψ_e	Definition 5.5
ψ	Eq. (5.36)
ψ_e	Definition 6.1
Π_S	Eq. (B.19)
$P \subset \mathbb{R}_+^K$	Eq. (4.6)
P_n	Eq. (4.6)
P_+	Eq. (6.10)
$\mathbf{q}(\mathbf{X})$	Eq. (1.2)
$Q \subset \mathbb{R}$	Sect. 1.3.1
\mathbb{R}	Real numbers
$\mathbb{R}_+ \subset \mathbb{R}$	Sect. A.4
$\mathbb{R}_{++} \subset \mathbb{R}_+$	Sect. A.4
\mathbb{R}^K	Sect. A.1
$\mathbb{R}^{K \times K}$	Sect. A.2
$\mathbb{R}_{++}^K(\Omega)$	Sect. 2.1
$\sigma(A)$	Matrix spectrum; Definition A.7
$\rho(\mathbf{X})$	Spectral radius; Definition A.7
S_K	Sect. 1.2
$S_K(\mathbf{X})$	Eq. (1.3)
$\text{SIR}_k(\mathbf{p})$	Eq. (4.2)
$S \subset \mathbb{R}^K$	Eq. (6.11)
S	Sect. 4.1
$\text{trace}(\mathbf{X})$	Matrix trace; Sect. A.2
$\theta(\mathbf{p})$	Eq. (6.30)
$\mathbf{u} \leq \mathbf{v}$	Partial ordering; Sect. A.1
$\mathbf{p}, \mathbf{q}, \mathbf{s}, \mathbf{u}, \mathbf{v}, \mathbf{z}, \dots$	Vectors; Sect. A.1

\mathbf{V}	Eq. (4.4)
$W_K(\mathbf{X})$	Eq. (1.14)
$X_K \subset N_K$	Definition A.21
$X_K(\Omega)$	Definition 1.32
$X_{K,\Gamma}(\Omega)$	Eq. (1.49)
$X_{K,\Gamma}^s(\Omega)$	Sect. 1.4.1
$X_{K,\Gamma}^p(\Omega)$	Sect. 1.4.2
$X_{K,\Gamma}^0(\Omega)$	Sect. 1.5
$\mathbf{0}$	Zero vector; Sect. A.1