

#### LECTURES ON PARALLEL COMPUTATION



# Cambridge International Series on Parallel Computation

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Cambridge International Series on Parallel Computation: 4

#### LECTURES ON PARALLEL COMPUTATION

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CAMBRIDGE UNIVERSITY PRESS

Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press The Edinburgh Building, Cambridge CB2 2RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org

Information on this title: www.cambridge.org/9780521415569

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First published 1993

This digitally printed first paperback version 2005

A catalogue record for this publication is available from the British Library

ISBN-13 978-0-521-41556-9 hardback ISBN-10 0-521-41556-X hardback

ISBN-13 978-0-521-01760-2 paperback ISBN-10 0-521-01760-2 paperback

The article by U. Vishkin is reproduced by permission of Springer-Verlag from the Proceedings of 18th ICALP 1991, Lecture Notes in Computer Science, Volume 510, edited by J. Leach Albert, B. Monien and M. Rodríguez.



### **Preface**

One friend in a lifetime is much. Two are many. Three are hardly possible. Friendship needs a certain parallelism of life, a community of thought, a rivalry of aim.

Henry Brooks Adams

This volume is essentially based on a series of lectures delivered at the Spring School of Parallel Computation held at the University of Warwick. The School was organised under the general aegis of the ALCOM (Algorithms and Complexity) project of the ESPRIT II Basic Research Actions programme of the European Community. Invited lecturers of the school were:

David Evans (Loughborough University of Technology)

Alan Gibbons (University of Warwick)

Torben Hagerup (Max Planck Institute, Saabrücken)

Zvi Kedem (Courant Institute of Mathematical Sciences, New York)

David May (Inmos Ltd., Bristol)

William McColl (University of Oxford)

Colm Ó Dúnlaing (Trinity College, Dublin)

Vijaya Ramachandran (The University of Texas at Austin)

Paul Spirakis (University of Patras)

Gerard Tel (University of Utrecht)

Jacobo Torán (Universitat Politècnica de Catalunya, Barcelona)

Uzi Vishkin (University of Maryland and Tel Aviv University)

Harry Wijshoff (University of Utrecht)

Most of the invited speakers generously found the time to contribute to this volume, as have several additional authors who have helped to widen and enrich the material:

Andrew Chin (Texas A&M University)

Costas Iliopoulos (King's College, London)

Krishna Palem (IBM Research Division, Yorktown Heights, New York)

Arvind Raghunathan (University of California, Davis)

Almost exclusively, this book is concerned with the foundations of parallel computation. The pre-dominant interest is in the efficiency of computation. There has been some effort to ensure that the bulk of the contributed chapters form a coherent stream taking the reader from a position of having little prior knowledge of the subject to a position of being familiar with leading-edge material taken from a variety of contemporary research and pre-occupations. For this reason, a few chapters are concerned with the presentation of basic material of broad interest, others present details of particular specialisations and yet others



provide a ranging but advanced perspective. The book may therefore function at the same time as a source of teaching material and as a reference for researchers.

Unencumbered with engineering details, the Parallel Random Access Machine model of parallel computation (the so-called P-RAM) has played a central rôle in studies over the last decade or so of how inherent parallelism within problems can be exploited for efficient computation. The P-RAM, which is a shared-memory model, is therefore a significant vehicle for the enquiries of this volume. The initial chapters justify and define the model and subsequent chapters use it in the development of efficient parallel algorithmic design in a variety of application areas. Return visits to chapter 1 after those ensuing chapters which are concerned with detail will provide wider appreciation of its ranging perspective. Apart from the development of deterministic algorithms, there are also contributions which exploit randomisation and investigate algorithmic resilience in the face of processor failures. Some problems with efficient sequential solutions seem inherently to resist attempts at parallelisation and this intransigence is also studied.

The second half of the book extends our enquiries into distributed memory models of computation which bear a closer relationship to extant machines and machines that are likely to be built using current technology. For such machines, both special purpose network topologies (as exemplified by dedicated systolic arrays) and networks which have been advocated for general purpose computation are reviewed. The question of efficiently implementing P-RAM algorithms on general purpose networks is addressed as are the immensely interesting prospects for general purpose parallel computers. One important strand of the latter concerns efficient emulation of the P-RAM model by distributed memory machines in a machine independent way. Studies in this area show that there seems to be no hindrance to scalable, efficient and practical parallel computation by this means. The coherent approach thus provided from the basis of the P-RAM model further justifies its study. Today the P-RAM is generally accepted at least as a model of a programming environment for general purpose parallel machines.

We thank Lesley Sims, Somasundaram Ravindran, Ben Dessau and Nick Holloway who were energetic members of the local team organising the ALCOM Spring School of Parallel Computation. Jan van Leeuwen, of the University of Utrecht and the erstwhile co-ordinator of the ALCOM project, is to be thanked for the enabling of funds. We thank the invited lecturers who so generously gave of their time to make a success of the School. They and the additional authors are also to be thanked for their final and excellent contributions to this volume. We owe a special debt of gratitude to Toula Pantziou in Patras and especially to Nick Holloway in Warwick who worked hard on technical preparation prior to publication.

Alan Gibbons, University of Warwick Paul Spirakis, University of Patras August 1992.



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