CAMBRIDGE STUDIES IN
ADVANCED MATHEMATICS 1

Algebraic automata theory

Algebraic automata theory

W.M.L.HOLCOMBE
Department of Pure Mathematics, The Queen's University of Belfast

CAMBRIDGE UNIVERSITY PRESS
CAMBRIDGE
LONDON NEW YORK NEW ROCHELLE
MELBOURNE SYDNEY

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York NY 10011–4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

First published 1982
First paperback edition 2004

A catalogue record for this book is available from the British Library

Library of Congress catalogue card number: 81-18169

ISBN 0 521 23196 5 hardback
ISBN 0 521 60492 3 paperback

To Jill, Lucy, and my mother, and in fond memory of my father and grandfather

Contents

Introduction

In recent years there has been a growing awareness that many complex processes can be regarded as behaving rather like machines. The theory of machines that has developed in the last twenty or so years has had a considerable influence, not only on the development of computer systems and their associated languages and software, but also in biology, psychology, biochemistry, etc. The so-called 'cybernetic view' has been of tremendous value in fundamental research in many different areas. Underlying all this work is the mathematical theory of various types of machine. It is this subject that we will be studying here, along with examples of its applications in theoretical biology, etc.

The area of mathematics that is of most use to us is that which is known as modern (or abstract) algebra. For a hundred years or more, algebra has developed enormously in many different directions. These all had origins in difficult problems in the theory of equations, number theory, geometry, etc. but in many areas the subject has taken on its own momentum, the problems arising from within the subject, and as a result there has been a general feeling that much of abstract algebra is of little practical value. The advent of the theory of machines, however, has provided us with new motivation for the development of algebra since it raises very real practical problems that can be examined using many of the abstract tools that have been developed in algebra. This, to me, is the most exciting aspect of the subject, the ability of using algebra in a useful and meaningful way to tackle some of the fundamental questions facing us today: What can machines do? How do we think and speak? How do cells repair themselves? How do biochemical systems function? How do organisms grow and develop? etc. We will not be able to answer these questions here, that is neither possible nor the aim of this book. What we will be doing is to lay the foundations for the algebraic study of machines, by looking at various types of machines, their properties, and ways in which complex machines can be simulated by simpler machines joined together in some way. This will then provide a theoretical framework for the more detailed analysis of the applications of machine theory in these subjects, with the ultimate aim of explaining many natural and artificial phenomena. Perhaps a later work devoted to the applications of machine theory would make use of the developments outlined here.

We begin with some elementary material concerning the theory of semigroups. This is presented as concisely as possible; it may be omitted by those readers familiar with the material. Others could easily start by reading the first few pages of chapter 2, which introduces the state machine, before returning (hopefully better motivated) to the details of chapter 1.

The second chapter examines many elementary properties of the state machine, the ways in which it can be connected together with others, and finishes with some applications. I have tried to include as wide a variety as possible and I have not treated them in great depth because the required background knowledge in biology, biochemistry, etc. may not be available. For those interested, the references provide sources of further reading.

Chapter 3 develops the idea of a covering, by which state machines can be simulated by other, perhaps simpler, state machines in various configurations. This area represents a major change in philosophy in algebra since we do not attempt to describe the machine exactly but rather what it can do. There are some general results that enable us to start with an arbitrary state machine and simulate it with simpler machines constructed from finite simple groups and elementary 'two-state' machines connected up suitably. The best known method for doing this, the holonomy decomposition, is examined in chapter 4. However, this process leads to simulating machines that can be very large and thus relatively inefficient. In specific situations it is possible to develop much better simulators and a variety of techniques for doing this are examined in chapter 3.

The theory of recognizers is intimately connected with the theory of state machines and is of considerable importance in the theory of computers. This area is studied in chapter 5.

Finally we end with a more practical and realistic type of machine and apply the previous results to this situation in chapter 6.

Some of this material would be suitable for an advanced undergraduate course on applied algebra or automata theory and I have indeed given such a course for some years at Queen's University, Belfast. The more advanced material (chapters 4, 6) would be suitable for a graduate course.

I hope that this book can help forge links between pure mathematicians, computer scientists and theoretical biologists. There are great benefits in a dialogue between practitioners of these subjects and although I realize that the approach here is rather mathematical, I hope that it will not prevent others from making use of the material. With this in mind I have included as an appendix a computer program for evaluating the semigroup of a state machine. This has been developed for me by Dr A. W. Wickstead (Pure Mathematics, Q.U.B.) and I would like to take this opportunity of thanking him for his help. The program is suitable for use on a microcomputer, something that is becoming readily available these days.

My other thanks go to many of my colleagues at Queen's who have helped me with various questions. As usual, though, I have to take responsibility for any errors that may occur in this work.

Sheila O'Brien (Q.U.B.) made an excellent job of typing my manuscript and I would like to record my gratitude here.

I must also thank Dr E. Dilger (Tübingen) for reading the manuscript and Dr B. McMaster for helping me with the proofs.

Michael Holcombe (*July 1981*)