FUNDAMENTALS OF SEISMIC WAVE PROPAGATION

CHRIS H. CHAPMAN
Schlumberger Cambridge Research

CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York, NY 10011–4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

First published 2004

Printed in the United Kingdom at the University Press, Cambridge

Typeface Times 11/14 pt. *System* LaTeX 2ε [TB]

A catalogue record for this book is available from the British Library

Library of Congress Cataloguing in Publication data

Chapman, Chris H., 1945–
Fundamentals of seismic wave propagation / Chris H. Chapman.
p. cm.
Includes bibliographical references and indexes.
ISBN 0 521 81538 X
1. Seismic waves. 2. Wave-motion, Theory of. 3. Seismology – Mathematics. I. Title.

QE538.5.C48 2004
551.22 – dc22 2003062528

ISBN 0 521 81538 X hardback

Contents

1
Introduction

Numerical simulations of the propagation of elastic waves in realistic Earth models can now be calculated routinely and used as an aid to survey design, interpretation and inversion of data. The theory of elastodynamics is complicated enough, and models depend on enough multiple parameters, that computers are almost essential to evaluate final results numerically. Nevertheless a wide variety of methods have been developed ranging from exact analytical results (in homogeneous media and in homogeneous layered media, e.g. the Cagniard method), through approximations (asymptotic or iterative, e.g. ray theory and the WKBJ method), transform methods in stratified media (propagator matrix methods, e.g. the reflectivity method), to purely numerical methods (e.g. finite-difference, finite-element or spectral-element methods), in one, two and three-dimensional models. Recent extensions of approximate methods, e.g. the Maslov method, quasi-isotropic ray theory, and Born scattering theory and the Kirchhoff surface integral method applied to anisotropic, complex media have extended the range of application and/or validity of the basic methods.

Although the purely numerical methods can now be used routinely in modelling and interpretation, the analytic, asymptotic and approximate methods are still useful. There are three main reasons why the simpler, approximate but less expensive methods are useful and worth studying (and developing further). First, complete numerical calculations in realistic Earth models are as complicated to interpret as real data. Interpretation normally requires different parts of the signal to be identified and used in interpretation. Signals that are easy to interpret are usually well modelled with approximate, inexpensive theories, e.g. geometrical ray theory. Our intuitive understanding of wave propagation normally corresponds to these theories. As no complete, robust, non-linear inverse theory has been developed, we must use simple modelling theories to interpret real data and understand (and check) numerical calculations. Secondly, the analytic and approximate modelling methods allow the properties of different parts of the signals to be analysed independently. Again for survey design, interpretation and inversion, this reduction of the properties and sensitivities of different signals to different parameters of the model is invaluable. Finally, although numerical solutions are possible in realistic Earth models, practical limitations still exist. Calculations in two dimensions are now inexpensive enough that they can be used routinely and complete surveys simulated, but in three dimensions this is only possible with compromises. Although computer speeds and memory have and continue to increase dramatically, this limitation in three dimensions is unlikely to disappear soon. To simulate a three-dimensional survey, the number of sources and receivers normally increases quadratically with the dimension of the survey (apart from the fact that acquisition systems are improving rapidly and the density of independent sources and receivers is also increasing). More importantly, there is a severe frequency limitation on numerical calculations. The expense of numerical methods rises as the fourth power of frequency (three from the spatial dimensions as the number of nodes in the model is related to the shortest wavelengths required, and one through the time steps or bandwidth required to model the highest frequency). Currently and for the foreseeable future, this places a severe limitation on the numerical modelling of high-frequency waves in realistic, three-dimensional Earth models. Analytic, asymptotic and approximate methods, in which the cost is independent or not so highly dependent on frequency, are and will remain useful. This book develops these methods (and does not discuss the purely numerical methods).

Although the analytic, asymptotic and approximate methods have limited ranges of validity, recent extensions of these methods have been very successful in increasing their usefulness. This book discusses four extensions of asymptotic ray theory which are inexpensive to compute in realistic, three-dimension Earth models: Maslov asymptotic ray theory extends ray theory to caustic regions; quasi-isotropic ray theory extends ray theory to the near degeneracies that exist in weakly anisotropic media; Born scattering theory that models signals scattered by small perturbations in the model and importantly allows signals due to errors in the ray solution to be included; and the Kirchhoff surface integral method which allows signals and diffractions from non-planar surfaces to be modelled at least approximately. Although these methods are widely used, limitations exist in the theories and further developments are needed. The future will probably see the development of hybrid methods that combine these and other extensions of ray theory with one another, and with numerical methods.

The foundations of elastic wave propagation were available by the beginning of the 20th century. Hooke's law had been extended to elasticity. Cauchy had developed the theory of stress and strain, each depending on six independent components, and Green had shown that 21 independent elastic parameters were necessary in general anisotropic media. In isotropic media this number reduces to two (the Lamé, 1852, parameters) and the existence of *P* and *S* waves was known. Love (1944, reprinted from 1892) gave an excellent review of the development of elasticity theory. Rayleigh (1885) had explained the existence of the waves now named after him, that propagate along the surface of an elastic half-space. Finally Lamb (1904), in arguably the first paper of theoretical seismology, was able to explain the *excitation* and *propagation* of *P* and *S* rays, head waves and Rayleigh waves from a point source on a homogeneous half-space. The paper contained the first theoretical seismograms.

Developments after Lamb's classic paper were initially slow. Stoneley (1924) established the existence of interface waves, now bearing his name, on interfaces between elastic half-spaces. Only with Cagniard (1939) was a new theoretical method developed which was a significant improvement on Lamb's method, although it was not until de Hoop (1960) that this became widely known and used. Bremmer (e.g. 1939; van der Pol and Bremmer, 1937*a*, *b*, etc.) in papers concerning radio waves developed methods that would become useful in seismology. Pekeris (1948) studied the excitation and propagation of guided waves in a fluid layer, calculating theoretical seismograms (although the existence of the equivalent Love waves had been known before). Lapwood (1949) studied the asymptotics of Lamb's problem in much greater detail, and Pekeris (1955*a*, *b*) developed another analytic method equivalent to Cagniard's. After a slow start in the first half of the 20th century, rapid developments in the second half depended on computers and improvements in acquisition systems to justify numerical simulations. This book describes these developments.

Chapter 2: *Basic wave propagation* introduces our subject by reviewing the basics of wave propagation. In particular, the properties of plane and spherical waves at interfaces are described. Ray results in stratified media are outlined in order to describe the morphology of travel-time curves. The various singularities, discontinuities and degeneracies of these ray results are emphasized, as it is these regions that are of particular interest throughout the rest of the book.

This introductory chapter is followed by two review chapters: Chapter 3: *Transforms* and Chapter 4: *Review of continuum mechanics and elastic waves*. The first of these reviews the various transforms – Fourier, Hilbert, Fourier–Bessel, Legendre, Radon, etc. – used throughout the rest of the book. This material can be found in many textbooks and is included here for completeness and to establish our notation and conventions. Chapter 4 reviews continuum mechanics – stress, strain, elastic parameters, etc. – and the generation of plane and spherical elastic waves in homogeneous media – *P* and *S* waves, point force and stress glut sources, radiation patterns, etc. – together with some fundamental equations and theorems of elasticity – the Navier wave equation, Betti's theorem, reciprocity, etc.

The main body of the book begins with Chapter 5: *Asymptotic ray theory*. This develops asymptotic ray theory in three-dimensional acoustic and anisotropic elastic media, and then specializes these results to isotropic elastic media and one and two-dimensional models. The theory for kinematic ray tracing (time, position and ray direction), dynamic ray tracing (geometrical spreading and paraxial rays) and polarization results is described. These are combined with the results from the previous chapter, to give the ray theory Green functions.

Chapter 6: *Rays at an interface* extends these results to models that include interfaces, discontinuities in material properties. The additions required for kinematic ray tracing (Snell's law), dynamic ray tracing and polarizations (reflection/transmission coefficients) are developed for acoustic, isotropic and anisotropic media, for free surfaces, for fluid media and for differential coefficients. Finally these are combined with the results of the previous chapter to give the full ray theory Green functions for models with interfaces.

The results of these two chapters on ray theory break down at the singularities of ray theory, i.e. caustics, shadows, critical points, etc. The next three chapters develop transform methods for studying signals at these singularities but restricted to stratified media. The first chapter, Chapter 7: *Differential systems for stratified media*, reduces the equation of motion and the constitutive equations to one-dimensional, ordinary differential equations for acoustic, isotropic and anisotropic media. Care is taken to preserve the notation used in the previous chapters on ray theory to emphasize the similarities and reuse results. The chapter then develops various solutions of these equations, in homogeneous and inhomogeneous layers, using the propagator and ray expansion formalisms. The WKBJ and Langer asymptotic methods, and the WKBJ iterative solutions, are included. The second chapter of this group, Chapter 8: *Inverse transforms for stratified media*, then describes the inverse transform methods that can be used with the solutions from the previous chapter. These include the Cagniard method in two and three dimensions, the WKBJ seismogram method and the numerical spectral method. These methods are then used in the final chapter of the group, Chapter 9: *Canonical signals*, which describes approximations to various signals that occur in many simple problems. These range from direct and turning rays, through partial and total reflections, head waves and interface waves, to caustics and shadows. These results link back to the introductory Chapter 2 where the various singularities, discontinuities and degeneracies of ray results had been emphasized. Particular emphasis is placed on describing the signals using simple, 'standard' special functions.

The final chapter, Chapter 10: *Generalizations of ray theory*, describes recent extensions of ray theory which increase the range of application and validity and include some of the advantages of the transform methods. The methods are Maslov asymptotic ray theory which extends ray theory to caustic regions; quasi-isotropic ray theory which extends ray theory to the near degeneracies that exist in weakly anisotropic media; Born scattering theory which models signals scattered by small perturbations in the model and importantly allows signals due to errors in the ray solution to be included; and the Kirchhoff surface integral method which allows signals and diffractions from non-planar surfaces to be modelled at least approximately.