Relevant Logic

This book introduces the reader to relevant logic and provides the subject with a philosophical interpretation. The defining feature of relevant logic is that it forces the premises of an argument to be really used ('relevant') in deriving its conclusion. The logic is placed in the context of possible world semantics and situation semantics, which are then applied to provide an understanding of the various logical particles (especially implication and negation) and natural language conditionals. The book ends by examining various applications of relevant logic and presenting some interesting open problems. It will be of interest to a range of readers including advanced students of logic, philosophical and mathematical logicians, and computer scientists.

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A Philosophical Interpretation

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Contents

Preface

This book is a philosophical interpretation of relevant logic. Relevant logic, also called 'relevance logic', has been around for at least half a century. It has been extensively developed and studied in terms of its mathematical properties. So relevant logic is a highly developed and mathematically well-understood branch of non-classical logic. But what is it good for and why should we adopt it? I think that it is a good tool for understanding ordinary deductive reasoning and that it provides us with the tools to understand conditionals. And that is what this book is all about.

Unlike intuitionist logic, relevant logic does not come packaged with its own philosophy. There are intuitionists, and they all share a large number of important philosophical views that non-intuitionists reject. Although some relevant logicians have talked about 'the relevantist', relevantism is not a well-developed view, nor one that is widely held even by relevant logicians. By and large, we are free to adopt their own philosophical interpretation of relevant logic.

Historically, my own view developed out of my acquaintance with the possible worlds approach to semantics. When I was a graduate student, I studied modal logic and Montague grammar and found the framework of possible worlds to be a very intuitive, elegant and powerful framework in which to do semantics. Later, after I had become immersed in relevant logic, I wanted to give others the same sort of feeling of being at home in relevant logic that I had felt when I was first exposed to possible world semantics. This book is the latest product of that attempt.

I begin with the possible worlds framework. It provides both the basis for my semantics and the ontology that underpins that semantics. The central notion in my interpretation of relevant logic is that it shows us how to make inferences about what *situations* hold at worlds. I have borrowed the notion of a situation from the situation semanticists – Jon Barwise, John Perry and their followers. But I have constructed these situations out of the elements available in possible world semantics.

What I want to do in this book is be able to give philosophers and others an intuitive grasp of what's going on in relevant logic and its semantics, and to show that it is viable and useful.

I have three main aims in this book. First, I try to show that relevant logic and its semantics are intelligible and intuitive. Second, I argue that relevant logic is useful. It provides us with a theory of inference, is the basis for a good theory of conditionals, and has several other uses. Third, I try to demonstrate that relevant logic does not force any untoward philosophical commitments on us. I argue this last point by constructing a model for relevant logic out of possible worlds, individuals and structures available in Peter Aczel's non-well-founded set theory. The latter is a very elegant theory and I recommend it to everyone working in logic.

Possible world semantics is the current paradigm in philosophical logic. If I can show that the same entities (pretty much) that are used in possible world semantics can be used to construct the model that I intend for relevant logic, I can undercut any metaphysical argument against relevant logic.

Although I do not assume any previous familiarity with relevant logic, this book is not a textbook in relevant logic. Rather, it is a philosophy book about relevant logic. I have tried to make the ideas in it accessible to people familiar with natural deduction. I have not tried to provide a survey of relevant logics, nor of the mathematical results about relevant logic. There are various good surveys of this sort. Among the shorter surveys are (Dunn 1986), (Mares and Meyer 2001), and (Mares 2002b). For readers with a good deal of logical sophistication, there are the *Entailment* volumes (Anderson and Belnap 1975) and (Anderson *et al.* 1992), and the volumes of *Relevant Logics and their Rivals* (Routley *et al.* 1982) and (Brady 2003). More recently, there are two excellent textbooks on substructural logics. Relevant logics are substructural logics, and so are treated in these books. The books are Greg Restall's *An Introduction to Substructural Logics* (Restall 2000) and Francesco Paoli's *Substructural Logic: A Primer* (Paoli 2002). I recommend Restall's and Paoli's books for readers who want to explore the technical properties of relevant logics and compare them to other sorts of logics. For the reader who would like a good and very readable general introduction to the topic of non-classical logics, there is Graham Priest's *An Introduction to Non-Classical Logic* (Priest 2001).

One last note. I use the term 'relevant logic' rather than 'relevance logic' because it sounds more natural. I find that North Americans tend to use 'relevance logic' and Australasians and Europeans use 'relevant logic'. I haven't canvassed the views of people from other continents, so I can't speak with authority about what they would say. Some people have tried to link different attitudes towards logic to the use of these different terms, but my usage is merely one of convenience. In fact, in an early draft of this book I used the two terms interchangeably. I was told by my editor and three referees to choose one term and stick with it. So I chose 'relevant logic'.

The plan of this book

The book is divided into three parts. In the first, I outline relevant logic and its model-theoretic semantics. While doing so, I give both philosophical motivations. In so doing, I do not discuss every aspect of the logic and its semantics. Rather, I paint a picture of the logic using a fairly broad brush. I leave 'housekeeping' details for appendices that appear at the end of the book. I have a chapter on each of the more problematic connectives – implication, negation, and modal operators. In addition, there is a chapter introducing the principal ideas behind relevant logic, a chapter introducing possible world semantics and the Routley–Meyer semantics, and a chapter discussing the metaphysical implications of adopting relevance logic.

The second part uses the semantics and ideas from the first part to develop a theory of conditionals. There are two chapters in this section – one on indicative conditionals and the other on counterfactuals.

The third part of the book goes into more detail about the theory of deductive inference. Chapter 9 gives some technical details concerning the nature of premises and logical consequence. Chapter 10 uses these details, along with the theory of conditionals developed in part II, to give an analysis of the rule of disjunctive syllogism. In the final chapter, further uses of relevant logic and some interesting open problems are discussed.

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I have discussed issues that crop up in this book with almost everyone I know and most of the people I have met. I can't list all of their names, but I will try to get in everyone with whom I have had substantive discussions or who has read drafts of this book. Among these, three stand out. Mike Dunn first taught me relevant logic when I was a graduate student. I have learned a great deal from his own work and from his comments on my work. Bob Meyer was my mentor when I was a Visiting Fellow at the Automated Reasoning Project at the Australian National University. We wrote a series of papers together and continue to work together on projects in relevant logic. I cannot express how much I have learned from Bob or the debt of gratitude that I feel I owe him. Max Cresswell read an early draft of this book and made very valuable comments. He also talked about logic (relevant, modal, and otherwise) with me on a great many occasions and I have also learned much from him.

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