London Mathematical Society Student Texts

Managing editor: Professor J. W. Bruce, Department of Mathematics
University of Liverpool, Liverpool L69 3BX, United Kingdom

Titles are available from http://www.cambridge.org

3 *Local fields*, J. W. S. CASSELS
4 *An introduction to twistor theory, second edition*, S. A. HUGGETT & K. P. TOD
5 *Introduction to general relativity*, L. P. HUGHSTON & K. P. TOD
7 *The theory of evolution and dynamical systems*, J. HOFBAUER & K. SIGMUND
9 *Automorphisms of surfaces after Nielsen and Thurston*, A. CASSON & S. BLEILER
11 *Spacetime and singularities*, G. NABER
12 *Undergraduate algebraic geometry*, MILES REID
13 *An introduction to Hankel operators*, J. R. PARTINGTON
15 *Presentations of groups, second edition*, D. L. JOHNSON
17 *Aspects of quantum field theory in curved spacetime*, S. A. FULLING
18 *Braids and coverings: Selected topics*, VAGN LUNDSGAARD HANSEN
20 *Communication theory*, C. M. GOLDIE & R. G. E. PINCH
21 *Representations of finite groups of Lie type*, FRANÇOIS DIGNE & JEAN MICHEL
22 *Designs, graphs, codes, and their links*, P. J. CAMERON & J. H. VAN LINT
23 *Complex algebraic curves*, FRANCES KIRWAN
24 *Lectures on elliptic curves*, J. W. S. CASSELS
26 *An introduction to the theory of L-functions and Eisenstein series*, H. HIDA
27 *Hilbert space: Compact operators and the trace theorem*, J. R. RETHERFORD
28 *Potential theory in the complex plane*, T. RANSFORD
29 *Undergraduate commutative algebra*, M. REID
31 *The Laplacian on a Riemannian manifold*, S. ROSENBERG
32 *Lectures on Lie groups and Lie algebras*, R. CARTER, G. SEGAL & I. MACDONALD
33 *A primer of algebraic D-modules*, S. C. COUTINHO
34 *Complex algebraic surfaces*, A. BEAUVILLE
35 *Young tableaux*, W. FULTON
37 *A mathematical introduction to wavelets*, P. WOJTASZCZYK
38 *Harmonic maps, loop groups and integrable systems*, M. GUEST
39 *Set theory for the working mathematician*, K. CIESIELSKI
40 *Ergodic theory and dynamical systems*, M. POLLICOTT & M. YURI
41 *The algorithmic resolution of diophantine equations*, N. P. SMART
42 *Equilibrium states in ergodic theory*, G. KELLER
43 *Fourier analysis on finite groups and applications*, A. TERRAS
44 *Classical invariant theory*, P. OLVER
45 *Permutation groups*, P. J. CAMERON
47 *Introductory lectures on rings and modules*, J. BEACHY
48 *Set theory*, A. HAJNÁL & P. HAMBURGER
49 *An introduction to K-theory for C^*-algebras*, M. RØRDAM, F. LARSEN & N. LAUSTSEN
50 *A brief guide to algebraic number theory*, H. P. F. SWINNERTON-DYER
51 *Steps in commutative algebra, second edition*, R. Y. SHARP
52 *Finite Markov chains and algorithmic applications*, O. HÄGGSTRÖM
53 *The prime number theorem*, G. J. O. JAMESON
54 *Topics in graph automorphisms and reconstruction*, J. LAURI & R. SCAPELLATO
55 *Elementary number theory, group theory, and Ramanujan graphs*, G. DAVIDOFF, P. SARNAK & A. VALETTE
56 *Logic, induction and sets*, T. FORSTER

London Mathematical Society Student Texts 60

Linear Operators and Linear Systems
An Analytical Approach to Control Theory

JONATHAN R. PARTINGTON
University of Leeds

CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York, NY 10011-4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

First published 2004

Printed in the United States of America

Typeface Computer Modern 9.5/12 pt. *System* LaTeX 2$_\varepsilon$ [AU]

A catalog record for this book is available from the British Library.

Library of Congress Cataloging in Publication data

Partington, Jonathan R. (Jonathan Richard), 1955–
Linear operators and linear systems : an analytical approach to control theory / Jonathan R. Partington.
p. cm. — (London Mathematical Society student texts ; 60)
Includes bibliographical references and index.
ISBN 0-521-83734-0 – ISBN 0-521-54619-2 (pbk.)
1. Linear operators. 2. Linear systems. I. Title. II. Series.
QA329.2.P37 2003
515′.7246—dc22 2003066665

ISBN 0 521 83734 0 hardback
ISBN 0 521 54619 2 paperback

Contents

Preface *page* vii

1 Operators and Hardy spaces 1
1.1 Banach spaces and bounded operators 1
1.2 Hardy spaces on the disc and half-plane 4
1.3 Inner and outer functions 8
1.4 Vector-valued Hardy spaces 13

2 Closed operators 17
2.1 The graph of an operator 17
2.2 Semigroups 22
2.3 The gap metric 30

3 Shift-invariance and causality 37
3.1 Invariant subspaces 37
3.2 Invariant operators 44
3.4 Causality 49
3.4 The commutant lifting theorem 53

4 Stability and stabilization 63
4.1 Stability theory 63
4.2 Robustness 72
4.3 The chordal metric 81

5 Spaces of persistent signals 89
5.1 Almost-periodic functions 89
5.2 Power signal spaces 105
5.3 Spectral distribution functions 109

6 Delay systems 121
6.1 Background and classification 121
6.2 Stability 129
6.3 Rational approximation 134
6.4 Stabilization 144

Bibliography 151
Index 162

Preface

It should be emphasised at the start that this book does not claim to be an exhaustive treatise on either linear operators or linear systems, but it presents an introduction to the common ground between the two subjects, one pure mathematical and one applied, by regarding a linear system as a (causal) shift-invariant operator on a Hilbert space such as $\ell^2(\mathbb{Z}_+)$ or $L^2(0,\infty)$. It therefore includes material on Hardy spaces, shift-invariant operators, the commutant lifting theorem, and almost-periodic functions, which might traditionally be regarded as "pure" mathematics, and is suitable for those working in analysis who wish to learn more advanced material on linear operators.

At the same time, it is hoped that students and researchers in systems and control will find the approach taken attractive, including as it does much recent material on the mathematical side of systems theory, which cannot easily be found elsewhere: these include recent developments in robust control, power signal spaces, and the input–output approach to time-delay systems. Parts of this book have been expounded in graduate courses and other lectures at that level and could be used for a similar purpose elsewhere.

Chapter 1 begins with a review of basic operator theory without proofs. All this material can be found in any introductory course and many textbooks, and so is included mostly for reference. The other main topic of this chapter, which is treated in considerably more detail, is that of Hardy spaces, which are Banach spaces of analytic functions on the disc or half-plane. Our treatment covers the essential ideas (in particular inner and outer functions) that will be needed later.

In Chapter 2 we begin with material that will be unfamiliar to many readers, namely, the study of unbounded closed operators. The approach is to study an operator by means of its graph, and we also introduce semigroups and the gap metric. Most of the material is fairly standard, although its approach is slanted towards the applications to be encountered later. We also include a brief discussion of admissibility, a topic of active research that has not yet reached the standard textbooks.

Chapter 3 establishes the link between linear systems and shift-invariant operators. The standard Beurling–Helson–Lax–Wiener theorems on invariant subspaces are presented, with the most elementary proofs available; then we go on to shift-invariant operators, which can be thought of as operators with shift-invariant graphs. The other property that distinguishes a linear system is the idea of causality, and we include a discussion of the now-notorious Georgiou–Smith paradox. We conclude with a gentle treatment of the commutant lifting theorem, an abstract theorem in operator theory with many attractive applications.

Chapter 4 brings together the ideas of the previous chapter to discuss robust control (i.e., stabilization by feedback) from a graph point of view using coprime factorizations. This is by no means a new idea, but the mathematical developments of the 1990s that link it with the idea of shift-invariance have put this on a much more rigorous footing, and it is time to present the material in a more operator-theoretic way. We also discuss the chordal metric, originating in complex analysis, which under the name "pointwise gap metric" provides another useful way of measuring the distance between linear systems.

Chapter 5 presents the topic of spaces of persistent signals. These interest engineers greatly, but the subject is a minefield, in that several errors are reproduced in the literature. We start conventionally with an easy introduction to almost-periodic functions, presented as the simplest persistent signals; there are many accounts of these, but no recent ones seem to be as clear as the original ones of Bohr and Besicovitch; these in turn suffer from using what is now a rather old-fashioned notation. After that we move on to bounded-power signals, where we draw largely on papers written within the last five years. We conclude with non-stochastic approaches to white noise, correlation, and parts of Wiener's generalized harmonic analysis, all treated from an operator-theory viewpoint.

Chapter 6 begins with a brief discussion of finite-dimensional systems, but the main topic is that of delay systems, which we may think of as the simplest and most important example of an infinite-dimensional system, having the greatest interest from an analytic point of view. The four themes of this chapter are the classification of delay systems into retarded, neutral and advanced types; stability, a question that resolves itself into asking when delay systems represent operators of multiplication by bounded functions, and how to locate the poles of their transfer functions; rational approximation, which can now be presented in an elementary fashion using shift operators; and finally stabilization, where we apply the ideas of Chapter 4 to this concrete situation. In all cases we give a presentation with the minimum of unnecessary technical detail.

The book includes approximately 100 exercises, which are mostly intended to be quite easy and to give further practical illustrations of the main results in the text.

Since this book's theme is the link between operator theory and systems theory, we deliberately omit certain topics that do not fit into this approach, for example, the spectral theory of non-compact operators. The reader wishing to learn more advanced functional analysis and operator theory is recommended to consult the books [25, 67, 117, 146], for example. Likewise, most treatments of systems theory tend to concentrate on the finite-dimensional case and then go into more details of control design. Of these, the texts [37, 49, 66, 75, 132] may be recommended.

We take the opportunity to mention some other interesting books that take an operator-theoretic approach to linear systems, for example, [31, 32, 38]. These seem to be pitched at a more advanced level, and their selection of material is rather different. This last comment is appropriate for the monographs [20, 151], which we may expect to become classics.

I am grateful to Birgit Jacob, Romesh Kumar, Pertti Mäkilä, Denis Matignon and Martin Smith for their helpful comments on the first draft of this manuscript. Some of the book was written when I was visiting the University of Lyon I, and I thank Isabelle Chalendar and Monique Gaffier for their help in making this possible. I also wish to thank Roger Astley and Elise Oranges for their advice on the production of this book. Finally, this book is dedicated affectionately to Andrew, Chris and Kate, who between them have put up with me for more than a century.