

Introduction

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The first *International Handbook on Mathematics Education* was published by Kluwer Academic Publishers in 1996. However, most of the writing for that handbook was done in 1995 and generally reflected the main research and development foci prior to 1994. There were four sections, 36 chapters, and some 150 people contributed to the final volume either as author, reviewer, editor, or critical friend. The task was a monumental one, attempting to cover the major research and practice developments in the international field of mathematics education as it appeared to the contributors in 1995. Inevitably there were certain omissions, some developments were only starting to emerge, and some literatures were only sketchy and speculative. However that *Handbook* has had to be reprinted three times, so it clearly fulfilled a need and I personally hope that it lived up to what I wrote in its Introduction:

The *Handbook* thus attempts not merely to present a description of the international 'state-of-the-field', but also to offer synthetic and reflective overviews on the different directions being taken by the field, on the gaps existing in our present knowledge, on the current problems being faced, and on the future possibilities for development.

(Bishop et al., 1996)

Since that time there has been even more activity in our field, and now seems a good time to take stock again, to reflect on what has happened since 1995, and to create a second *Handbook* with the same overall goals. Two of the four yearly International Congresses on Mathematics Education have taken place, ICME-8 in Seville, Spain in 1996 and ICME-9 in Tokyo, Japan in 2000. As well, many other regular specialist conferences have continued, including the International Group for the Psychology of Mathematics Education (PME), the Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques (CIEAEM), the History and Pedagogy of Mathematics (HPM), the European Congress of Mathematics (ECM), the International Congress of Mathematicians (ICM), the International Congress on the Teaching of Mathematical Modelling and Applications (ICTMA), the Conferencia Interamericana de Educación Matemática (CIAEM) and the Mathematics Education and Society conference (MES).

So with all this continuing international conference activity, how has the field developed? What are the new issues and trends? What do we know now that

we didn't know then? Also, has international collaboration increased? Is there more collaborative research now taking place? Is research being restructured through these collaborations? These were some of the main questions facing the authors and editors of this *Handbook*.

However, there has also been a large number of regional conferences, and although we cannot mention them all here, perhaps we can pick out as representative the following: The Second Asian Technology Conference in Mathematics which took place in Penang, Malaysia in 1997, CERME, the European group for Research in Mathematics Education, PME-NA, the annual meeting of the North American Chapter of PME, the East Asian regional conference on mathematics education (ICMI-EARCOME), the Southeast Asian Conference on Mathematics Education (SEACME), and the Mediterranean Conference on Mathematics Education. We have also seen the first Interdisciplinary Conference of the International Society for the Arts, Mathematics, and Architecture, ISAMA 99 in San Sebastian, Spain, and the conference on the Future of Science and Mathematics Teaching and the needs of Arab society in Tunisia, 1996. There was a meeting of mathematics educators from Francophone countries in July 2000 in Grenoble, France, under the title "Espace mathématique francophone". The first all-Russia conference on Mathematical Education took place in Dubna, Russia in 2000, with the theme "Mathematics and Society: Mathematical Education at the frontier of centuries".

So does this developing regional activity mean that there is a greater representation of diversity, of both people and ideas, in our knowledge field now? Is there more involvement in research and development of colleagues from countries other than those well represented in the first *Handbook*? Is there more of a regional emphasis to research and development? Is interdisciplinary research growing? These were also some of the other questions provoking those of us who have been involved in putting this *Handbook* together.

In addition, several new books and journals have been published, demanding our attention, and in October, 2000, a famous journal in our field even celebrated its one hundred years' birthday, at a symposium in Geneva with the title: "L'Enseignement Mathématique: moments of mathematics education in the 20th century". Another significant contribution to the recent research literature in our field has come from the several international studies that ICMI has initiated:

- Perspectives on the teaching of geometry for the 21st century, in 1995 in Catania, Italy.
- The role of the history of mathematics in the teaching and learning of mathematics, in 1998 in Luminy, France.
- The teaching and learning of mathematics at university level, in 1998 in Singapore.
- The future of the teaching and learning of algebra, in 2001 in Melbourne, Australia.
- Mathematics education in different traditions: a comparative study of East Asia and the West, in 2002 in Hong Kong.

Thus, our main intention for this *Handbook* was to represent what we editors saw as the most significant changes and developments that have happened in our field over the last ten years, and we did not want to merely up-date the first *Handbook*. The other editors and myself met twice to discuss the format of this present handbook, and it was with these overall thoughts in our minds that we contacted the various authors and critical friends to solicit their help and contributions. As with the first *Handbook* I have been very fortunate to have had as section editors, four internationally respected scholars from different regions of the world: Christine Keitel from Berlin, Germany; Frederick Leung from Hong Kong S.A.R., China; Jeremy Kilpatrick from Athens, Georgia, USA; and Ken Clements from Brunei Darussalam. One of the strengths of the first *Handbook* was its international coverage and we have tried to achieve this level again. One method we used before was to arrange for some joint authoring of chapters, and another was to arrange for an internationally spread group of reviewers for each chapter. Despite some individual conflicts felt by some authors and reviewers we believe that these procedures worked well in the first *Handbook*, and we have therefore used them again as much as possible.

The arrangement of the sections and chapters in this *Handbook* is however rather different from the first. With the first *Handbook* it was necessary to achieve as much coverage of our field as we could, with chapters on, for example, elementary, secondary, tertiary, further and distance mathematics education. Here we have taken a different approach, and rather than attempting to cover everything, we have attempted to structure what we have seen as the significant trends and developments in our field that have happened in the last ten or so years. In particular, we have considered the increased international activity in our field and the increased range of publications and educational media now available. We have therefore organised this *Handbook* into four sections which we believe represent the most significant areas of our field at this time. Also within each section we have chosen foci for the chapters which we hoped would enable the authors to reflect on what has happened in the last ten years, and on what might develop in our field in the near future. As with the first *Handbook*, we intend this *Second Handbook* to be an essential resource for those who are serious students and researchers of the field of mathematics education, as well as being a resource for those in governments and educational systems who shape educational provision and curricular policy in mathematics education, in order to ensure that the voice of research enters fully into that decision-making.

One recent and highly significant factor affecting developments in our field has been the increase in global communication of all kinds. Migration has increased into most countries, and globalisation, that familiar if not well understood phenomenon, is affecting education at all levels. It is easy to state that mathematics education is an international field, but it is less clear what to make of the effects on it of increasing globalisation, not to speak of the ways our field, and that of mathematics itself, is contributing to the influences of globalisation on other aspects of our complex lives. Globalisation is thus an explicit or an implicit issue in several of the chapters of this *Handbook*. Of course we can feel

that these influences are not always positive, but by what academic criteria should they be judged? How can research contribute to clarifying these criteria, influences, and other attempts to interpret our changing global context?

As a research-based contribution to the new global discussions, the first section of this *Handbook*, edited by Christine Keitel from Berlin, focuses on policy dimensions of mathematics education. Global economic and political tensions have increased, creating the need for more effective inter-national and inter-cultural education. How is our field responding to these challenges? For example, the last ten years have been marked by heated policy debates about the role of mathematics in education, the place of numeracy in the curriculum, the quality of teacher education, etc. What other developments can we see that are enabling our students and educators to deal with the challenges that the new global society is creating? More specifically what are the policy implications of the new global situation? The first section of this *Handbook* launches out into these uncharted waters, and the six chapters there represent an important beginning in coming to grips with the key issues involved.

As a second driver of our field, technology has also continued to develop and influence significantly, yet in a way almost stealthily, our professional lives. For example, it is hard to realise that when the first *Handbook* was being created the mobile phone was in its infancy. Now it plays a taken-for-granted central role in many people's lives, as well as strengthening peer-group cultures and fostering new modes of communication such as text messaging. Likewise the internet has provided educators, parents and politicians with an amazing array of material and information, some of which can have direct applications for mathematics education. Statistical data-bases, graphical representations of all kinds, games, puzzles, architectural programs, geographic and demographic information, communication possibilities with schools and students everywhere, all are potentially interesting for mathematics educators. But how should this new technology be used to maximize its benefits and minimize its drawbacks? What can be gained but what might be lost by engaging new technologies in the mathematics education endeavour?

As might be expected therefore, technology is also present in several chapters, but it is also a major theme in the second section of this *Handbook*. It is edited by Frederick Leung from Hong Kong S.A.R., and has the title "Responses in mathematics education to technological developments". This section deals with the different technological influences being felt by mathematics educators, and it includes a first chapter based on a significant research project attempting to map the burgeoning literature in IT/mathematics education. This we hope will help to organize what is a rather disparate if active part of our research field at present.

One of the main reasons for creating this second *Handbook* is that our field has moved on in various ways, and new areas of research and developmental activity are attracting international attention, due to the new global situation in which we now all operate. In relation to this research, the third section of the

Handbook is edited by Jeremy Kilpatrick from Athens, Georgia, and has the title “Issues in research in mathematics education”.

However, as well as new areas of research developing, there has also been a strongly growing interest in the different introspections and self-analyses of the research field itself, in terms of both people and processes. For example, ethics and values clearly play a strong role in any research endeavour, but they have become more significant as more research is government and corporation funded, raising questions from many scholars about these institutions’ agendas in commissioning this research. Also the fact that research is taking place in very different cultural and social contexts, including situations of social and political conflict, means that the research methods themselves that are being currently used are also being critiqued and analysed for their appropriateness in these ‘new’ situations.

Moreover, realising the increasing significance of research in helping to come to terms with ever-increasingly complex social and political contexts has meant that there is correspondingly more concern with who is doing this research, the quality of their training, and with their motivation for doing it. For example, PhD level research exists now in many countries, alongside mathematics education as a university development but is research just to be pursued in universities? What role do, and should, teachers and other practitioners play in the research process? What kinds of preparation do they need to be recognized contributors to the research process? The chapters of Section 3 deal with some of the complex issues raised by the self-analysis and critiques of research practices referred to above.

The fourth and final section of the *Handbook* is edited by Ken Clements (Brunei Darussalam) and has the title “Professional practice in mathematics education”. It is the largest section, and reflects the enormous growth in research and interest in professional issues, especially those related to policy, and professional development. As the growing global situation creates more conflicts and challenges for us in mathematics education, as technology offers us more and more educational decision-making choices, and as more and more practitioners are engaging in different ways with research, so mathematics educators are having to develop their professionalism. If we want our professional voices to be heard in the policy-dominant arenas, then we must look to our own professional standards, professional practices, and professional futures.

Principally of course it is what happens in the mathematics classroom that remains of most concern in most countries. One often hears at international conferences discussions about why classroom practices don’t change in line with research developments, and about the supposedly increasing gap between research and practice. The seven chapters in this last section deal in different ways with aspects of this issue, but they all discuss interesting developments in research and practice that seem to be aimed at reducing this gap. Whether it is the teachers who are the focus, or teacher educators, or mathematicians involved in some way in teacher education, the research and practice being reviewed here deserve to be known by all practitioners involved with mathematics education.

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Mathematics, Mathematics Education and Economic Conditions

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ABSTRACT

This chapter looks at the rhetoric which surrounds the relationship of mathematics to the economic assumptions in modern societies. It is concerned on the one hand with the economic language which has invaded educational principles and on the other its converse, in which the language of mathematics is used to justify and authenticate political and economic arguments. Economic conditions are now globally managed – through inter-dependant markets, through overt political pressures from such bodies as the World Bank and through dominance and pressures from global corporations – and so too education is becoming an internationally uniformly conditioned commodity. The chapter also looks at the issues raised by international testing as measures of educational success and the supposed dominant variable in national economic success. After reviewing the dilemmas raised by ICT and new technologies in the context of disparate world resource divisions it also looks at the limited studies available on the impact of poverty on mathematics education in classrooms.

1. INTRODUCTION

This chapter looks at the rhetoric which surrounds the relationship of mathematics to the economic assumptions in modern societies. It is concerned with the economic language which has invaded educational principles and with the manner in which the language of mathematics is used to justify and authenticate political and economic arguments. Economic conditions are now globally managed – either through inter-dependant markets, through overt political pressures from such bodies as the World Bank and through dominance and pressures from global corporation. It looks at the issues raised by international testing as measures of educational success, and also reviews the potential impact of ICT and new technologies in the context of disparate world resource divisions. It finishes by looking at the limited studies on the impact of poverty on mathematics education in classrooms. It is also related to other chapters in Section 1 of the second *International Handbook of Mathematics Education*, in particular to: Atweh

et al. on 'Mathematics Education in International and Global Contexts'; Gates and Vistro-Yu on 'Is Mathematics for All' and 'Lifelong Mathematics Education' by FitzSimons, Coben and O'Donoghue. There is also a close connection with the chapter in the first Handbook by Jacobsen (1996) on 'International Co-operation in Mathematics Education' which looked at disparity in resources between nation states and the impact in this of the World Bank and other international organisations.

2. CONTEXT

Mathematics has a peculiarly special position in the social and political discourse across the world – to use a now dangerous word it holds global significance. As the 'language of science' it had long assumed power and influence as the terminology of science but during the last half century it has permeated many of the social sciences, including not only economics but also such social areas of debate as wealth distribution (poverty and affluence) or crime and its causes and consequences. Mathematics is initiated into political/social debate as a contribution to the conduct of rational debate – much as forensic science has become recognised in crime detection – fuelling and being fuelled by the 'myth of cold reason' (Taylor, 1996). It has been used in this form within 'economics' during its rise to political prominence. Economists use mathematics not to formulate, not to theorise, but essentially to describe their world. They thereby invoke all the security and certainty that is embedded in popular conceptions of mathematics but with no axiomatic basis and little predictability. Yet by the use of mathematics as a language of 'the market' so mathematics has become entwined and identified with market economics.

This is, in the words of Ian Stronach (1999) essentially a 'cultural performance' containing all the irrational intuitions and imagery with which plays, films, dance and writing capture the hearts and minds of their audiences. No longer an arbiter of truth, mathematical language has become the *creator* of truth. Stronach's article offers

"... a deconstruction of global evaluation discourses concerning school effectiveness and improvement. It portrays these discourses in anthropological terms, as 'cultural performances' and examines the ways in which technical discourses obscure elements of ritual, philosophy, myth and shamanism. The author concludes that such discourses, especially in their mediated forms – as league tables – are a form of contemporary 'spectacle'. They are our form of Olympic Games." (p. 173)

We shall return to this issue later when the impact of, and response to, the TIMSS activity is discussed.

This use of TIMSS as a politicised view of international comparative league tables was a much criticised issue at ICME 9 by delegates from a number of

countries. It is important not just for the impact and political import within the countries themselves, i.e. those in the developed world, but it displays an attitude of collective mind, of national priorities and of a style of national argument which colours the international project (“Our aim is to lead all other nations in the achievement of our pupils in mathematics and science”, USA Goals 2000). It brings about, it is argued, the reduction of internationalism to global capitalistic competition in which the inequities of the current world resource distribution are not merely maintained but exacerbated (see Atweh, Clarkson, & Nebres in Section 1).

Locally it also, of course, affects teachers and their actions. The constant context of assessment and critical comparison engenders a protective relationship with the activity. The act of teaching becomes that of a highly-skilled artisan, delivering the goods designed and owned by someone else. It discourages risk-taking and de-professionalises the activity. Dunne (1999) described teachers taking up a ‘positioned neutrality’ in their classroom assessments and in a similar manner the developed world also takes up a ‘positioned neutrality’ in terms of the assessments of resource provisions and resource disparities, sympathetic but dispassionate and self-absorbed.

At the opening session of ICME 9 in Tokyo, serious concerns were also raised in regard to the technological divide which increasing dependence on the new technologies is likely to create, and which could serve to exacerbate the existing differentials both globally between nations and locally between pupils. ICT was not seen as problematical in terms of its presence or power, there was universal recognition that it was already a significant arbiter and provider of the curriculum. It is the implications of economic imperialism (however altruistic the intention) coupled with epistemological imperialism which will increase the distance between global societies (Koblitz, 1996; Dubinsky & Noss, 1996; Woodrow, 2001). At a micro-level it does indeed increase the distance between those individual pupils with access and those for whom access is unavailable. The difficulties arise from the unequal distribution of resources, which the use of ICT in the curriculum enhances and increases. Yet who would gainsay its use where it improves learning and understanding.

As with the proselytisation of the ‘new maths’ in the 1970’s, when the developed countries exported their resource (altruistically – but not without profit) to the developing world and imposed an often apparently inappropriate mathematics curriculum, so now there is a real danger that they will repeat this pattern with the ICT reforms and we will arrive at a global curriculum designed by (and thus maintaining) those already holding power and resource. Some would see this as disempowering; see for example the theoretical underpinnings of the ethnomathematics movement pioneered by D’Ambrosio (1997) and Gerdes (1996) amongst others, however, Kuku (1997) and Sawiran (1995) argue for greater globalisation with its potential for raising resources and utilising the gains made by the ‘front runners’ to help those who are intent on ‘catching up’. Vithal and Skovsmose (1997) present a detailed critique of the political and social arguments relating

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Is Mathematics for All?

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ABSTRACT

Mathematics for All began as a programme in the early 1980s when concerns about pupils' access to mathematics education heightened due to the many issues surrounding the mathematics classroom and the mathematics student. The following chapter highlights these important issues by appropriately discussing the contexts within which these issues arise and may be resolved. The issues include curriculum content and assessment practices, equity among subgroups classified by gender, race, and socio-economic status, the use of mathematics as a selection device, democracy in the mathematics classroom, and the value of culture in the teaching of mathematics. The chapter, likewise, echoes the voices of marginalized groups in the mathematics classroom that are products of undemocratic pedagogical practices, societal perceptions, and cultural realities. Prerequisites for a successful Mathematics for All programme are put forward and directions for further research are offered.

1. INTRODUCTION

It can hardly be contested that many of us live in an uneven and unjust society where access to education and to justice depend on the capital one can appropriate and accumulate – particularly through the benefits the education system bestows on some individuals. There is ample evidence in the academic and research literature in education to support this contention such that it is hardly now arguable.

Many mathematicians and mathematics teachers would see their discipline as an important means through which individuals can make sense of the world; that mathematics is an empowering force in solving life's problems. The guidance to the UK National Curriculum put this argument in the following way:

Mathematics provides a way of viewing and making sense of the world. It

is used to analyse and communicate information and ideas and to tackle a range of real life tasks and real life problems.

(NCC, 1989, p. A2)

Yet, there are others who posit a more critical approach to understanding the importance of mathematics that is portrayed in that relatively value free contention. Ole Skovsmose argues that mathematics plays a more socially defining role:

Mathematics not only creates ways of describing and handling problems, it also becomes a main source for the reconstruction of that reality.

(Skovsmose, 1994, p. 52)

Mathematics thus acts as a 'gatekeeper' to social progress. A qualification in mathematics seems to be considered as vital for many careers as it is for entry to University (Harris, 1991; Stake & Easley, 1978), even when the subject to be studied bears little connection to mathematics. Furthermore, a qualification in mathematics tends to result in higher salaries in later life, so what we do in schools actually does affect not only the here-and-now existence of our pupils, it also affects their future prospects (Volmink, 1994).

In 1990, the World Conference on Education for All (WCEFA) was convened by the World Bank, UNESCO, UNICEF and UNDP. Participants in this important event included some 1500 individuals representing governments, intergovernmental bodies, non-government organisations, institutes and foundations. The conference was organized in response to the widespread concern over the deterioration of education systems during the 1980s (The Consultative Group on Early Childhood Care and Development, 1993). At its conclusion, the Conference unanimously adopted two texts that signified the world community's renewal of its commitment to ensure the rights of all people to education and knowledge: the 'World Declaration on Education for All' and a 'Framework for Action to Meet Basic Learning Needs'.

The initiative of the WCEFA was unique in two aspects: the breadth of its definitions on what is needed to make education available to all and its focus on action. Subsequent documents on each and every participant country in the conference testify to the second aspect. On what it means to make education available to all, the Conference affirmed what most educators believe:

In addition to calling for universal access to schooling for all children, the declaration reaffirms that "every person – child, youth and adult – [should] be able to benefit from educational opportunities designed to meet their basic learning needs." These include "both essential learning tools, such as literacy, oral expression, numeracy and problem solving and the basic learning content (knowledge, skills, values and attitudes) required by human beings to be able to survive, to develop their full capacities, to live and

work in dignity, to participate fully in development, to improve the quality of their lives, to make informed decisions and to continue learning.”

(UNESCO, 1990, p. 1)

To strengthen its claims, the WCEFA noted in the preamble of the *World Declaration on Education for All: Meeting Basic Learning Needs* the following realities that persist to this day:

- More than 100 million children including at least 60 million girls, have no access to primary schooling;
- More than 960 million adults, two-thirds of whom are women, are illiterate and functional illiteracy is a significant problem in all countries, industrialised and developing;
- More than one-third of the world's adults have no access to printed knowledge, new skills and technologies that could improve the quality of their lives and help them shape and adapt to, social and cultural change; and
- More than 100 million children and countless adults fail to complete basic education programmes; millions more satisfy the attendance requirements but do not acquire essential knowledge and skills.

(UNESCO, 1990, p. 1)

It is in light of the above facts that we want to question the existing conditions of mathematics education in the world. In support of the WCEFA and the *World Declaration on Education for All: Meeting Basic Learning Needs*, we want to ask, yet again, “*Is Mathematics for all?*”

1.1. *A Historical Perspective*

In the late 70s and 80s, past presidents of the Commission Internationale pour l'Etude et l'Amelioration de l'Enseignement des Mathematique (CIEAEM) tried to “end the ‘noble isolation’ of mathematics and mathematics education and its orientation towards pure mathematics only and to connect mathematics education closer to other sciences, to the social reality and to the social mathematical practice” (CIEAEM, 2000, p. 2).

The demand for a ‘*Mathematics for All*’ programme was inevitable. First, there was the demand to give access to mathematics not just to some, but to all pupils, to make school mathematics beyond the primary phase compulsory and sufficiently comprehensible in order to gain mathematically literate citizens. At the same time, a broad international audience (ICME-6 1988, 5th Day Special Programme and ICME-5 1984, Theme Group I ‘*Mathematics for All*’) acknowledged the problems of cultural selectivity and unequal distribution of mathematics education as a legitimate challenge and as a matter of world-wide consciousness and recognition. One important focus was on analysing conditions

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Mathematical Literacy

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ABSTRACT

This chapter investigates different perspectives on mathematical literacy that vary with the values and rationales of the stakeholders who promote it. The central argument is that it is not possible to promote a conception of mathematical literacy without at the same time – implicitly or explicitly – promoting a particular social practice. It is argued that mathematical literacy focussing on citizenship also refers to the possibility of critically evaluating aspects of the surrounding culture – a culture that is more or less colonised by practices that involve mathematics. Thus the ability to understand and to evaluate these practices should form a component of mathematical literacy.

1. INTRODUCTION

There is an expanding body of literature referring to the terms ‘numeracy’ and ‘mathematical literacy’, although sometimes these terms are used only as a synonym for mathematical knowledge. On the other hand, much of the literature does not refer specifically to ‘mathematical literacy’, but is relevant because of its concern with issues such as the goals of mathematics education, mathematics for all, the public image of mathematics, or with the role of mathematical knowledge for scientific and technological literacy. Accordingly, the references given in this chapter are neither comprehensive nor restricted to the discussion of numeracy or mathematical literacy.

Section 2 of this chapter briefly deals with the development of the terms ‘numeracy’ and ‘mathematical literacy’. One main part of this chapter is divided into five subsections. It provides a critical account of different perspectives on mathematical literacy. The central argument is that it is not possible to promote a conception of mathematical literacy without at the same time – implicitly or explicitly – promoting a particular social practice. Section 3.1, *Mathematical Literacy for Developing Human Capital* considers attempts of developing a cross-cultural definition of mathematical literacy for the purpose of generating measurable standards. Section 3.2, *Mathematical Literacy for Cultural Identity* reviews literature on ethnomathematics with respect to its implications for conceptualising mathematical literacy. Section 3.3, *Mathematical Literacy for Social Change*

deals with a conception of mathematical literacy that promotes the use of mathematical knowledge for analysing critical features of societal reality within a process of pursuing a different reality. Section 3.4, *Mathematical Literacy for Environmental Awareness* considers the possibilities of linking mathematical literacy not only to an individual's capacity to solve personal and local problems, but also to global environmental concerns. Section 3.5, *Mathematical Literacy for Evaluating Mathematics* schematically develops a conception of mathematical literacy that reflects the view of the author more than the previous sections. It is argued that mathematical literacy focussing on citizenship should refer to the aim of critically evaluating aspects of the surrounding culture – a culture that is more or less colonised by practices that involve mathematics. Thus the ability to understand and to evaluate these practices should form a component of mathematical literacy.

2. FROM NUMERACY TO MATHEMATICAL LITERACY

It is indisputable that in today's society the ability to deal with numbers and to interpret quantitative information is an important component of literacy in addition to speaking, writing and reading. At the same time, however, it is difficult to say what the distinct meanings of 'numeracy' and 'mathematical literacy' are.

There are a number of perspectives on numeracy or mathematical literacy that vary with respect to the culture and the context of the stakeholders who promote it. It may be seen as the ability to use basic computational and geometrical skills in everyday contexts, as the knowledge and understanding of fundamental mathematical notions, as the ability to develop sophisticated mathematical models, or as the capacity for understanding and evaluating another's use of numbers and mathematical models. These different interpretations reflect different rationales and values of proponents, such as the desire to standardise and measure the output of formal mathematics education, popularise academic mathematics, vocationalise general mathematics education, or educate critical citizens.

According to the 1959 edition of the Webster's Collegiate Dictionary, in which the term appears for the first time, 'numerate' means "marked by the capacity for quantitative thought and expression". This definition reflects the meaning of numeracy from the Crowther Report (DES, 1959); this report was concerned with the education of students in the 15–18 age group. Being 'numerate', meant to have a rather sophisticated understanding of mathematics and science (see Brown et al., 1998). Noss (1997) observes a narrowing of meaning in the Cockcroft Report (DES/WO, 1982) towards the ability to perform basic arithmetic operations and to decode information given in the form of graphical representations. He sees this narrow, number-based interpretation which excludes important ideas of mathematics (e.g., geometry, algebra and proof) as linked to the culture of utility.

'Innumeracy' and 'mathematical illiteracy' became more familiar terms in 1988 when John A. Paulos published his book 'Innumeracy. Mathematical Illiteracy and Its Consequences', a national best-seller in the USA. He shows many authentic examples of innumeracy and develops a conception of 'numeracy' as being able to understand better the quantitative aspects of one's environment. Relatively simple number facts and some elementary ideas from probability and statistics are used for estimating, for understanding large numbers by linking them to concrete examples of things, for building relationships between quantities, and for evaluating chances. However, looking at the quantitative aspects of our environment does not automatically make us see something of interest.

The volume 'On the Shoulders of Giants: New Approaches to Numeracy' (Steen, 1990), developed under the auspices of the 1989 Mathematical Sciences Education Board Curriculum Committee, provides an example of a different interpretation of numeracy that is informed by the practice of mathematics in research and in science. It starts with deep mathematical ideas, such as dimension, quantity, uncertainty, shape and change and shows a vision of the richness of mathematics as the language of patterns. This interpretation of numeracy does not primarily stress the idea of critically evaluating another's use and misuse of data and numbers, an ability that should be seen as a central ingredient of a 'liberating literacy' (Cremin, 1988).

Definitions of numeracy commonly include 'number sense' and 'symbol sense', which are asserted a mediating role between symbolic (numeric or algebraic) representations and their interpretations. Number sense refers to informal aspects of quantitative reasoning, such as the knowledge of situation-specific quantities, common sense in employing numbers as measures, and the ability to make order-of-magnitude approximations (McIntosh, Reys, & Reys, 1992). Symbol sense includes being comfortable in using and interpreting algebraic expressions, an ability that relies upon generating numeric, graphic or computer representations of algebraic expressions (Fey, 1990; Arcavi, 1994).

The National Council on Education and the Disciplines (Steen, 2001) prefers to speak of 'quantitative literacy' in stressing the importance of inquiring into the meaning of numeracy in a society that keeps increasing the use of numbers and quantitative information. Nevertheless, the term 'numeracy' is still widely used in adult mathematics education programs, even if the interpretation goes far beyond the mere functional use of numerical and technical skills to process, communicate, and interpret numerical information (Benn, 1997; Gal, 2000; FitzSimons et al., 1996).

'Mathematical literacy' and 'numeracy' cannot be literally translated into many languages so their meanings have to be paraphrased. In German, for example, there is not even a common word for 'literacy', but only for 'illiteracy'. 'Numeracy', however, connotes numbers and calculations with numbers. Hence in this chapter the term 'mathematical literacy' is used intentionally to focus attention on its connection to mathematics and to being literate. Thus it refers metaphorically to a mathematically educated and well-informed individual.