CAMBRIDGE MONOGRAPHS ON APPLIED AND COMPUTATIONAL MATHEMATICS

16 Topology for Computing

The *Cambrïdge Monographs on Applied and Computational Mathematics* reflects the crucial role of mathematical and computational techniques in contemporary science. The series publishes expositions on all aspects of applicable and numerical mathematics, with an emphasis on new developments in this fast-moving area of research.

State-of-the-art methods and algorithms as well as modern mathematical descriptions of physical and mechanical ideas are presented in a manner suited to graduate research students and professionals alike. Sound pedagogical presentation is a prerequisite. It is intended that books in the series will serve to inform a new generation of researchers.

Also in this series:

1. A Practical Guide to Pseudospectral Methods, *Bengt Fornberg*
2. Dynamical Systems and Numerical Analysis, *A. M. Stuart and A. R. Humphries*
3. Level Set Methods and Fast Marching Methods, *J. A. Sethian*
4. The Numerical Solution of Integral Equations of the Second Kind, *Kendall E. Atkinson*
5. Orthogonal Rational Functions, *Adhemar Bultheel, Pablo González-Vera, Erik Hendiksen, and Olav Njåstad*
6. The Theory of Composites, *Graeme W. Milton*
7. Geometry and Topology for Mesh Generation, *Herbert Edelsbrunner*
8. Schwarz–Christoffel Mapping, *Tofin A. Driscoll and Lloyd N. Trefethen*
9. High-Order Methods for Incompressible Fluid Flow, *M. O. Deville, P. F. Fischer, and E. H. Mund*
10. Practical Extrapolation Methods, *Avram Sidi*
11. Generalized Riemann Problems in Computational Fluid Dynamics, *Matania Ben-Artzi and Joseph Falcovitz*
12. Radial Basis Functions: Theory and Implementations, *Martin D. Buhmann*
13. Iterative Krylov Methods for Large Linear Systems, *Henk A. van der Vorst*
14. Simulating Hamiltonian Dynamics, *Ben Leimkuhler and Sebastian Reich*
15. Collocation Methods for Volterra Integral and Related Functional Equations, *Hermann Brunner*

Topology for Computing

AFRA J. ZOMORODIAN

Stanford University

CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York, NY 10011-4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

First published 2005

Printed in the United States of America

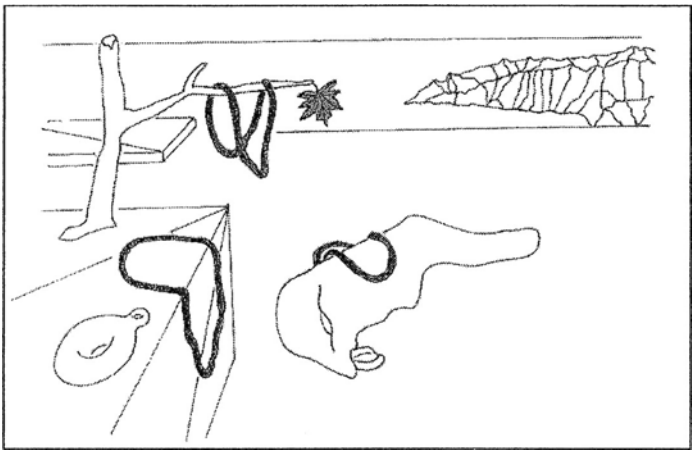
Typeface Times Roman 9/11 pt. *System* LaTeX 2_ε [AU]

A catalog record for this book is available from the British Library.

Library of Congress Cataloging in Publication Data

Zomorodian, Afra J., 1974–
Topology for computing / Afra J. Zomorodian.
p. cm. – (Cambridge monographs on applied and computational mathematics ; 16)
Includes bibliographical references and index.
ISBN 0-521-83666-2 (hardback)
1. Topology. I. Title. II. Series.
QA611.Z65 2004
514–dc22
2004047311

ISBN 0 521 83666 2 hardback



— *Persistence of Homology* — Afra Zomorodian (After Salvador Dali)

To my parents

On the left, a double-torus and a 1-cycle lie on a triangulated 2-manifold. There is a box-shaped cell-complex above. An unknot hangs from the large branch of the sapless withering tree. Through some exertion, the tree identifies itself as a maple by bearing a single green leaf. A deformed two-sphere, a torus, and a nonbounding loop form a pile in the center. Near the horizon, a 2-manifold is embedded by an associated height field. It divides itself into regions using the 1-cells of its Morse-Smale complex.

Contents

Preface

My goal in this book is to enable a non-specialist to grasp and participate in current research in computational topology. Therefore, this book is not a compilation of recent advances in the area. Rather, the book presents basic mathematical concepts from a computer scientist's point of view, focusing on computational challenges and introducing algorithms and data structures when appropriate. The book also incorporates several recent results from my doctoral dissertation and subsequent related results in computational topology.

The primary motivation for this book is the significance and utility of topological concepts in solving problems in computer science. These problems arise naturally in computational geometry, graphics, robotics, structural biology, and chemistry. Often, the questions themselves have been known and considered by topologists. Unfortunately, there are many barriers to interaction:

- Computer scientists do not know the language of topologists. Topology, unlike geometry, is not a required subject in high school mathematics and is almost never dealt with in undergraduate computer science. The axiomatic nature of topology further compounds the problem as it generates cryptic and esoteric terminology that makes the field unintelligible and inaccessible to non-topologists.
- Topology can be very unintuitive and enigmatic and therefore can appear very complicated and mystifying, often frightening away interested computer scientists.
- Topology is a large field with many branches. Computer scientists often require only simple concepts from each branch. While there are certainly a number of offerings in topology by mathematics departments, the focus of these courses is often theoretical, concerned with deep questions and existential results.

Because of the relative dearth of interaction between topologists and computer scientists, there are many opportunities for research. Many topological questions have large complexity: the best known bound, if any, may be exponential. For example, I once attended a talk on an algorithm that ran in quadruply exponential time! Let me make this clear. It was

$$O\left(2^{2^{2^{2^n}}}\right).$$

And one may overhear topologists boasting that their software can now handle 14 tetrahedra, not just 13. But better bounds may exist for specialized questions, such as problems in low dimensions, where our interests chiefly lie. We need better algorithms, parallel algorithms, approximation schemes, data structures, and software to solve these problems within our lifetime (or the lifetime of the universe.)

This book is based primarily on my dissertation, completed under the supervision of Herbert Edelsbrunner in 2001. Consequently, some chapters, such as those in Part Three, have a thesis feel to them. I have also incorporated notes from several graduate-level courses I have organized in the area: *Introduction to Computational Topology* at Stanford University, California, during Fall 2002 and Winter 2004; and *Topology for Computing* at the Max-Planck-Institut für Informatik, Saarbrücken, Germany, during Fall 2003.

The goal of this book is to make algorithmically minded individuals fluent in the language of topology. Currently, most researchers in computational topology have a mathematics background. My hope is to recruit more computer scientists into this emerging field.

Stanford, California A. J. Z.
June 2004

Acknowledgments

I am indebted to Persi Diaconis for the genesis of this book. He attended my very first talk in the Stanford Mathematics Department, asked for a copy of my thesis, and recommended it for publication. To have my work be recognized by such a brilliant and extraordinary figure is an enormous honor for me. I would like to thank Lauren Cowles for undertaking this project and coaching me throughout the editing process and Elise Oranges for copyediting the text.

During my time at Stanford, I have collaborated primarily with Leonidas Guibas and Gunnar Carlsson. Leo has been more than just a post-doctoral supervisor, but a colleague, a mentor, and a friend. He is a successful academic who balances research, teaching, and the mentoring of students. He guides a large animated research group that works on a manifold of significant problems. And his impressive academic progeny testify to his care for their success.

Eleven years after being a freshman in his "honors calculus," I am fortunate to have Gunnar as a colleague. Gunnar astounds me consistently with his knowledge, humility, generosity, and kindness. I continue to rely on his estimation, advice, and support.

I would also like to thank the members of Leo and Gunnar's research groups as well as the Stanford Graphics Laboratory, for inspired talks and invigorating discussions. This book was partially written during a four-month stay at the Max-Planck-Institut. I would like to thank Lutz Kettner and Kurt Mehlhorn for their sponsorship, as well as for coaxing me into teaching a mini-course.

Finally, I would like to thank my research collaborators, whose work appears in this book: Gunnar Carlsson, Anne Collins, Herbert Edelsbrunner, Leonidas Guibas, John Harer, and David Letscher. My research was supported, in part, by ARO under grant DAAG55-98-1-0177, by NSF under grants CCR-00-86013 and DMS-0138456, and by NSF/DARPA under grant CARGO 0138456.