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0521604931 - Fredholm Theory in Banach Spaces: (Theori Fredholm Yng Ngofodau Banach)

Anthony F. Ruston

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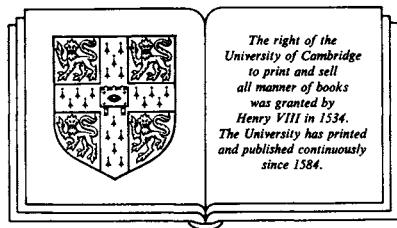
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To
FRANK SMITHIES,
*who first directed my feet
along these paths*

Gwell dysg na golud
(*Better learning than wealth*)
Welsh proverb

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Preface

At the meeting of the University of Wales Pure Mathematics Colloquium held in the delightful setting of Gregynog Hall, the University conference centre in the old county of Montgomeryshire (now part of Powys), in 1972, I gave a short talk on ‘Generalized Fredholm theory in Banach spaces’. In thanking me, Jeffrey Weston expressed the hope that I should some day collect my contributions to Fredholm theory in Banach spaces into a single book. Other friends have made similar suggestions from time to time. This tract is the outcome.

In this tract we present analogues, for operators on a Banach space, of Fredholm’s solution of integral equations (of the second kind). In the first place we imitate Fredholm’s construction for operators of finite rank. Then, ignoring questions of their construction, we consider formulae which have a structure similar to that of Fredholm’s formulae. In studying these, we use the Riesz theory. We study further the relations between these formulae and the Riesz theory, in particular obtaining bases for the finite-dimensional subspaces figuring in the Riesz theory. Then we return to the study of specific constructions for various classes of operators. A fuller summary of the tract appears in §1.2.

The theorems, lemmas and (formal) definitions are separately numbered in each section, so that for instance Definition 1.3.6 is the sixth (formal) definition in the third section of the first chapter. On the other hand, corollaries (numbered if necessary) are always quoted with the theorems to which they are attached, for example, ‘Theorem 3.1.6 Corollary’ or ‘Corollary 2 to Theorem 3.3.8’. Theorems, lemmas, definitions and corollaries are placed in logical order – for instance, lemmas used in the proof of a theorem are placed before the theorem, and not embedded in the proof. Expressions being defined, formally or informally, are printed in bold type, **roman** or *italic* according to context. Similarly, an expression may be printed in bold type when a direct or indirect reference to a definition is given. Certain obvious conventions will be observed without comment. For instance, $x^{(1)} \otimes \cdots \otimes x^{(r-1)} \otimes x^{(r+1)} \otimes \cdots \otimes x^{(n)}$ will be taken to mean $x^{(2)} \otimes \cdots \otimes x^{(n)}$ when $r = 1$ and $x^{(1)} \otimes \cdots \otimes x^{(n-1)}$ when $r = n$.

I have made no attempt to avoid appealing to such high-powered theorems as the open mapping theorem, even though it might appear to be ‘using a

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Preface

sledgehammer to crack a nut' when a more elementary, though often longer, argument is available.

It is impossible to acknowledge all my debts, often unconscious. Clearly I owe much to my teachers and colleagues, and to my other friends, among whom I include the mathematician that I have known longest, Professor C.A. Rogers, F.R.S. In particular, my debt to Frank Smithies, both when he was my research supervisor in the University of Cambridge and since, is immense. Other specific debts are mentioned in the text, mostly in Chapter 8.

On the personal level, I clearly owe a great deal to my parents, the late Colonel A.F.G. Ruston and Mrs C.M. Ruston, and also to my sisters and brother, Mrs U.M.V. Hopegood, Mrs G.C.A. Jones and the Right Reverend J.H.G. Ruston. I am also indebted to my aunt, the late Miss M.D. Symonds, who provided me with a home for the four years that I was a lecturer at King's College London. She always took an interest in my work, even though she did not pretend to understand the technical details, and I know that she was disappointed not to live to see this tract in print.

I am also grateful to my medical advisers, especially Dr D.A. Jones, and latterly Dr C. Tillson, for encouragement.

Finally, I want to express my thanks to my wife, Irene Brenda, who has cared for me with great understanding and forbearance for thirty years.

Bangor, Gwynedd

TONY RUSTON