2

Introduction

Establishing stops (or stations) within a transportation network is fundamental for offering public transportation service, since stops are an important part of the PTN. But it is not clear in advance, how many stops are reasonable, and where they should be built. Let us consider the effects of stops on the customers:

- On the one hand, many stops are advantageous from the customers' point of view, since they increase the accessibility of the trains or buses. Establishing a new stop may hence attract new customers and increase the demand. In bus transportation, the *covering radius* is often assumed to be 400 m, meaning that a customer will think about using a bus, only if the next bus stop is within a distance of at most 400 m. In rail transportation, the covering radius is larger, and is usually assumed to be 2 km.
- On the other hand, each additional stop increases the transportation time (e.g., by two minutes in rail transportation) for all trains or buses stopping there. This makes the transportation service unattractive to customers.

Moreover, this additional running time of trains (or buses) is costly for the transportation company, and also fixed costs arise for establishing a new stop.

In the *continuous stop location problem* we deal with the location of new stops along a given track system. This means, we assume that the tracks for the trains are already built, or the routes for the buses are already fixed. For the sake of simplicity we will use the wordings "stops" and "tracks" in the following, but keep in mind that the models and algorithms presented can also be applied for bus transportation.

We further assume a (possibly empty) set of already existing stops or stations. As input data we also need the locations of the potential customers, given as points or as regions in the plane, and the traffic load along the edges of the given tracks. An example for a set of demand points is depicted in Figure 2.1. Our goal is to locate additional stops along the tracks such that

- as many (potential) customers as possible live closer than a given radius r to their nearest stop, and such that
- the increase of travel time caused by the new stops is as small as possible.

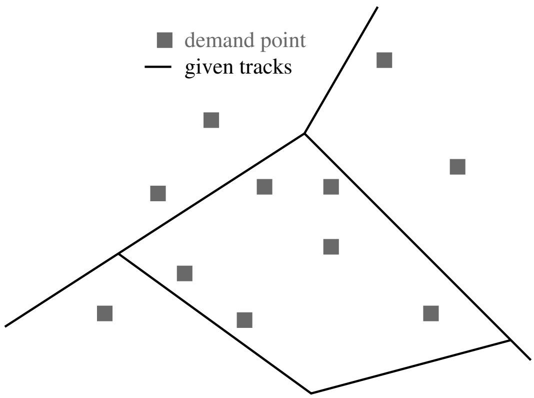

Fig. 2.1. The set of tracks $\mathcal{T}$ and a set of demand points $\mathcal{D}$ in the plane.

The result we obtain by solving the continuous stop location problem defines the PTN which is the basis for many subsequent optimization models in public transportation planning. Establishing no stop at all means that the additional travel time is minimal, but for none of the customers does the accessibility increase. The other extreme is to open stops until the complete demand is covered. The following optimization problems will be treated in this chapter.

- In the *complete cover stop location problem (CSL)* we want to cover all potential customers with as few stops as possible, or with as few costs as possible. The problem will be treated in Chapter 3 for the case that the demand is given at points and in Section 5.1 for the case of demand regions.
- The *bicriteria stop location problem (BSL)* focuses on minimizing the additional travel time and on maximizing the covered demand simultaneously. This provides solutions between the two extremes of covering the complete demand and of establishing no (additional) stop at all. (BSL) is discussed in Chapter 4.
- In the *door-to-door travel time stop location problem (DSL)* we investigate the door-to-door travel time over all customers. The door-to-door travel time for a customer is given by the time he needs to get to the first station of his trip plus the time of the trip itself plus the time he needs to reach his final destination after leaving the public transportation system. (DSL) will be considered in Section 5.2.

Chapter 2 is structured as follows: We start by presenting the application which motivated us to deal with continuous stop location problems. A literature review on stop location is given next. Then we present a model for the continuous stop location problem, enabling us to evaluate the interesting objective functions.

2.1 Application

When comparing railway systems all over Europe, it turns out that Switzerland has a higher amount of rail transportation than other countries. Among others, one reason could be that in Switzerland the number of stops compared to the overall length of the track system is significantly higher than in other countries. The interesting question arising by this observation is, if it is an advantage or a disadvantage to have many stops. To come to an answer, we consider a customer-oriented point of view. A quality criterion for the customers which is influenced by the number of stops is the door-to-door travel time of their journeys, including the time they need to get from home to their departure stations and the time they need to reach their final destinations. A priori it is not clear if this time will increase or decrease by opening new stops along the track system.

Note that by a stop we do not mean a fully equipped station, but just a stopping point for the trains, which is relatively cheap for the railway company. Our results and some of our algorithmic approaches have been implemented and tested using data of the largest German railway company, *Deutsche Bahn*. Here we located new stops along the track system, relevant for regional trains, i.e., all regional trains are supposed to stop while the fast long-distance trains pass through. Our real-world data is described next.

- We use 30 637 demand regions, given as polygons with an average of 45 nodes per polygon. These polygons are not identical with the borders of the communities and also do not form a partition of Germany. They represent the population distribution better than community borders since green land is excluded. This means that most of the data is very accurate; even relatively small towns are given as a set of more than 10 different demand regions.
- The PTN we used represents the network of *Deutsche Bahn*. It has a size of 6 828 stations and 8 724 edges.
- For each demand region we furthermore know the number of inhabitants, and for each edge we got an approximation of the traffic load, i.e., the number of customers using the edge.

Moreover, *Deutsche Bahn* specified some of the necessary parameters for our models. The time needed for an additional stopping activity of a regional train was estimated as two minutes. For the covering radius, a distance of 2 km is often used in rail transportation.

2.2 Literature Review

The importance of planning stops carefully and different customer-oriented criteria for bus stop location were already discussed in the case study of Demetsky et al, see [DAL82]. Among the many possible objective functions one goal is to establish as few stops as possible in such a way that all customers are covered. This was done in [Gle75] and in [MDSF98, Mur01a, Mur01b]. In the latter papers, the public transportation network in Brisbane, Australia was analyzed in detail and it turned out that 84.5 % of the stops are not necessary in terms of covering a set of given demand points within a Euclidean distance of 400 m, i.e., closing them would not decrease the actual number of covered customers. The stop location problem was treated in a discrete setting in these papers, i.e., the authors either considered only the actual stops, or they assumed that a finite candidate set of new stops is given. This leads to an unweighted set covering problem, also called *location set covering problem* which was introduced in [TSRB71, TR73]. In the context of stop location this problem has been solved by [Mur01a] using the Lagrangian-based set covering heuristic of [CFT99]. A new discrete stop location model was developed by Laporte et al. [LMO02]. They investigate which candidate stops along one given line in Seville should be opened, taking into account demand regions and constraints on the inter-station space. The coverage of a new stop is determined using a gravitation model. Finally, they solved the problem by a longest path algorithm in an acyclic graph. Their model resembles the *maximum coverage location problem* originally presented in [CR74, WC74].

The difference between the continuous stop location problem considered here and most papers published so far is that in the continuous stop location problem we do *not* choose the stops from a known set of possible candidates, but allow establishing a new stop anywhere along the given railway tracks (or along the given bus routes). The covering information can hence not be given explicitly but must be calculated by some (geometric) formula. The first approaches dealing with a continuous candidate set were given in [HLS$^+$01] and [SHLW02]. They are described in more detail in Section 5.2 and in Chapter 3. The results of [RS04, Sch05c, SS03] are based on these two papers and can be found in Section 3.6, Chapter 9, and Section 5.1. The research of [KPS$^+$03] was also motivated by this research. They deal with a variant of the continuous stop location problem, aiming to cover as much demand as possible with a given number of new stops, see Section 4.1. In [MMW04] the stop location problem has been investigated and solved for the case of two intersecting lines. Solving the stop location problem by data reduction of the underlying covering problem has been studied in [Mec03] and in [MW04].

2.3 A Model for Continuous Stop Location

Let $G = (V, E)$ be a finite, planar graph with straight-line embedding in the plane. In real-world data sets, the nodes of G represent either existing stations or important breakpoints. We identify each edge $e \in E$ by a line segment in the plane. Moreover,

- c_e is the traffic load along edge $e \in E$, i.e., the number of customers using edge e, and
- c_v is the traffic load through station $v \in V$, i.e., the number of customers passing through station v (and not getting on or off there).

Both parameters can be given, for example, in customers per day.

Definition 2.1. *Given $G = (V, E)$ define the* **track system**

$$\mathcal{T} = \bigcup_{e \in E} e = \{x \in \mathbb{R}^2 : x \in e \text{ for some } e \in E\} \subseteq \mathbb{R}^2$$

as the set of points on edges of the planar embedding of G.

Our goal is to establish stops (or stations), which are represented by points in $\mathcal{T}$. The evaluation of a set $S \subseteq \mathcal{T}$ is described next.

Additional Travel Time

To calculate the additional travel time induced by some set of stations $S \subseteq \mathcal{T}$ we take the number of customers affected by the additional stopping activities and multiply them by the time t_{stop} which is needed for an additional stop. According to *Deutsche Bahn*, t_{stop} can be assumed to be two minutes, independent of the location of the stop. This is specified in the following notation:

Definition 2.2. *Given $s \in \mathcal{T}$ let*

$$g(s) = \begin{cases} s \text{ if } s \in V \\ e \text{ if } s \in e, s \notin V. \end{cases}$$

Furthermore, given a finite set $S \subseteq \mathcal{T}$ we define

$$f_{\text{time}}(S) = \sum_{s \in S} t_{\text{stop}} c_{g(s)}.$$

For an infinite set S we define $f_{\text{time}}(S) = \infty$.

Since t_{stop} is a constant, e.g., two minutes in rail transportation, it can be neglected for the optimization process. Furthermore, note that $f_{\text{time}}(S) = |S|$ if all traffic loads $c_{g(s)} = 1$, i.e., if we assume that each edge is used by exactly one customer. Hence, we will refer to the *unweighted problem* if we deal with the special case of minimizing the number of stations.

The Cover of a Set of Stops

To deal with the accessibility of potential customers, we next assume that $\mathcal{D} \subseteq \mathbb{R}^2$ is a finite set of either

- demand points, or of
- pairwise disjoint demand regions

representing important points or regions such as settlements, industrial areas, shopping centers, or leisure parks.

Notation 2.3. *For $\mathcal{D}$ let*

$$D_{total} = \bigcup_{D \in \mathcal{D}} D$$

be the **demand set***. Note that $D_{total} = \mathcal{D}$ if $\mathcal{D}$ consists of demand points.*

We now introduce the notion of *covering* with respect to a distance measure γ. We may specify different distance measures for each of the elements of $\mathcal{D}$, i.e., for each of the demand points or regions. As distance measure γ_D we allow any norm or gauge (see Appendix C); readers who are not familiar with gauges may simply imagine γ_D as the Euclidean distance. For $d \in \mathbb{R}^2$, $S \subseteq \mathbb{R}^2$, let (as usual)

$$\gamma_d(d, S) = \min_{s \in S} \gamma_d(d, s).$$

Notation 2.4. *Let $d \in D_{total}$. Then γ_d denotes the distance measure associated with d.*

If $\mathcal{D}$ consists of demand regions, and $D \in \mathcal{D}$, then we require for all points $d_1, d_2 \in D$:

$$\gamma_{d_1} = \gamma_{d_2} = \gamma_D.$$

A demand point is covered, if the distance to its closest station is smaller than or equal to a given radius r, where the used distance need not be the same for all demand points. Formally, this is specified below.

Definition 2.5. *Given $r > 0$, and $S \subseteq \mathcal{T}$.*

1. *A point $d \in D_{total}$ is* **covered** *by S if $\gamma_d(d, S) \leq r$.*
2. *Furthermore, the* **cover** *is S is $\operatorname{cover}_{\mathcal{D}}(S) = \{d \in D_{total} : d$ is covered by $S\}$.*

If it is clear to which set $\mathcal{D}$ we refer, we just write $\operatorname{cover}(S)$. Furthermore, for $s \in S$ we use $\operatorname{cover}(s)$ for $\operatorname{cover}(\{s\})$. Note that if $\gamma_d = \gamma$ for all $d \in D_{total}$ we obtain

$$\operatorname{cover}_{\mathcal{D}}(S) = \{d \in \mathbb{R}^2 : \gamma(d, S) \leq r\} \cap D_{total}.$$

The cover of a point is illustrated in Figure 2.2. The small rectangles in parts (a) and (b) represent the demand points $d_1, \ldots, d_6$, while we consider two demand regions D_1 and D_2 in parts (c) and (d). All elements of $\mathcal{D}$ in parts (a) and (c) are assumed to have the Euclidean distance associated with them. In part (b), $\gamma_{d_1}, \gamma_{d_2}$, and γ_{d_3} equal the rectangular distance, while the remaining elements $d \in D_{total}$ again have γ_d as Euclidean distance. In part (d), we assume γ_{D_1} as rectangular distance and γ_{D_2} as Euclidean. In parts (a) and (b) the cover consists of the filled small rectangles, in parts (c) and (d) the cover is given by the dashed area.

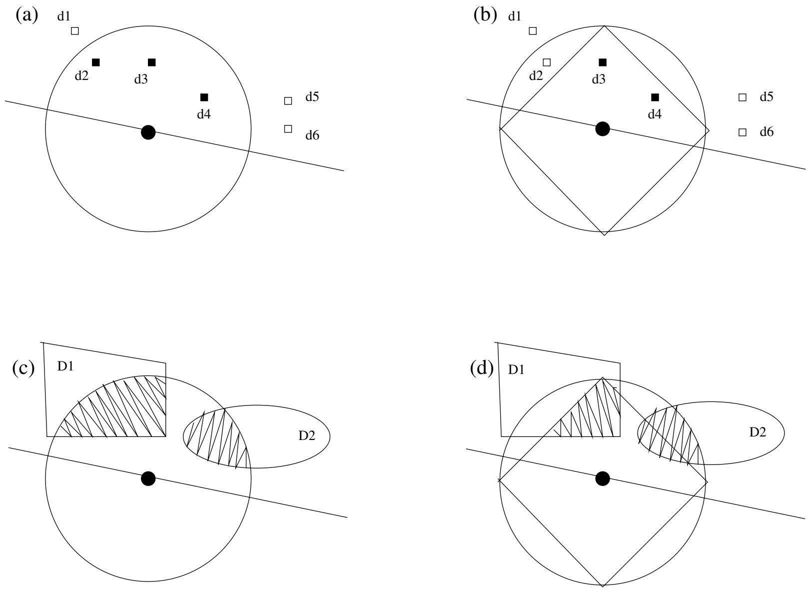

Fig. 2.2. The cover for demand points (see (a) and (b)) and for demand regions (in (c) and (d)), both for the Euclidean distance (see (a) and (c)) and for mixed rectangular and Euclidean distances (in (b) and (d)).

We further need the following notation. Consider $d \in D_{total}$ with associated distance function γ_d. Let $B_d = \{x \in \mathbb{R}^2 : \gamma_d(x) \leq 1\}$ be the unit ball associated with γ_d, see Appendix C. Using the denotation

$$B_d^r = d + rB_d,$$

we get

$$\gamma_d(d, x) \leq r \text{ if and only if } x \in B_d^r.$$

Hence, we obtain:

Lemma 2.6. *Let $d \in D_{total}$ and $S \subseteq \mathcal{T}$. Then d is covered by S if and only if $S \cap B_d^r \neq \emptyset$.*

We refer to Figure 2.3 for an illustration.

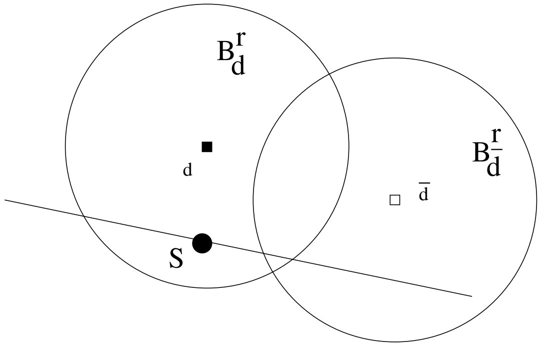

Fig. 2.3. $B_d^r \cap S \neq \emptyset$, hence d is covered by S. On the other hand, $B_{\bar{d}}^r \cap S = \emptyset$, hence $\bar{d}$ is not covered by S.

We will often use this *dual view* of the stop location problem, not considering the cover of some points $S \subseteq \mathcal{T}$ but starting from one point $d \in D_{total}$. For $d \in D_{total}$ we determine the set of points on $\mathcal{T}$ which can be used to cover d, i.e., those points where the location of a new stop would attract the demand in d.

Notation 2.7. *Let* $d \in D_{total}$. *Then* $\mathcal{T}(d) = \{s \in \mathcal{T} : \gamma_d(d, s) \leq r\}$.

$\mathcal{T}(d)$ can be calculated by intersecting the unit ball B_d^r of the gauge γ_d (with radius r) centered at the demand point d with the set of tracks $\mathcal{T}$, as the following lemma shows. For an illustration, see Figure 2.4.

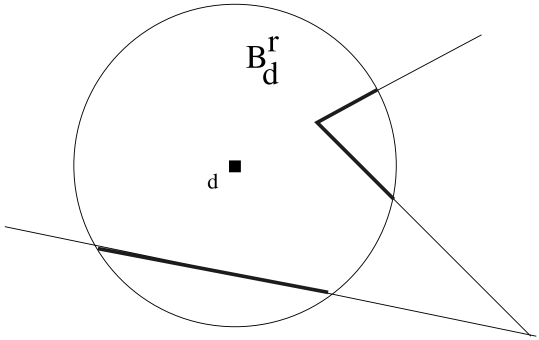

Fig. 2.4. The set $\mathcal{T}(d) = B_d^r \cap \mathcal{T}$ (the thick part of the tracks).

Lemma 2.8. *Let $d \in D_{total}$. Then $\mathcal{T}(d) = \mathcal{T} \cap B_d^r$.*

Proof.

$\Longrightarrow$: Let $s \in \mathcal{T}(d)$. Per definition $s \in \mathcal{T}$ and $\gamma_d(d,s) \leq r$, i.e., $\gamma_d(s-d) \leq r$. The latter means that

$$s - d \in rB_d, \text{ i.e., } s = d + (s-d) \in d + rB_d = B_d^r.$$

$\Longleftarrow$: Now let $s \in B_d^r = d + rB_d$. This yields $s - d \in rB_d$, hence $\gamma_d(d,s) = \gamma_d(s-d) \leq r$. Since s also is in $\mathcal{T}$ the result follows. □

With the notation of $\mathcal{T}(d)$ we can reformulate Lemma 2.6 as follows.

Lemma 2.9. *Let $d \in D_{total}$ and $S \subseteq \mathcal{T}$. Then d is covered by S if and only if $S \cap \mathcal{T}(d) \neq \emptyset$. In particular, d can be covered, if $\mathcal{T}(d) \neq \emptyset$.*

The Number of Covered Customers

The second objective function we are interested in gives the number of customers living closer than the distance of r to their nearest station. Denoting w_D as the number of (potential) customers located at demand point or demand region $D \in \mathcal{D}$, we are now in the position of defining the second objective.

For the case of demand points we investigate

$$f_{\text{cover}}(S) = \sum_{d \in \text{cover}(S)} w_d.$$

In the case of demand regions, let $\lambda(D)$ denote the area of a (measurable) set $D \subseteq \mathbb{R}^2$. Assuming that the demand is equally distributed within each set $D \subseteq \mathcal{D}$, we get the number of covered customers by calculating the percentage of D which is covered and multiplying it with the demand w_D of the respective set. By summing up these values over all $D \in \mathcal{D}$ we obtain

$$f_{\text{cover}}(S) = \sum_{D \in \mathcal{D}} w_D \frac{\lambda(\text{cover}(S) \cap D)}{\lambda(D)}$$

for demand regions.

We distinguish the following two types of problems.

(SL) Planning stations from scratch: Given $\mathcal{D}$, $\mathcal{T}$, and Q_{cover}, $Q_{\text{time}} \in \mathbb{R}$ find a set $S^* \subseteq \mathcal{T}$ such that $f_{\text{cover}}(S^*) \geq Q_{\text{cover}}$ and $f_{\text{time}}(S^*) \leq Q_{\text{time}}$.

(SL') Opening additional stations: Given $\mathcal{D}'$, $\mathcal{T}'$, a set of already existing stations $S^{ex} \subseteq \mathcal{T}'$ and $Q'_{\text{cover}}, Q'_{\text{time}} \in \mathbb{R}$, find a set $S^* \subseteq \mathcal{T}'$ such that $f_{\text{cover}}(S^* \cup S^{ex}) \geq Q_{\text{cover}}$ and $f_{\text{time}}(S^*) \leq Q_{\text{time}}$.

In (SL) the goal is to plan the set of stations from scratch, i.e., we assume that no station has been opened so far, whereas in (SL') a set of already existing stations has to be taken into account and we just add some new stations within the already existing network. For our analysis, both problems are equivalent, such that we can – for the sake of simpler notation – restrict ourselves to the problem of planning the stations from scratch. This means, we assume in the following that the set of already existing stations S^{ex} is empty. The justification for this assumption is given in the next lemma.

Lemma 2.10. *(SL) and (SL') are equivalent.*

Proof. To transfer a problem instance of (SL) to a problem instance of (SL') define $S^{ex} = \emptyset$ and leave everything else as it is, i.e., $\mathcal{T}' = \mathcal{T}$, $\mathcal{D}' = \mathcal{D}$, $Q'_{\text{cover}} = Q_{\text{cover}}$, and $Q'_{\text{time}} = Q_{\text{time}}$.
For the reduction from (SL') to (SL) let W_{cover} be the number of customers in $\mathcal{D}'$ who are already covered by existing stops, i.e., $W_{\text{cover}} = f_{\text{cover}}(S^{ex})$ where cover is meant with respect to $\mathcal{D}'$. To obtain an instance of (SL) we set

$$\mathcal{D} = \mathcal{D}' \setminus \text{cover}_{\mathcal{D}'}(S^{ex})$$
$$Q_{\text{cover}} = Q'_{\text{cover}} - W_{\text{cover}},$$

and leave the set of tracks $\mathcal{T} = \mathcal{T}'$ and $Q_{\text{time}} = Q'_{\text{time}}$ as they are. □