# 2 Quantify Values and Risk Attitudes

As in any decision process, the starting point of technology portfolio planning and management is the assessment of the *subjective* values of the decision maker. These values are also often expressed in terms of goals and objectives for the technology portfolio decision as they represent what the decision maker would like to achieve with the portfolio.

As discussed in Chapter 1, eliciting and articulating values are usually difficult. In many cases, because values are complex, multidimensional, emotion-laden, and often evolving, a decision maker may not even be fully aware of his or her true values. In other cases, due to the potential conflict of interests between the decision maker and other stakeholders in the decision, the decision maker may be reluctant to reveal his or her own values. Although highly interesting, the value extraction process is beyond the scope of this book. We shall assume that the decision maker is both willing and able to clearly identify and define his or her subjective values for the technology portfolio planning and management decision.

A technology generally possesses many different values, represented by the characteristics or attributes of the technology that are desirable to the decision maker. Furthermore, the values of individual component technologies in a portfolio need to be integrated into an overall value for the portfolio, so that these overall values of different portfolios can be systematically compared. Thus, once the desired values of the decision maker have been identified and defined, they will still need to be assessed, quantitatively if possible, to make the process of choosing the best alternative, i.e., the resource allocation among these technologies that will yield the optimal overall value, more explicit and definitive. This quantitative assessment of values is especially important in integrating many less tangible values, such as prestige, good will, and other social considerations, together with the more directly measurable financial return and physical output values, such as cost, profit, and production quantity, into an overall value for a technology portfolio.

In this chapter, we will discuss three widely applied approaches for the quantitative assessment of the decision maker's values. All these approaches are based on the common observation and assumption that the levels of importance of different values, as often represented by the criteria or outcomes of alternative technology choices, are reflected in and can be quantified through the degrees of *subjective preference* of these values by the decision maker.

## 2.1. Simultaneous Rating Approach

The simplest approach is to simultaneously compare and rate the degrees of subjective preference of various values by the decision maker. This can be done through either setting an ordinal rating of the values or by further converting the ordinal ratings into semiquantitative measures. As a specific example, assume that the three major values for a prospective technology portfolio are profitability, quality, and prestige. These values can be further defined as follows:

Profitability: The expected present value of the net profit achieved from the development or application of the technology portfolio in the next 10 years

Quality: The precision, reliability, and durability of the technologies in the portfolio

Prestige: Recognition of the innovativeness of the technologies and impact on the general reputation of the company in the business community

Among these values, only the profitability may be readily quantified, for example, by assuming that the degree of preference is proportional to the amount of expected profitability. Even there, the quantification process may be complicated, as the financial return of the technology portfolio could involve many assumptions and relationships, such as the accounting procedure used and the sequence of cash flows over time. Furthermore, in reality, the degree of preference by the decision maker may not be directly proportional to the amount of expected profitability, which adds to the complexity of estimation. Thus, a qualitative rating is often used to measure the degrees of preference of these values.

The typical qualitative rating measures are Low (L), Medium (M), and High (H). They can be further elaborated into many more ordinal ratings, such as LL, LM, LH, ML, MM, MH, HL, HM, and HH. These ordinal ratings can then be converted into numerical measures, such as 1 for LL, 2 for LM, …, and 9 for HH. There are also many other semi-quantitative rating scales. The popular ones are: 1 through 3, 1 through 5, 1 through 10, and 1 through 100.

In the above example, the decision maker may rate qualitatively the degrees of preference for profitability as High, for quality as Medium, and for prestige as Low, and assign numerical measures 9, 6, and 3 to these degrees of preference, respectively.

The advantages of this approach are in its simplicity and intuitive appeal. The main disadvantage is in its imprecision, which rises largely from the inherent inability for humans to precisely differentiate several values at the same time.

## 2.2. Analytic Hierarchy Process

The analytic hierarchy process (AHP) was developed by Thomas Saaty in the early 1970s based on the following observations of the relative preference of values by a decision maker:

(a) Various types of values in a decision process can be divided into different levels or hierarchies of details, so that the degrees of relative preference of the values within a hierarchy by the decision maker are all within the same order of magnitude, i.e., within a factor of 10 of one another.

(b) Human judgments can best differentiate the degrees of relative preference of the values within a hierarchy through pair-wise comparisons of these values

(c) The overall degrees of preferences of these values and their consistency can be assessed from the pair-wise comparisons of these values through the use of matrix theory.

Using AHP to assess values takes the following steps:

**(1) Set up the Hierarchy of Values**

In this first step, we will classify the values into various levels of detail, so that all values that are comparable within the same order of magnitude, i.e., within a factor of 10, in their degrees of relative preference by the decision maker, belong to the same level of detail or hierarchy.

As a specific example shown in Figure 2.1, the Profitability, Quality, and Prestige values of a prospective technology portfolio can generally be regarded to be in the same order of magnitude, and thus the same hierarchy. On the other hand, Revenues and Expenses as components of Profitability value are not in the same order of magnitude in terms of the level of detail as those of Quality and Prestige values. In this case, the higher hierarchy values can be decomposed into lower hierarchy values that are comparable within the same order of magnitude. For example, Quality value of a technology may be decomposed into lower hierarchy values of Precision, Reliability, and Durability, which are then within the same order of magnitude in the degree of relative preference as that of Revenues and Expenses.

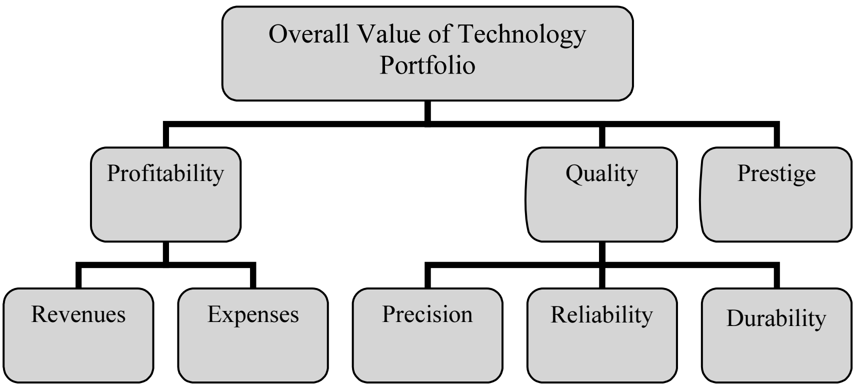


**Figure 2.1.** Example of the Hierarchy of Values for a Technology Portfolio

The number of hierarchies of values is judgmentally determined to provide sufficient detail so that meaningful and effective pair-wise comparisons can be made for values within a hierarchy.

**(2) Set up a Standard Scale for the Pair-wise Comparison**

Saaty has set up the following standard scale for the pair-wise comparison, $w_{ij}$, of the degrees of preference of Value i relative to that of Value j by the decision maker. Specifically, $w_{ij}$ will be:

1 — if Value i is equally preferred to Value j
3 — if Value i is moderately preferred to Value j
5 — if Value i is strongly preferred to Value j
7 — if Value i is very strongly preferred to Value j
9 — if Value i is extremely strongly preferred to Value j

Even-numbered comparison measures 2, 4, 6, and 8 can be used for $w_{ij}$ lying in between these odd-numbered measures. For example, $w_{ij}$ would be 4 if Value i is moderately to strongly preferred to Value j.

**(3) Develop an nxn Pair-wise Comparison Matrix W for the n Values in a Hierarchy.**

In this step, the decision maker will use judgments to estimate $w_{ij}$, the relative preference of Value i to Value j in a given hierarchy. Note that $w_{ii} = 1$ for all i, as it is Value i compared to itself. Furthermore, by definition, $w_{ij} = 1/w_{ji}$. In other words, the relative preference of Value i to Value j is the *reciprocal* of the relative preference of Value j to Value i. Thus, out of the nxn elements in the pair-wise comparison matrix, the decision maker needs only to estimate the relative preferences of n(n–1)/2 pairs of values that are neither the diagonal elements nor the reciprocals of each other.

As an example, for the three values of a prospective technology portfolio, the decision maker needs only estimate by judgment the relative preference through 3 pair-wise comparisons of these values that are not reciprocals of one another. In this specific example, the decision maker may judge that:

- Profitability (Value 1) is moderately to strongly preferred to Quality (Value 2); $w_{12} = 4$
- Profitability (Value 1) is extremely preferred to Prestige (Value 3); $w_{13} = 9$
- Quality (Value 2) is moderately to strongly preferred to Prestige (Value 3); $w_{23} = 4$

Then the 3x3 pair-wise comparison matrix, $W=[w_{ij}]$, would be:

| | Profitability | Quality | Prestige |
|---|---|---|---|
| Profitability | 1 | 4 | 9 |
| Quality | 1/4 | 1 | 4 |
| Prestige | 1/9 | 1/4 | 1 |

**(4) Estimate the Average Preferences or Weights of the n Values in a Hierarchy**

Notice that $w_{ij}$'s in the $j^{th}$ column are the relative preferences of Values, i = 1,2,..,n to Value j. Thus, the ratios, $w_{ij}/\Sigma_k w_{kj}$, represents the normalized relative preference of Value i by the decision maker using the relative preference of Value j as the base. As an example, for i = 2 and j =1,2,3, $w_{ij}/\Sigma_k w_{kj}$ are ¼ /(1+ ¼ +1/9)=0.1837, 1/(4+1+ ¼)=0.1905, and 4/(9+4+1)=0.2857 respectively. If the decision maker is totally consistent in these pair-wise comparisons, in that $w_{kj} = w_{ki}$ ($w_{ij}$) for any i, j, and k, then the normalized relative preferences of Value i, $w_{ij}/\Sigma_k w_{kj}$, will all be identically equal to the constant ratio, $1/\Sigma_k w_{ki}$, for all j. However, because of fluctuations in judgments particular when the relative preferences for the values are close to one another, these pair-wise comparisons are often not totally consistent, such as the case in the example. Since we do not know which comparison causes the inconsistency, these ratios for different j's are simply different estimates of the true normalized relative preference for Value i. To smooth out the errors introduced by inconsistency, we will use the row average of the ratios by summing the ratios for each row i and divide them by n as the best estimate for the true normalized relative preference for Value i. The computational formula is given below.

Normalized Relative Preference of Value i = $RAV_i = \Sigma_j(w_{ij}/\Sigma_k w_{kj})/n$ for i = 1, 2,…, n.

For the above example, we have

| | Profitability | Quality | Prestige | RAV=(row sum)/n |
|---|---|---|---|---|
| Profitability | 1/(1+1/4+1/9) | 4/(4+1+¼) | 9/(9+4+1) | 0.71 |
| Quality | 1/4/(1+1/4+1/9) | 1/(4+1+¼) | 4/(9+4+1) | 0.22 |
| Prestige | (1/9)/(1+1/4+1/9) | (1/4)/(4+1+¼) | 1/(9+4+1) | 0.07 |

**(5) Check Matrix Consistency**

As discussed earlier, human judgments can often be inconsistent, especially when the decision maker feels ambivalent about two values. As a result, the decision maker may, for example, prefer Value i to Value j and Value j to Value k in one set of comparisons and then contradictorily prefers Value k to Value i in another set of comparisons. It is thus important to check the consistency of *each* pair-wise comparison matrix.

Let W be the nxn comparison matrix, RAV = ($RAV_i$) be the nx1 column vector of row averages, and $RRAV^T$ be the 1xn row vector (with the superscript T signifying the transpose of the column vector RRAV, which changes it into a row vector), where the $i^{th}$ component is the reciprocals of $RAV_i$, the $i^{th}$ component of RAV. Then based on matrix theory, the consistency of W can be checked by first estimating the largest eigenvalue $\lambda_{max}$ of the matrix through the formula below:

$$\lambda_{max} = RRAV^T (W) (RAV)/n$$

If W is *totally* consistent, then $w_{ik}=w_{ij}(w_{jk})$ for all i, j, and k. As a result, its columns (as well as rows) must be *proportional to one another*, i.e., any two columns i and j, $w_{ik}= c(w_{jk})$ for k=1,2,…,n with $c= w_{ij}$ being the proportionality constant. In this case, it can be easily shown that $\lambda_{max} = n$. The consistency of W can then be assessed by the consistency index CI, which measures the deviation of the estimated $\lambda_{max}$ from n as follows:

$$CI = |(\lambda_{max}-n)/(n-1)|$$

It has been determined *empirically* that if CI $\leq$ 0.05[3], then W is sufficiently consistent in that there is no significant contradiction among the relative preferences of the values and the ratios $w_{ij}/\Sigma_k w_{kj}$ for each column j are close to each other as well as to $RAV_i$, the average normalized relative preference of Value i. Otherwise, W is sufficiently inconsistent and revisions will need to be made for the pair-wise comparisons. For the above example, we have

$$RAV^T = (0.71, 0.22, 0.07)$$

$$W(RAV)=[0.71(1)+0.22(4)+0.07(9),\ 0.71(1/4)+0.22(1)+0.07(4),\ 0.71(1/9)+0.22(1/4)+0.07(1)]^T$$

$$RRAV^T= (1/0.71, 1/0.22, 1/0.07)$$

$$CI = \{[(1/0.71, 1/0.22, 1/0.07)W(RAV)/3]-3\}/(3-1) = 0.021 < 0.05$$

Thus, W is sufficiently consistent.

**(6) Revise the pair-wise comparison matrix for consistency[4]**

If the pair-wise comparison matrix W turns out to be insufficiently consistent, then it can be revised to total consistency through the following procedure:

(a) Let $C_0$ be the set of n diagonal elements, $w_{ii}$'s of W.

(b) Let $w_{ij}$ and $1/w_{ij}$, with j>i, be the first off-diagonal reciprocal pair of elements in W of which the decision maker is *most* confident. Then $C_1= \{C_0, w_{ij}, 1/w_{ij}\}$.

(c) In the $i^{th}$ iteration, let $w_{hk}$ and $1/w_{hk}$ be the next most confident reciprocal pair not yet in $C_i$. If $C_i$ already contains $w_{fk}$, $w_{hg}$, and $w_{fg}$, (note that one of these may be a diagonal element) for some f and g, so that a rectangle or square formed by $w_{hk,}$ $w_{hk,}$ $w_{hk}$, and $w_{hk}$ exists in the matrix, then to be consistent, $w_{hk}$ must either equal to or be revised to $w_{hg}(w_{fk}/w_{fg})$. Otherwise, $w_{hk}$ remains unchanged and $C_{i+1}=\{C_i, w_{hk}, 1/w_{hk}\}$.

[3] Saaty has used empirically determined requirements on CI that vary with n; however, CI$\leq$0.05 is a stronger requirement that satisfies Saaty's other requirements for all values of n.

[4] This section is based on original work by the author.

(d) Continue until all reciprocal pairs have been included and thus all columns become proportional to one another.

Specifically, in the above example, suppose that the decision maker desires to make the pair-wise comparison matrix totally consistent, and he or she is most confident about $w_{12}$=4, second most confident about $w_{13}$=9, and least confident about $w_{23}$=4. Then following the above procedure, we have:

$$C_0 = \{ w_{11}=1, w_{22}=1, w_{33}=1\}$$

$$C_1 = \{ w_{11}=1, w_{22}=1, w_{33}=1, w_{12}=4, w_{21}= 1/4 \}$$

Since $w_{13}$ does not form a rectangle or square with any elements in $C_1$, it does not need to be revised and

$$C_2 = \{ w_{11}=1, w_{22}=1, w_{33}=1, w_{12}=4, w_{21}= 1/4 , w_{13}=9, w_{31}=1/9\}$$

Finally, since $w_{23}$ forms a square in the matrix with $w_{22}$, $w_{13}$, and $w_{12}$ in $C_2$ (in this case, h=2, k=3, f=1, and g=2.), it needs to be revised to $w_{22}(w_{13}/w_{12}) = 9/4 = 2.25$. With this change, the matrix takes the following revised form:

| | Profitability | Quality | Prestige |
|---|---|---|---|
| Profitability | 1 | 4 | 9 |
| Quality | 1/4 | 1 | 9/4 |
| Prestige | 1/9 | 4/9 | 1 |

As a result, all columns of the matrix become proportional to one another. Then CI=0, and the matrix becomes totally consistent.

**(7) Distribute the Relative Preference of a Value to Values in a Sub-hierarchy**

If value $V_i$ in a given hierarchy can be decomposed into a set of component values, $V_{i1}$, $V_{i2}$, …, $V_{iN}$, in a sub-hierarchy, then the relative preference of $V_i$ should be distributed to all component values by multiplying it to the relative preferences of the component values obtained from the pair-wise comparison matrix of $V_{ik}$'s.

In the above example, Quality has a relative preference of 0.22 and is decomposed into three component values: Precision, Reliability, and Durability. Assume that by pair-wise comparison matrix analysis we have obtained the relative preferences (i.e., the RAVs) of these component values as 0.2, 0.5, and 0.3 respectively. Then the overall relative preference for these component values would be 0.22x0.2=0.044, 0.22x0.5=0.11, and 0.22x0.3=0.066 respectively.

***Major Advantages and Disadvantages***

The major advantages of AHP are:

1. It provides a psychologically sound basis for making a more precise assessment of values through hierarchical structuring and pair-wise comparisons of values within the same hierarchy.
2. It provides a mathematically sound basis for checking the consistency of human judgments.
3. It is simple, intuitive, and easily programmable on a computer
4. The weights or relative importance of the values resulting from the analysis are numerically stable for small inconsistencies in human judgments in pair-wise comparisons
5. In addition to value assessment, it can also be used for forecasting, as well as alternative selection and resource allocation as to be discussed in Chapters 3 and 4, respectively
6. The method has been widely applied and accepted by major business corporations and government agencies throughout the world

On the other hand, AHP has the following pitfalls:

1. The pair-wise comparison may be distorted by human perception. For example, human judgments of light intensity at different distances have been shown to be inconsistent with the physical law that the intensity diminishes with the square of the distance. This may also be caused by the fact that the 1–9 comparison scale is not numerically proportional. For example, although in the preference scale, 5 lies at the exact mid-point of 1 and 9, but numerically the ratio of 5 to 1 is much more significant than the ratio of 9 to 5, which could cause distortions in the final comparison results.
2. If the values to be compared are highly correlated, the comparison results can also be distorted. As an extreme example, if two of the three values are totally correlated, then they should be combined into a single value. By being two separate values, the same value would be over-rated in the analysis.
3. The hierarchy is one-directional and it is difficult to accommodate feedback relationships between lower hierarchies and higher hierarchies[5].
4. The values are highly aggregated and difficult to reflect the degree of uncertainty in the estimation of the values.
5. It is also difficult to develop the relationships between alternatives and values in resource allocation applications where the measure of a value changes with the amount of resources allocated as in the case that profitability increases usually nonlinearly with the amount of investment.

## 2.3. Utility Theory Approach

Utility theory was first formalized in modern context by John von Neumann and Oskar Morgenstern in their 1944 classic, "Theory of Games and Economic

[5] Professor Saaty has recently developed the concepts of Analytic Network Process to remedy this deficiency.

Behavior." The theory uses a set of axioms as the mathematical basis for quantifying the degree of relative preference as the utility of a particular value for the decision maker. These axioms, or commonly accepted truths, are stated in a simplified version below:

Let U(i) be the utility or the degree of relative preference by the decision maker for Value i among a set of competing values.

**Axiom (1) Completeness and Rankability:**

For values i and j, a decision maker can have only one of the following three preferences:

(i) Value i is preferred to Value j, then $U(i)>U(j)$
(ii) Value j is preferred to Value i, then $U(i)<U(j)$
(iii) Value i is equally preferred to Value j, then $U(i)=U(j)$

**Axiom (2) Transitivity and Consistency:**

If $U(i)>U(j)$ and $U(j)>U(k)$, then $U(i)>U(k)$.

*Comments:* Although this axiom seems intuitive, it is not unusual that some decision makers display cyclical preferences, i.e., Value i is preferred to Value j, and Value j is preferred to Value k, but Value k is preferred to Value i. This generally occurs when the decision maker has considerable ambivalence about the values and/or when the preferences are made over different periods of time. However, for a rational decision maker, this axiom should hold for preferences made *simultaneously in time*.

**Axiom (3) Substitutability:**

For Values i and j, if U(i) = U(j), then these values are totally substitutable for each other.

*Comments:* Some decision makers may have difficulty with this axiom, especially when the two values are very different in nature, such as a tangible monetary value and an intangible environmental value. The difficulty arises generally because of different implicit assumptions made in the comparisons. For example, the utilities of a ton of greenhouse gas reduction and of $50,000 to the decision maker may be the same under one set of assumptions but different under another set of assumptions. If the utilities are indeed the same under all assumptions, then these values should be truly substitutable.

**Axiom (4) Computability of Expected Utility:**

If a lottery L has two possible outcomes, e.g., one has probability p of achieving Value i and the other has probability 1–p achieving Value k, then the utility of the lottery is defined and computed by the expected value $U(L) = pU(i) + (1-p)U(k)$.

This axiom can be generalized to n possible outcomes, with probability $p_i$ of achieving Value i and all $p_i$'s summing to 1. Then $U(L)=\Sigma_i p_i U(i)$.

**Axiom (5) Continuity of Expected Utility:**

For Values i, j, and k, if U(i)>U(j)>U(k), then there exists a probability p* that U(j) = p*U(i) + (1–p*)U(k). In this case, the lottery with probability p* of yielding Value i and probability 1–p* of yielding Value k is the *indifference lottery* to Value j and Value is the *certainty equivalent* of the indifference lottery.

With these axioms, a decision maker can now quantify the relative preferences of different values. Specifically, the decision maker will first identify two extreme values i and k, one the decision maker prefers the most, such as a huge monetary gain, or even an intangible value like great ecstasy, and the other the decision maker prefers the least, such as a monetary loss, or an intangible value like extreme agony. The decision maker artificially sets a very high utility for the most preferred Value i and a very low utility for the least preferred Value k. Then these axioms can be applied to quantify all other values in between the two extremes through a lottery scheme as illustrated by the following examples[6].

**Example 1: Utility of Monetary Value**

To determine the utility of a monetary value, say $500,000, between the two extremes of $0 and $1 million, the decision maker can assign U($1 million)=100 and U($0)=0. Then the decision maker is asked to choose between two alternatives: (1) $500,000 for sure and (2) a two-outcome lottery with probability p yielding $1 million and probability 1–p yielding $0.

Assume that the utility of money is a non-decreasing function of the amount of money; i.e., the decision maker will not prefer less money to more money. By Axioms (1) and (2), clearly, if p=0, then the decision maker will choose the alternative of $500,000 for sure. On the other hand, if p=1, then the decision maker will certainly choose the lottery. By Axiom (5), there exists a probability p* such that the decision maker will be indifferent between $500,000 for sure and the lottery with probability p* yielding $1 million and probability 1–p* yielding $0. By Axioms (3) and (4), U($500,000)=p*U($1 million)+(1–p*)U($0) =100p*.

**Example 2: Utility of an Intangible Value**

To determine the utility of an intangible value, such as the Prestige of national recognition, in relation to the utility of net monetary profit, the decision maker needs to identify a high monetary value, say $100 million in net profit, that is more

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[6] Like temperature expressed in terms of degree of Fahrenheit or Celsius and like altitude expressed in terms of foot or meter, utility is a measure of value in terms of the degree of relative preference by the decision maker. Thus, the utility of a particular value can change with the utilities assigned to the extreme values.

preferred to this intangible Prestige value, and a monetary value, say $0 in net profit, that is less preferred to this intangible value. Again by Axiom (5), there exists a probability p* for which the decision maker will become indifferent between such a Prestige value and a lottery that yields $100 million net profit with probability p* and $0 net profit with probability 1–p*, and by Axiom (4), U(Prestige) = 100p*.

On the other hand, assuming that a decision maker prefers the value of Profitability to the value of Quality and the value of Quality to the value of Prestige, the decision maker can then estimate the utility for Quality in terms of the utilities of Profitability and Prestige. Specifically, the decision maker could assign a high utility say 50 for Profitability and a low utility say 10 for Prestige. By Axiom (5), there exists a probability p* for which the decision maker will be indifferent between having the Quality value alone for sure and a lottery that yields Profitability value alone with profitability p* and Prestige value alone with probability 1–p*, and by Axiom (4), U(Quality) = 50p* + 10(1–p*) = 40p*+10.

Note that if the decision maker assigns different utilities to Profitability and Prestige, even though the probability p* remains the same for the indifference lottery, as a relative measure, the utility for Quality may be different. Specifically, if U(Profitability)=60 and U(Prestige)=5, then by Axiom (4), U(Quality) = 60p* + 5(1–p*) = 55p*+5.

**Example 3: Utility of a Candidate Technology**

As an approach to measure the decision maker's degree of preference, utility theory can also be applied to objects other than value. For example, it can be used to determine directly the degree of preference for a candidate technology B by the decision maker. In this case, the decision maker needs to identify a technology A that is more preferable to B and set artificially U(A) to say 100, and a technology C that is less preferable to B and set artificially U(C) to say 0. Then by Axiom (5), there exists a probability p* for which the decision maker will feel indifferent between technology B and a lottery that yields technology A with probability p* and technology C with probability (1–p*), and by Axiom (4), U(B)=100p*. Again, if different utilities were assigned to technologies A and C, then the utility of technology B may vary even though the indifference probability p* remains the same.

## 2.4. Risk Attitude and Risk Premium

When utility theory is applied to monetary value, a decision maker generally will have utilities for money that are not proportional to the quantities of money. This non-proportionality reflects the decision maker's risk attitude towards money.

If a decision maker is *risk-avoiding*, he or she would prefer a small amount of sure money to a lottery that has a higher expected monetary value but carries a significant risk of yielding an outcome with little monetary value or even possibly a monetary loss.

As a specific example, such a risk-avoiding decision maker would be one who prefers $500,000 to a lottery that has a 60% probability of yielding $1 million and

40% probability of yielding \$0, even though the lottery has an expected monetary value of (0.6)(\$1 million)+(0.4)(\$0)= \$600,000, which is higher than \$500,000. If this risk-avoiding decision maker sets U(\$1 million)=100, and U(\$0)=0, and feels indifferent between getting \$500,000 sure money and a lottery that has a probability of 0.7 in yielding \$1 million and 0.3 in yielding \$0, then to this decision maker, U(\$500,000) = (0.7)(100)+(0.3)(0) = 70.

For such a risk-avoiding decision maker, the difference between the expected monetary value of the lottery and the sure amount of money that is equivalent in utility to the lottery is called the *risk premium*, i.e., it is the amount of expected monetary gain the decision maker is willing to give up in order to avoid the risk of getting the undesirable outcome from choosing the lottery. In the above example, the risk premium for this risk-avoiding decision maker is [(0.7)(\$1 million)+(0.3) (\$0)]-\$500,000=\$200,000.

In insurance, the risk premium is the extra amount of money that the insurance company charges an individual customer above its expected loss for the average customer in the same class of insurance risk. For example, a technology developer carries a \$10 million liability insurance policy for an annual insurance premium of \$2,000. For simplicity, assume that in a given year the insurance company has a 1/10000 probability of paying out \$10 million for liability damage claims for the developer and a 9999/10000 probability of paying out nothing. Then the risk premium for the technology developer is \$2,000 – [(1/10000)(\$10 million) + (9999/10000)(\$0)] = \$1,000.

On the other hand, the decision maker is *risk preferring* if he or she prefers a lottery that has a possible outcome of high monetary value but a low overall expected monetary value to the alternative of getting a sure monetary value that is greater than the expected monetary value of the lottery. In this case, the decision maker prefers the thrill of a risky gamble for the chance of a high monetary value outcome to accepting a lower amount of sure money.

As a specific example, such a risk-preferring decision maker would be one who prefers a lottery with a 40% probability of yielding \$1 million and a 60% probability of yielding \$0 to getting \$500,000 for sure, even though the lottery has a lower expected monetary value of \$400,000. If this risk-preferring decision maker sets U(\$1 million)=100 and U(\$0)=0 and becomes indifferent between a lottery with a 35% probability of yielding \$1 million and a 65% probability of yielding \$0 and the alternative of getting \$500,000 for sure, then U(\$500,000) = (0.35)(100) + (0.65)(0) = 35.

Finally, if the utility of money is proportional to the amount of money, then the decision maker is *risk neutral*. In other words, the decision maker would be indifferent between a lottery with an expected monetary value and that amount for sure. As a specific example, if this risk-neutral decision maker sets U(\$1 million)=100 and U(\$0)=0, then U(\$500,000)=50.

As shown in Figure 2.2, for a risk neutral decision maker, the utility function U(x) for monetary value x is a straight line with U(x)=cx, where c is a proportionality constant. On the other hand, U(x) will be a concave curve above the straight line for a risk-avoiding decision maker, and a convex curve below the straight line for a risk-preferring decision maker[7].

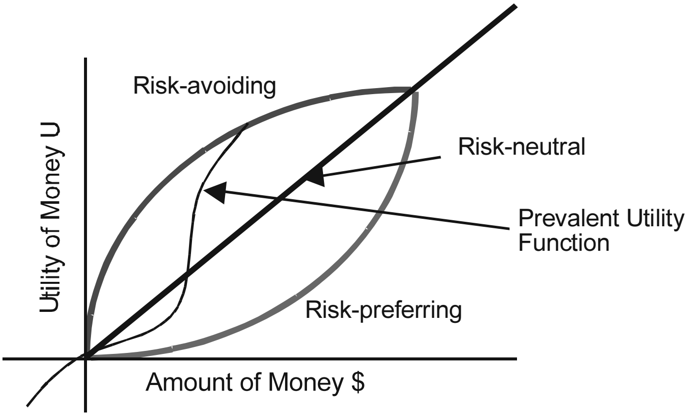


**Figure 2.2.** Utilities of and Risk Attitudes toward Money

The risk attitude of a decision maker generally depends on two factors: One factor is the decision maker's personality, whether the person enjoys the thrill of winning big despite unfavorable odds or is more comfortable in avoiding the risk for a possible loss even when the chance is small. The other factor is the amount of resources available to the decision maker. If there are large amounts of resources available, then most decision makers tend to be more willing to take risks and become at least risk-neutral. On the other hand, if there are little resources available, then decision makers tend to feel less willing to take the risk of a low return or a significant loss even though the probability may be small.

Because of the effect of resource availability, many decision makers would display a combination of these risk attitudes as shown by the S-shaped prevalent utility function in Figure 2.2. In this case, when the stakes are small, such as purchasing a lottery ticket that costs $1 but has an expected payoff of $0.5 in a game with 1 out of 6 million chances of winning the $3 million jackpot and $0 otherwise, many decision makers would be risk-preferring and take the gamble. On the other hand, when the stake is high, such as a $100,000 investment in a high-tech stock with a 0.2 probability of a net profit of $500,000 but a 0.8 chance of a total loss in a year, many decision makers would be risk-avoiding and prefer safe investments such as a federally insured bank savings account with a low annual return, say $5,000, which is much less than the 0.2($500,000)+0.8(-$100,000)=$20,000 expected net profit of the risky investment.

The risk attitude concept applies not only to monetary value but also to other values, such as the value of time, as illustrated by the following example. Assume

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[7] A concave curve is defined as a curve that all points on the straight line connecting two points on the curve lie either on or below the curve, and a convex curve is defined as a curve that all points on the straight line connecting two points on the curve lie either on or above the curve.

that there are two alternative methods of developing a technology: one is a sure but slow method that takes a year to develop the technology, and the other is a risky method that has a 60% probability of developing the technology in 0.5 year and a 40% probability of developing it in 1.5 years. If the decision maker prefers the sure and slow method to the risky method, then he or she is a risk-avoider as the risky method has actually a shorter expected development time of 0.9 year than the 1-year development time of the sure and slow method. On the other hand, if the decision maker prefers a risky approach that has a 40% probability of developing the project in 0.5 year and a 60% probability of developing it in 1.5 years to the sure but slow method, then he or she is risk-preferring, as the expected development time for the risky approach is 1.1 year, which is greater than the 1-year development time of the sure but slow method.

Because risk attitude is subjective and varies among decision makers, it will be difficult to standardize or generalize in a typical technology decision making process. For simplicity of discussing the various search methods for the best alternative to maximize the utility of the monetary value, in the remainder of this book, we will assume the decision maker to be *risk-neutral*, i.e., the utility for money of the decision maker is proportional to the amount of money. Thus, maximizing the expected utility of the monetary value of the decision maker is equivalent to maximizing the expected monetary value.

### *Major Advantages and Disadvantages*

The major advantages of the Utility Theory approach in quantifying values are in the intuitive appeal of the axioms and the ease of the utility estimation procedure.

However, the estimation procedure requires in-depth self examination, which may not appeal to some decision makers. Furthermore, it is based on two arbitrarily set extreme utilities. As a result, the intermediate utilities estimated vary with the extremes and the variation may not be internally consistent among different sets of extremes.

Specifically in Example 1 shown earlier in this chapter, the decision maker has estimated U($0.5 million) to be 100p*, based on the indifference between a lottery of probability p* for getting $1 million and probability 1–p* for getting $0 and the alternative of getting $0.5 million for sure.

Now, using a new lottery with outcomes $0 and $2 million and setting U($0)=0 and U($2 million)=200, a decision maker may be indifferent between a lottery of probability x for getting $2 million and probability 1–x for getting $0 and the alternative of getting $1 million for sure. Furthermore, the decision maker may be indifferent between a lottery of probability y for getting $2 million and probability 1–y for getting $0 and the alternative of getting $0.5 million for sure. Thus, for the decision maker using the new lottery, U($1 million)=200x and U($0.5 million)=200y. In this case, if the decision maker is totally consistent, then y should be equal to xp*, as indicated in Example 1. However, because of fluctuations in human perceptions and judgments of different extremes, y is often observed as not equal to xp*, and hence the utility estimates can be unstable.

Moreover, in some cases, actual human preferences may not follow the axioms of utility theory. One of the most famous observations of such violation is the Allais Paradox, which arose when people were surveyed about the following choices:

**A. Choice between**

A1: $1 million cash

A2: A lottery with a 10% chance of winning $5 million, an 89% chance of winning $1 million, and a 1% chance of winning $0.

**B. Choice between**

B1: A lottery with a 11% chance of winning $1 million and an 89% chance of winning $0

B2: A lottery with 10% chance of winning $5 million and 90% chance of winning $0.

**C. Choice between**

C1: $1 million cash

C2: A lottery with a 10/11 chance of winning $5 million and 1/11 chance of winning $0.

By the axioms, if the decision maker is risk-neutral, then the second alternative of each choice would be preferred because it yields a higher expected monetary value. On the other hand, if the decision maker is risk-avoiding, then by the axioms, to be consistent, the first alternative of each choice should be preferred. However, French scientist Maurice Allais observed that most people strongly preferred A1 to A2, and C1 to C2, but B2 to B1, which would be a contradiction of the axioms. This and other paradoxes may indicate that either the axioms are not complete in their description of the logic of human preferences or human perceptions may have produced distortions about the chances and payoffs of various alternatives that cause violations of these axioms. In any case, caution should be exercised in the application of utility theory to quantify values.

## 2.5. Equivalence and Reconciliation between the Analytic Hierarchy Process and the Utility Theory Approach

Both AHP and Utility Theory quantify values by measuring the degrees of relative preferences of these values to the decision maker. Thus, they are theoretically equivalent.

Specifically for the example in AHP, Profitability has the highest weight of 0.71 and Prestige has the lowest weight of 0.07. Since utility is a measure of the degree of relative preference, we can set U(Profitability)=71 and U(Prestige)=7. Then, theoretically U(Quality) should be 22. In other words, the decision maker should be indifferent between Quality for sure and a lottery with probability p*=(22–7)/(71–7)=0.234 of yielding Profitability and (1–p*)=0.766 of yielding Prestige. However, in reality, a decision maker would often be unable to produce a probability p* for the indifference lottery close to this totally consistent ideal value of 0.234.

In general, because AHP has the more rigorous basis of hierarchical structuring, pair-wise comparisons, and consistency check, the quantification of value process tends to be more reliable than that based on the Utility Theory. Thus, in the above example, if the probability p* determined by the indifference lottery is 0.3, then

U(Quality) = (0.3)(71) + (0.7)(7) = 26.2. But this U(Quality) can be reconciled with that obtained from AHP by adjusting p* to 0.234, which is always possible because 22 lies between 71 and 7 and p* is simply the interpolation ratio (22–7)/(71–7) obtained by solving the equation p*(71)+(1–p*)(7)=22.

However, if the two methods produce significantly different utilities for a value lying between identical extreme values, then it will be useful to re-examine the detailed implementations of both methods to be sure that they indeed reflect consistent judgments by the decision maker. Again, using the above example, if the decision maker applies the utility theory and becomes indifferent between Quality for sure and a lottery with a 70% probability of yielding Profitability and a 30% probability of yielding Prestige, then by Axioms (4) and (5) of utility theory, U(Quality) = (0.7)(71)+(0.3)(7) = 51.8, which is significantly different from the utility of 22 obtained from AHP. Although AHP is theoretically more reliable, it will still be useful to recheck the pair-wise comparison matrix of AHP as well as the estimation process based on utility theory to uncover the root cause of this significant differences in the utilities for Quality obtained by these two methods. In the end, with such large difference, it is possible that both estimations would require some adjustments to reconcile the utilities for Quality to maintain the consistency in the decision maker's judgments about the degree of relative preference of these values.

## 2.6. References

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## 2.7. Exercises

**Problem 2.1**

For the computer purchase exercise in Problem 1.4, apply both the Simultaneous Rating approach and the Analytic Hierarchy Process to the three major values: Affordability, Quality, and Performance, and compare the results for your relative preferences of these values.

**Problem 2.2**

For four values A,B,C, and D, a decision maker has the following pair-wise comparison matrix and rankings of the confidence in the comparisons:

| | A | B | C | D |
|---|---|---|---|---|
| A | 1 | 3 | 1/3 | 2 |
| B | | 1 | 2 | 1/2 |
| C | | | 1 | 4 |
| D | | | | 1 |

(1) B vs. C Highest
(2) B vs. D
(3) C vs. D
(4) A vs. B
(5) A vs. C
(6) A vs. D Lowest

Use the ranking to make the appropriate modifications of the pair-wise comparisons so that the matrix would become totally consistent.

**Problem 2.3**

Based on the application of the Analytic Hierarchy Process (AHP) in Problem 2.1, let V1 be your weight or rating for the most preferred value and V3 be your weight or rating for the least preferred value. Set the utilities of the most and least preferred values respectively as U(most preferred value)=V1 and U(least preferred value)=V3. Apply Axioms (4) and (5) of the Utility Theory and determine your subjective judgment for the probability p that will make a lottery of getting V1 with probability p and V3 with probability 1–p indifferent from the alternative of getting the value in between for sure. Then estimate the utility of this value between the most and least preferred values. Is this estimated utility equal to the weight or rating for the value in between obtained by AHP? If not, in which results are you more confident? Furthermore, if the two results are different, how can you reconcile them?

**Problem 2.4**

The CEO of a technology company is comparing two alternatives:

(a) improving an existing technology, that is *certain* to produce a moderate profit
(b) developing a new technology with the following possible outcomes:

(1) Successful development with major potential profit as well as *enhancing company image and future markets*, which would be most preferable for the CEO
(2) So-so development with moderate potential profit
(3) Unsuccessful development resulting in a significant financial loss, which would be least preferable for the CEO

The CEO believes the probabilities of outcomes (b1), (b2), and (b3) to be 25%, 50%, and 25% respectively.

The CEO prefers Outcome (b1) the most and Outcome (b3) the least. Furthermore, after much contemplation, the CEO feels that she would be indifferent between the outcome of Alternative (a) and a lottery with an 80% chance of yielding Outcome (b1) and a 20% chance of yielding Outcome (b3).

Furthermore, she would feel indifferent between Outcome (b2) and a lottery with a 70% chance of yielding Outcome (b1) and a 30% chance of yielding Outcome (b3).

The CEO assigns a utility of 100 to Outcome (b1) and –50 to Outcome (b3). Use the utility theory to estimate the utilities of Outcomes (a) and (b2).

**Problem 2.5**

For the ease of analysis, utility functions are often stylized into simple mathematical forms. For each of the following four utility functions of monetary value x for a decision maker, assume that the U($1 million)=100, and determine the utility of $500,000 and whether the decision maker is risk preferring or avoiding. Justify your answer.

(a) $U(x) = x^2$,
(b) $U(x) = x^3$,
(c) $U(x) = x^{½}$
(d) $U(x) = x^{1/3}$