

The Muqarnas Dome of the Hall of the Two Sisters in the Alhambra in Granada

JEAN-MARC CASTERA

The work of Jules Goury and Owen Jones

In 1834 two young architects, the English Owen Jones and the French Jules Goury, sojourned a long while in Granada in order to carry out an accurate study of the Alhambra Palace. Unfortunately it was a voyage without return for Jules Goury, who contracted cholera and died in Granada on 28 August 1834.

Plans, sections, elevations, surveys of decorations and inscriptions, rubbings and plaster casts furnished the material for this noteworthy work [1], published in a folio edition with 104 etched plates, of which a copy is housed in the Bibliothèque nationale de France.

Included is a ponderous study of the Hall of the Two Sisters with sections and plans of the dome. Those plates have been reproduced numerous times in various works dedicated to Islamic architecture.

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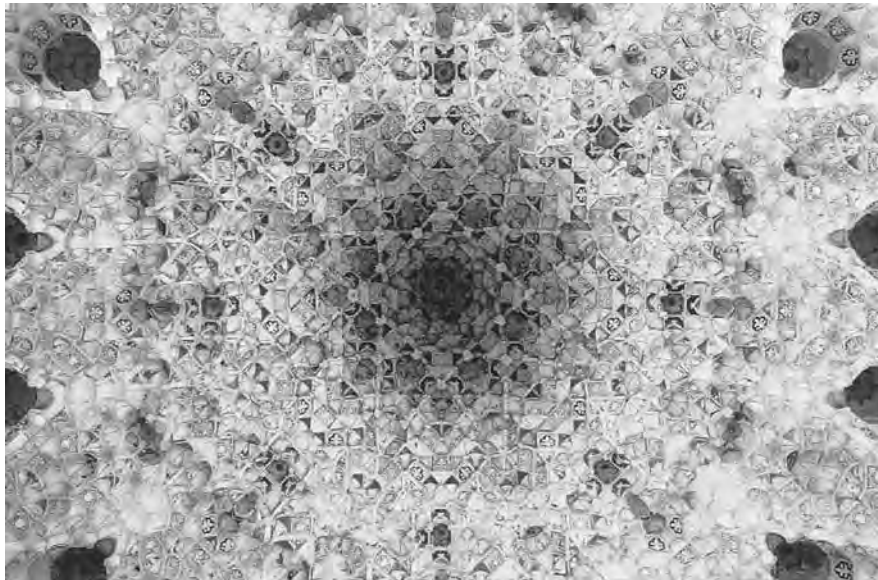


Fig. 1. Photograph of the dome. An 'avalanche' of geometry

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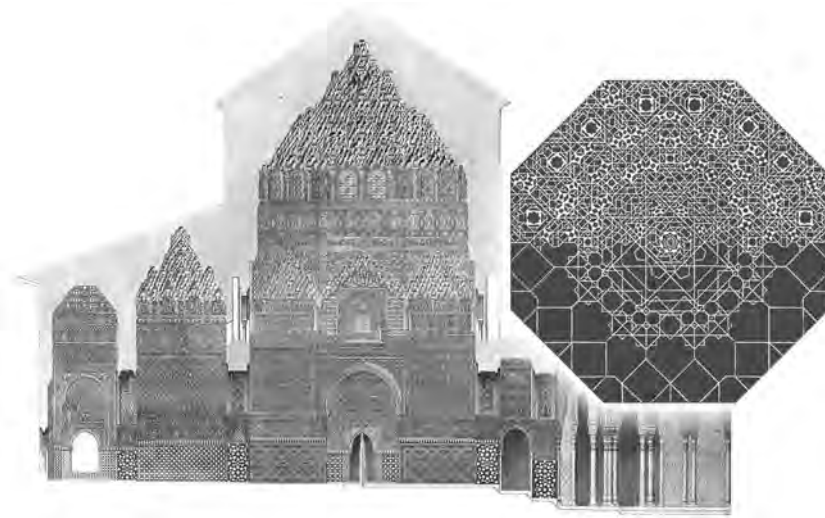


Fig. 2. Section and projective plane of the dome, according to Gourey and Jones

The study that Jones and Gourey undertook is typical of Western architectural investigations, which require two projected planes (plan and elevation) in order to account for the three-dimensional structure. We will see in what follows how this information can be contained in a single plane representation that includes a code for each piece of the muqarnas.

On the other hand, it is surprising that the authors of the study did not indicate the irregularity that manifests itself in the projective plane, even though they must have encountered it in the preparation of their plan.

Structures made of muqarnas – the principle

Muqarnas are among the principle characterizing elements of the Islamic architectural style. Naturally there exist regional differences: for example, they are generally constructed in successive layers of brick in Iran and in Iraq, but are often made in stone in Syria and Egypt. In the Arabian-Andalusian regions the constructions are modular, either of wood or in plaster. They are generally, with rare exceptions, based on an octagonal symmetry, though pentagonal symmetry is very common in the Persian style.

The modules can be in wood, usually cedar, or in plaster. They are thus assembled as if making an immense puzzle based on a modest number of different shapes. The use of wood in muqarnas, as the use of *zellij* for plane decorations imposes a very precise mark on each form. In contrast, the use of plaster permits a greater freedom in the assembly of the modules, allowing for small deformations in some pieces. The pieces are decorated (with paintings and sculpture) after having been assembled. The dome of the Hall of the Two Sisters was made in plaster, and required more than 5000 pieces. All the pieces are right prisms that have been cut at

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Fig. 3. The cuts of the four principle pieces, beginning with a triangular prism for the first two, and rhombic for the remaining two. The visible parts are shown in black

an extremity, usually in the form of a curved surface; these are the visible parts of the muqarnas. Because these visible surfaces are connected to each other in a continuous way, the muqarnas appear more like a skin than a volume. The detail of the design of an individual piece does not make any difference structurally, since the same assemblage can lend itself to the application of styles that look very different.

Four different forms are sufficient to construct an infinity of structures. Two are sections of a triangular prism (a half-square); the other two are sections of a 45° rhomboid prism. The cut begins with a single curve, applied to certain faces of the prism. This curve defines the profile of the muqarnas, and in general has an inclination of circa 30° with respect to the vertical.

Codification in the representation: in order to distinguish two pieces with the same section (triangular or rhomboid) on the projective plane, we identify with a dot the lower vertices of the piece. Such a point will be called the ‘foot’ of the module.

Rules for assembly. Assembly of muqarnas is accomplished by setting the curved edges together. Two edges will not fit together if they do not have the same orientation. If they do have the same orientation, there exists only one possible positioning. As a consequence, thanks to the codification introduced, a *plane representation* contains all the information about the structure: if we know the inclination of the muqarnas, or the height of only one module, the representation is complete.

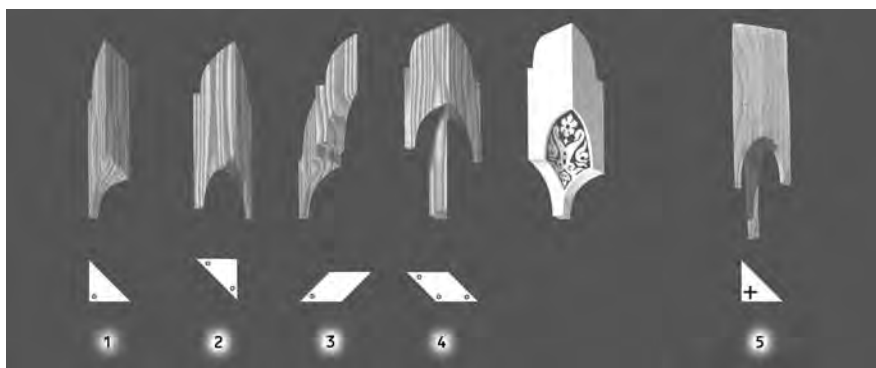


Fig. 4. The four principle pieces of the *muqarnas* and their plane representation. In order to simplify the reading on the plane, we have denoted piece number 4 with three points instead of only one. At its right, the same piece in plaster decorated in the Alhambra style. Piece number 5 is a variant of piece number 1.

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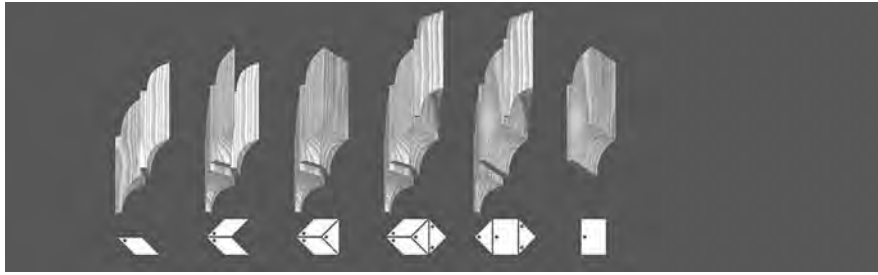


Fig. 5. Formation of a group of *muqarnas* and its variants with a new piece of rectangular section

The other pieces. In addition to the four main pieces, some secondary pieces exist, as for example the piece with a rectangular section shown in Fig. 5 and its variants. Two small pieces are derived from the decomposition of piece number 3. There are also half-pieces used to join angles, and some pieces with only one level (the small squares in Fig. 6; see *the colour section*). The *chechia*¹ is a little dome that replaces an group of pieces at the top of a dome (Figs. 9 and 15).

Elements for a symbolic reading of a domed hall

Structure

A domed hall is constituted of three principle elements:

- The hall, with a square plan;
- The dome, with a circular or polygonal base (with at least eight sides);
- The drum, an intermediate element, usually octagonal, which permits the passage from the square shape to the base of the dome. It serves thus as a base for the dome and is itself supported on the four walls of the hall.

This last element arises out of an architectonic necessity: a circle cannot rest solidly on a square. In their turn, the diagonal sides of the drum require support, and this purpose is usually served by squinches or pendentives.

From a symbolic point of view

The square represents the earth with its four cardinal points. It symbolises man's material aspect. It is also discontinuity, and by extension, the atomistic aspect of material, the separation of the individual consciences.

The circle is the sky, the line of the horizon as it appears when lying on the floor and lifting one's eyes towards the zenith. It is also the shape of the *astras** that live in the inaccessible sky, the Moon and the Sun. And it is, albeit approximately, the trajectory that the stars and planets draw in the sky. It is also continuity and infinite cyclical motion. And because the circle inherits the symbolism of the sky, it expresses the idea of unity. Regarding human beings, the circle is related to their non-material, aerial and spiritual qualities.

¹ *Chechia means hat.*

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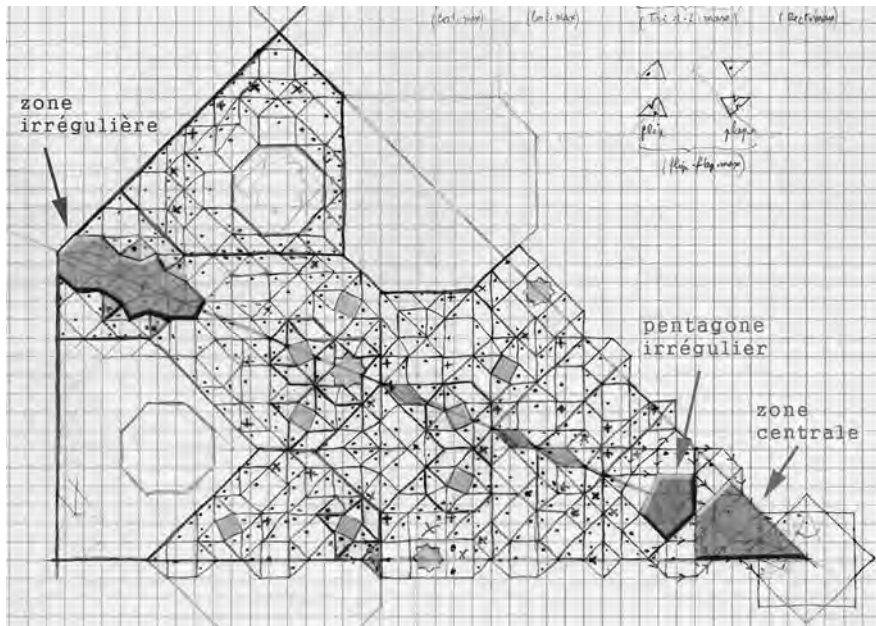


Fig. 6. Drawing of the dome in plan

Thus a hall culminating in a dome symbolises the alchemic passage from the square to the circle by means of its upwards motion. From earth to the sky, from material beings to spiritual ones, from individual conscience to the ideal of nonseparation, the cosmic conscience...

The passage from the square to the circle become possible through the octagon, which can be seen as the fruit of the loving encounter between two complementary squares rotated 45° with respect to each other. A connotation of love, with its multiple descriptions, definitions, its infinite poetic evocations and invocations, which are found in all forms of spirituality.²

² This symbolism (from the square to the circle, from earth to the sky) is strongly universal, apart from the connotations attributed to it by any given religion or philosophy. The first mosques, as were the first temples, were simple rectangular delimitations of space, open to the sky. The most minimal mosque is effectively a rectangular prayer rug. A basin of water was often placed at the centre to provide a natural mirror reflecting the image of the sky, represented by the dome: the presence of heaven on earth. It should be noted as well that sometimes are found double drums, made up of a sixteen-sided drum on top of an eight-sided drum (as, for example, in the Friday mosque in Isphahan, Iran).

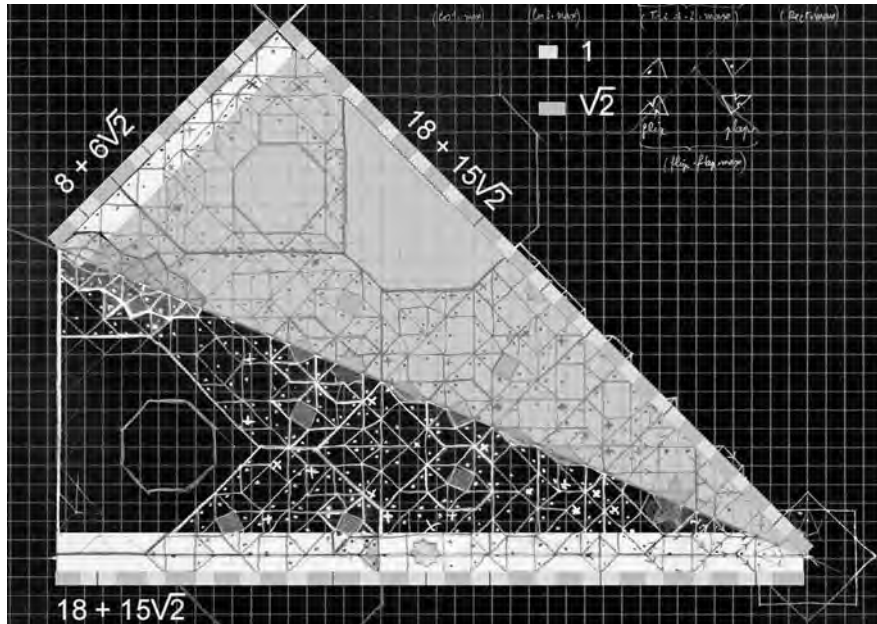


Fig. 7. Analysis of the dimensions: the impossible triangle

The Hall of the Two Sisters: a mapping

The cupola

The survey: an impossible triangle

The survey was accomplished starting with photographs, using a manual drawing technique on gridded paper (in which $\sqrt{2}$ is approximated by 1.5).

Drawing the map, one understands however that something is amiss: 'something doesn't fit!' We can confirm this by analysing the lengths of the sides of the triangle that defines the sixteenth part of the dome (see Fig. 7). Effectively, the pieces of the muqarnas lead to a mono-dimensional covering of these sides with two types of segments (in blue and yellow in the figure; see the section in colour). If the length of the small segment is adopted as unity, then the length of the large one is $\sqrt{2}$. It is sufficient to count the segments to obtain (with respect to the considered scale) the length of the sides of the right triangle, $8 + 6\sqrt{2}$ for the lesser, $18 + 15\sqrt{2}$ for the greater. It is equally amazing to verify that such a right triangle, with an angle of 22.5° is... impossible! We deduce that certain pieces have to be irregular.

The proposed solution

The simplest solution consists in concentrating the irregularities in the small group of coloured pieces in blue in the drawing in Fig. 6 (see the section in colour), which would then be deformed in the horizontal dimension. In any case further investigation would be necessary to determine if this was the solution adopted by the builders of the Alhambra.

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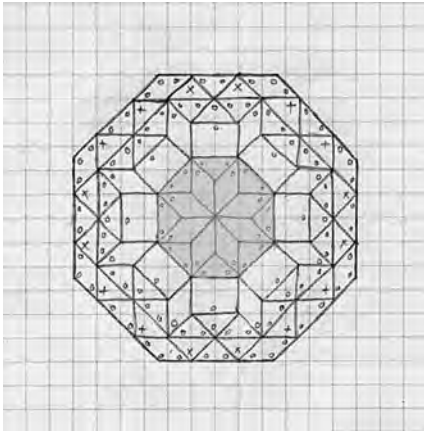


Fig. 8. A possible solution for the centre

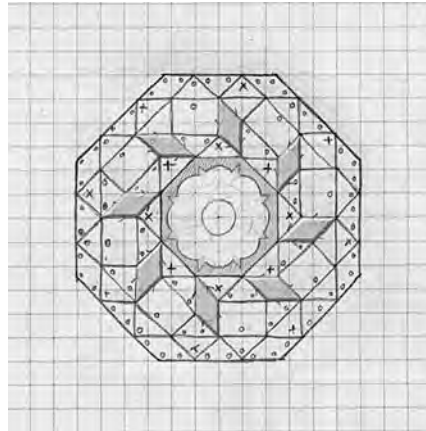


Fig. 9. The solution finally adopted

A pseudo-pentacle

Another oddity can be observed (Figs. 6 and 16) in the stellated pentagon which appears to be, on first sight, as regular. However, there cannot be regular pentagons in such a structure.

An unexpected symmetry break

Let us consider the central zone of the dome, shown in Fig. 6. Figure 8 shows how the muqarnas might have been assembled: a classic solution with standard pieces that respects the octagonal symmetry that dictates the rules in that story. In short, no surprises.

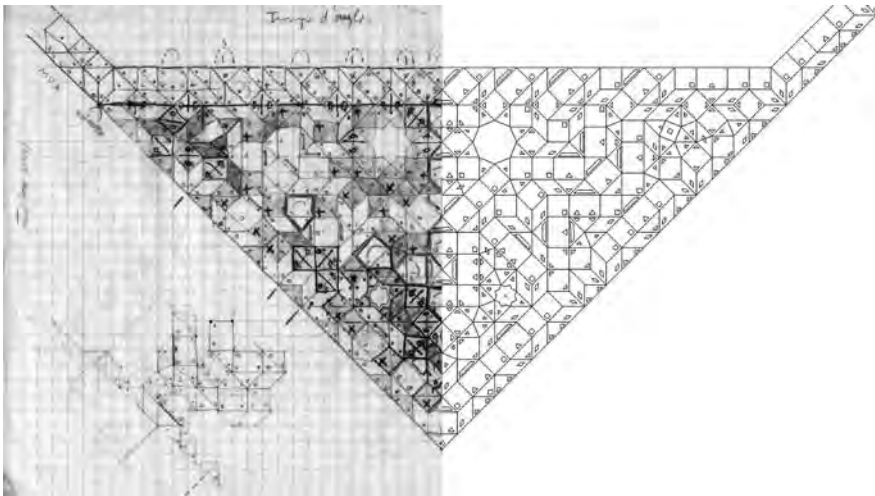


Fig. 10. Arrangement of muqarnas in the squinches: survey by hand and exact representation. All pieces are regular

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On the other hand, we are in the Alhambra and, consequently, nothing is as simple as it seems. Figure 9 shows the solution actually adopted: all pieces are regular, but the symmetry has been broken due to the totally original arrangement of some of the pieces (the grey rhombus in the drawing).

The law laid down by the octagonal symmetry is destroyed, in an almost invisible way to the observer, precisely underneath the *chechia* at the vertex of the dome. Can this be some kind of subversive allusion?

Friezes and corners

The lower part of the drum rests on the four walls of the hall. The empty portions of the four corners must be filled to redistribute the load of the dome (in fact there are two domes: the muqarnas dome, just decorative, and the structural dome, that can be seen only from the exterior and from which the muqarnas are suspended). This redistribution is achieved with an assemblage of muqarnas, and the four angles are connected with a very simple muqarnas frieze. No surprise, everything is regular. It should be noted that these muqarnas modules are on a smaller scale with respect to those used for the dome (perhaps because of considerations about perspective?).

3D modelling

The images that follow, extracted from a 3D animation, show a virtual reconstruction of the dome made from a plan, which was drawn by hand.



Fig. 11. Axonometric representation (3D modelling without decoration of the pieces of the *muqarnas*

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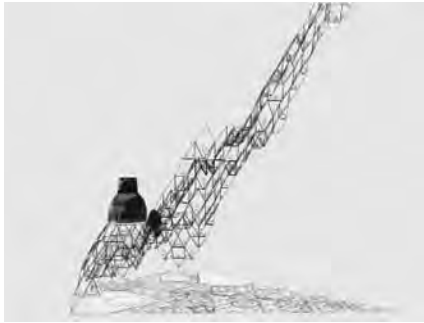


Fig. 12. Drawing in plan and in elevation. The pieces of the muqarnas were simplified to the extreme: simple quadrilaterals that will later be replaced with more realistic pieces. Some elements have already been represented by volumes

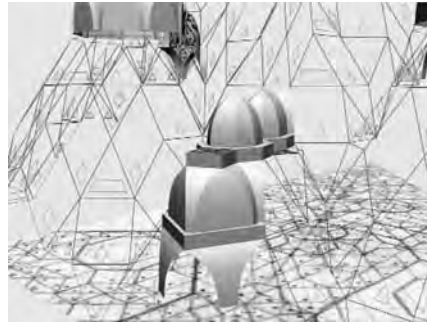


Fig. 13. Close-up view, from the exterior, of the arrangement of the irregular pieces



Fig. 14. Going up towards the top of the dome



Fig. 15. The top, seen from the interior. The symmetry of the *chechia* is sixteen-fold, and culminates in a circle. Note, immediately below, the symmetry break



Fig. 16. Pentagonal stars, though they seem to be regular, they are not

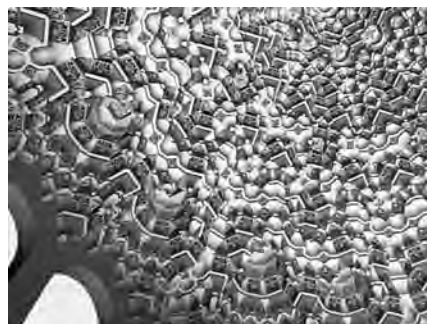


Fig. 17. The joining of the dome to the octagonal drum

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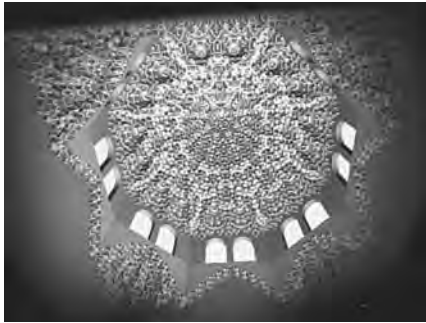


Fig. 18. The joining of the octagonal drum to the four walls of the hall, with the squinches filled with muqarnas

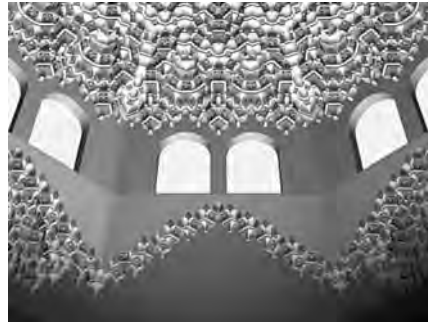


Fig. 19. Close-up view of the joining

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