Chapter 2.1

LESS CHALK, LESS WORDS, LESS SYMBOLS ... MORE OBJECTS, MORE CONTEXT, MORE ACTIONS

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Abstract: We will show how real objects, real places and real challenges may play an important role in the process of teaching mathematics by means of modelling and applications.

1. INTRODUCTION

Throughout this contribution we will defend the idea that realistic teaching is an appropriate method for quantitative literacy training (Steen, 1998, 2001).

An important consequence of teaching "via applications" is that the classical way of delivering lectures needs to be changed. Teaching with applications today means stopping the "talk & chalk" method; no longer using an old textbook and instead offering a very lively guiding program, based upon various information sources, which opens new windows to appreciate the context of the students and their creativity as individuals and as a group (Alsina, 1998b, 2003)

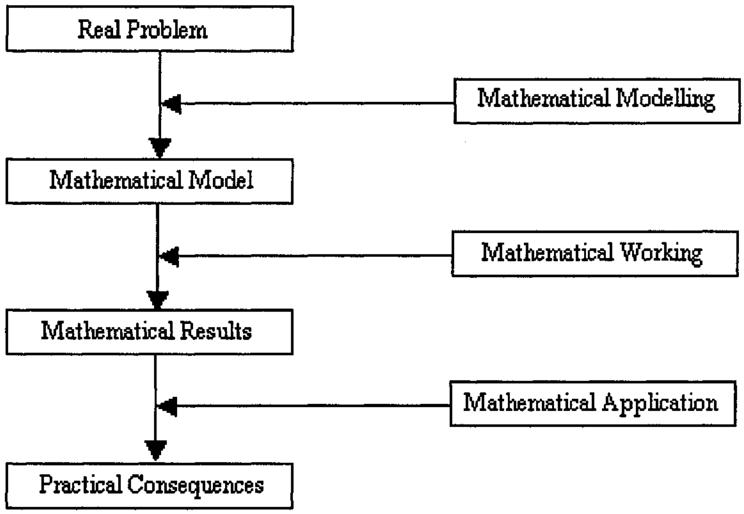
Following the discussion document of this ICMI Study: "by *real world* we mean everything that has to do with nature, society or culture, including everyday life as well as school and university subjects or scientific and scholarly disciplines different from mathematics."

So we will focus only on *objects* and *instruments*, on *everyday* situations, on *frequent* or *recent events*, on real challenges and showing how this realistic approach may play an important role in the process of teaching by means of modelling and applications (see, e.g., the educational projects "Modelling our world" (COMAP, 1998) and "Math in context" (Romberg & de Lange, 1998).

The ideas that we will present here come from our own experience and research in teaching mathematics as a service subject, training groups of teachers and doing workshops with high school students.

2. MATHEMATICAL TAKING OFF AND LANDING

The following diagram shows the classical way to deal, step by step, with the procedures for modelling-working-applying:



While most contributions in this field focus their attention on the central parts of this diagram, our aim here is to fix our views on the two boxes at either extreme: *the realities to be considered and skills for deriving practical consequences.* Too often, both ends become theoretical: word problems versus word solutions. If this is the case then we lose the possibility of motivating and providing students applied competencies (Niss, 1992, 2001).

3. LET US USE REAL OBJECTS

Our chief concern in this section is to pay attention to the wide range of daily life objects that may be used for teaching purposes, either for introducing new shapes and problems or for motivating a concrete visual approach to useful mathematics (Bolt, 1991; Steen, 1994; Alsina,1998a,b). All these objects constitute real applications of mathematics and by observing their characteristics and functional properties, students may appreciate the creativity behind them or may discover their limitations. As teachers may bring objects to the classroom and students may bring their own, this is a free material at our disposal.

Example: Modelling in the rain – with umbrellas

Today's umbrellas are sophisticated folding structures but they share with older ones a beautiful geometrical fact: the regular 8-gon determined by their extreme points. When you observe the moving octagonal pyramid of the structure you can discover how several articulated parallelograms change angles (and areas) but keep their perimeters. All these parallelograms determine a moving bipyramid. If you join two extreme points of the 8-gon you obtain a moving 7-gon which can be used to mark the 7-gon in a given circle (one of the impossible solutions with rules and compasses!).

It is interesting (Alsina, 2003) to collect umbrellas, fans, hats, etc, from all around the world (see also Gheverghese, 1996), i.e., daily life objects that exhibit flexibility.

Example: Polyhedra and polygons in context.

Nature exhibits a very restrictive collection of polyhedra. Only in some specific classes of minerals does one find basic shapes such as cubes or prisms. However, designers have produced a wide range of objects that have polyhedral forms. Packaging, logistics and beauty have motivated these designs.

Table 2.1-1. Polyhedra and daily life objects

CUBES	Dice, soup cubes, presents in boxes, hat boxes…
TETRAHEDRA	Tetra Pack ®, 3D puzzles, tripods, 4-faced dice...
OCTAHEDRA	Diamonds for cutting, table structures, kites, 8-faced dice...
ICOSAHEDRA	MAA logo, 20-faced dice, domes, sculptures…
DODECAHEDRA	Holders, 12-faced dice, no parking signs…
PRISMS	Toblerone packages, cookie boxes, pencils…
BOXES	TetraBrik ® packaging, cakes, Chanel nº 5 box, packages…
PYRAMIDS	Egyptian pyramids, the top of an obelisk, Sharkowski pieces…
BIPYRAMIDS	Whipping tops, jewels…
OTHER POLYHEDRA	Jewels, footballs, puzzles…

In our geometry education we anticipate the study of n-gons to the knowledge of polyhedra. This is, possibly, a mistake. Our visual experience goes, in general, from 3D to 2D.

Nevertheless, n-gons appear also by themselves in some planar objects or graphical designs.

Table 2.1-2. Polygons and daily life objects

TRIANGLES	Traffic signs, damage signals, musical instrument
QUADRILATERALS	Paper sheets, tile, cookies, cubes, brooches, snacks
PENTAGONS	Chrysler logo, napkin knot, tables
HEXAGONS	Tiles, plates, pencil sections, kite
OCTAGONS	Wind's directions, tables, trays, domes
n-GONS	Hours in a watch (12), cookies, commercial logos
STAR GONS	Sea star, star of David, tyre, clasps

Example: Curves in our life

Table 2.1-3. Curves in our life

Curve	Daily life examples
Straight line	Edge of a sheet of paper, string with a plumb-bob...
Circle	Plate, rim of a glass, coin, wheel, ring...
Ellipse	Profile of a hat, inclined liquid in a glass...
Parabola	Parabolic antenna, hand near ear...
Hyperbola	Profile of a bell, arcs in a hexagonal pencil...
Sinusoid	Snake's movement, sea waves, roofs...
Cycloid	Trajectory of a point in a wheel, pizza maker...
Catenary	Train wires, hanging chain...
Spirals	Classical discus, tape in a cassette, CDs...

Example: Transformations in daily life experiences

We may identify the basic geometrical transformations whose effects are seen in daily life movements: *translation* when walking down a street: *rotation* when the hands of a watch move or when we open a door; *symmetry* as a mirror effect: *similitude* when making reduced or enlarged Xerox copies; *affinities* when folding a box; *projectivities* when making shadows or photographs; *homeomorphisms* when folding a T-shirt, etc.

4. LET US EXPLORE REAL PLACES

Where are we teaching? Are we in a big city? Are we in a small village? Are we in a developed country...? We must be sensitive to our location. Some environments are rich in motivating contexts; others are not. We may take advantage of the location or, alternately, we may need to supply "additional motivation".

Do we have factories to visit? Are interesting measurements available in the area? Do we have notable buildings? How is public transportation organized? How is pollution measured...? If we get positive answers to these questions then we will have interesting places to generate mathematical activities at hand. Otherwise we can "bring" appropriate input to the classroom by means of Internet, books or pictures...

Wherever we are, in addition to geographical or architectural possibilities, there are social demands, social issues to be faced, cultural activities, etc.

We need to take these motivating situations into account as much as possible: working conditions, retirement plans, economic indices, inflation, theatre, television, book reading, local dances, music, musical instruments and cuisine—all social and cultural realities may have some mathematical interest.

On a compulsory level we prepare future citizens in a very specific social context. Our mathematics teaching may benefit from local characteristics and it is our goal to prepare students to be *critical citizens and good professionals in whatever their context.*

Examples of local applications	Examples of global applications
• Geometrical characteristics of the school • Distances from school; times • Geographical coordinates of the school • Cost of food at school • Ratios of ingredients in popular dishes • Geometry of particular buildings in town • Mathematics in folk dances • Different scales in local charts • Statistical study of minorities in local society	• Demographic issues: perspectives • Timing in travelling • Mathematics in democracy • Ecological problems: The Kyoto agreement • Mathematics and traffic (cars, roads, petrol) • Locations in the planet (GPS system) • Air traffic control and capacities • Digital images as messages • Implications of air conditioning in housing • Statistics on process: imports and exports

- History of calendars. Local holidays
- Mathematics and sports. World records
- Art exhibits in the town
- Mathematics in newspapers and magazines
- Mathematics in consumer issues. Indices
- Numbers and classical tales
- Numbers in popular sayings
- Mathematics and music
- Alcohol rates and driving: waiting times
- A visit to a car factory: sequential working
- A visit to a food factory: quality control
- Codes, phones, messages, Internet
- Mathematics and genomics
- Art: painting, sculptures, buildings
- Fair division: geometry and equity
- Mathematics and information: CDs, DVDs.

5. LET US FACE REAL CHALLENGES

In the previous sections we have been using objects and places to provide visual images and to make mathematics visible. Let us consider now the challenge of facing realistic problems and finding realistic solutions.

One may know a lot of things about cubes, observing minerals and houses, making cardboard models, enjoying interactive 3D-programs in the computer, etc., but occasionally it is useful to face the real problem of making a real cubic box, such as one that can be used to contain a present. Say, for example, that you want to open (and to close) just one face, and you want to transport the box – design problems may be very instructional.

Example: Function and design

Most shapes that we have around us are the result of a design process: houses, streets, cars, beds, bells, pencils... In this designed reality there is a strong mathematical component, from measurements to shapes. Most of these objects were designed to satisfy some desirable function. As part of the classical dialogue between form and function designers look for "optimal solutions". But "optimal" may hide different ideas: minimal quantity of element, low cost, ecological aims, transportability goals, etc.

It's interesting to know how designers work and how they find the best design solutions. Let us recount here, in some detail, the story of Jacob Rabinov.

Rabinov worked in New York, making 225 patents for all sorts of devices. When he retired, he wrote the acclaimed book *Inventing for Fun and Profit.* One of Rabinov's favourite topics was "screws and screwdrivers". He wanted to avoid the problem of so many screws being removed due to the fact that so many screwdrivers could be used for the same class of screw (even coins!). Thus, Rabinov took advantage of geometry and created a screw whose head made it impossible to manipulate with any conventional driver. Here we reproduce his description:

> *If you make a triangular depression with sides in the shape of three arcs, were each point of the triangle is the center of curvature of the opposite arc, you have a triangular hole that can be driven with a specially shaped screwdriver, but not by any flat screwdriver. If you insert a flat blade, the blade will pivot at each corner and slide over the opposite curved surface, hit the next corner and slide again, and so on. Such a screw should look very attractive and would be very difficult to open without the proper tool.*

These three arcs quoted above form a Reuleaux triangle, a convex figure of constant width, which is not a circle.

Example: Beware of the steps in a staircase!

This is an example to be studied with materials on a 1:1 scale, and which has a universal value: all humans need stairs which are easy to climb and almost all human beings use shoes. Stairs are important objects. Measure them! (e.g. using electronic measurements). The ideal steps have two important measurements: H (height) and D (depth), related by the affine equation, $2H + D = 63$ cm. The inclination $\tan A = H{:}D$ is also interesting. What are the upper and lower bounds for H, D and A? When is it convenient to have a ramp and not a stair? In vertical ladders (such as in submarines) you face the steps to go down, but in normal stairs you come down the other way around: when is it better to face the steps?

In houses, stairs, streets, singular buildings, parks, mountains or plains – not far away from the classroom site – we find 1:1 models to provide a rich setting in which to practice actual measuring, drawing techniques, indirect measurements, finding of data, etc. The best lecture in the blackboard on the inclination of streets can't replace the real experience (at least once!) of effectively measuring the inclination in a real street. We want real challenges, not artificial questions.

6. IMPLEMENTATION IS THE ANSWER – WHAT ARE THE QUESTIONS?

One major challenge in mathematics education is to achieve the goal that realistic modelling and realistic application be indeed implemented in courses. This, however, is not so easy. We have clear evidence that there are many difficulties entailed in introducing this approach to our daily teaching (Breiteig, Huntley, & Kaiser-Meβmer, 1993). We would like to make some remarks:

We need teachers who are confident

The main objection is that pre-service education does not provide teachers enough knowledge and experiences to be confident in dealing with applications and modelling. Thus, many improvements on pre-service and in-service training need to be made.

The level of learners

Clearly, the level of learners will orientate us as to which choice to make concerning applications. While a tender, fictional tale on numeracy may be appropriate in kindergarten, there is no way to tell the same story in a high school. Each generation of students has topics that are relevant to them – and we want them to be interested.

Often in recreational mathematics, problems are presented in a fiction-real context which insinuates that the result to students' discovery will be a crucial issue in their lives. Crossing rivers, climbing castles, covering chessboards with tetraminos – who does these things today? Useless mathematics cannot become useful even if it is presented in a fun way.

"Cooked" examples to illustrate some mathematical concepts or results are related to situations which are not interesting for the students, or even teachers. Let us recall the old problem "if 5 workers in a building will end the work in 3 weeks, when will the building be finished if 25 workers are assigned?"

Applications (and especially research activities related to them) are ideal for inducing cooperative work or teamwork. Good assessment, e.g., in setting up projects, needs to take into account individual and cooperative aspects. Preparation for cooperative work is a crucial goal in today's circumstances (Galbraith, 1998; deLange, 1996; Blum, Niss, & Huntley, 1981; Blum & Niss, 1991; Tanton, 2001).

The applied approach is time consuming

This is a key issue. Indeed, many teachers do not organize active learning visits, experiments, etc., because there is a lack of time. It seems there is no problem of time for chalkboard expositions. What is true is that our proposals imply a careful preparation of agendas.

Technology is not the solution

The growing power of technologies may induce some people to believe that these new devices are the essential tools for providing support for well-structured experiences; that simulations and images may be enough to eliminate completely the need for "real experiments" and hands-on materials. This is not possible. New software gives new mathematical insights but can't replace "learning by making". In our discussion here, technology serves to complement what we are presenting.

Hands-on materials are not always available

While textbooks and classroom materials are produced in big quantities, with a wide range of alternatives, very few kites can be purchased in the market. Of course, many materials can be easily made and real objects are everywhere. New commercial initiatives, however, would truly be welcome. Fortunately, many "free" opportunities exist around us.

Realistic activities need to be properly integrated

There is the risk that realistic experiences can become isolated, like islands in the sea of formal instruction. Such experiences are not useful if they are not combined with the usual activities of everyday teaching practice. These actions serve to assist students to learn important concepts, and of giving them new opportunities to develop new skills.

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