# A Unified Approach for the Analysis of Networks Composed of Transmission Lines and Lumped Circuits*

A. Maffucci[1] and G. Miano[2]

[1] D.A.E.I.M.I, Università di Cassino, Via G. Di Biasio 43, 03043 Cassino, Italy, maffucci@unicas.it
[2] D.I.EL., Università di Napoli Federico II, Via Claudio 21, 80125 Napoli, Italy, miano@unina.it

**Abstract** The use of transmission line models in high-speed circuit analysis is here reviewed, by means of a unifying approach which allows getting insight on both the numerical simulation and theoretical investigation. Starting from a detailed analysis of the physical meanings of the transmission line models, the paper analyzes the effects of electrical interconnects on signal propagation by using a suitable time-domain equivalent circuit representation of the lines. Qualitative and quantitative analysis are carried out, with particular emphasys to nonlinear dynamics.

## 1 Introduction

Transmission line (TL) theory is a classic topic of Electromagnetics and several well-assessed analysis techniques are available to study through TL models the effects of propagation in a very wide class of problems, *e.g.,* [1]. Many of such techniques are only suitable for linear problems or for steady-state solutions. However, there are applications such as *high-speed electronic circuits* where the presence of nonlinear devices and the interest on fast transients require time-domain analysis of systems made by distributed and lumped elements. Due to the high operating frequencies and small sizes of such circuits, a reliable design must account for the signal distortion due to the propagation along the *electrical interconnects*, present at various hierarchical levels, *e.g.*: [2]-[6].

Under suitable hypotheses, the interconnects may be described by means of TL models. The TLs of practical interest have losses, parameters depending on the frequency and may be spatially non-uniform. In many cases the physical parameters of the lines are uncertain and a description of statistical type is required, [4]. Lumped circuits may contain dynamic elements (*e.g.,* inductors, capacitors, transformers), resistive elements that may be nonlinear and time-varying (*e.g.,* diodes, transistors, operational amplifiers, logic gates, inverters) and integrated circuits. The interactions between these devices and the TLs, and between the TLs themselves, are described by continuity conditions for voltages and currents at the 'boundaries' between the TLs and the lumped circuit elements, and between the TLs themselves.

To analyze such systems, coupled problems of a profoundly different nature have to be studied: TLs are described by linear and time-invariant partial differential equations, while lumped circuits are modeled by algebraic-ordinary differential equations, eventually time-varying and nonlinear. For such reasons, TL model has recently received renewed attention, focused on important issues concerning both the *qualitative* (well-posedness of the models, convergence of numerical solutions, study of nonlinear dynamics,...) and the *quantitative* point of view (efficient simulation of large systems, model-order reduction,...), *e.g.*: [2]-[6].

Here we present a unifying approach to get an insight on all the above questions. In Sect. 2.1 we first focus on some important *physical properties* of the TL models, in order to highligth the limits of the standard TL model and to suggest a way to *generalize* it. Then in Sect. 2.2 a general method is presented to characterize the terminal behavior of TLs lines, in order to study of networks composed of TLs and

*Invited paper at SCEE-2004

This work is supported in part by Italian Ministry of University under a Program for the development of Research of National Interest (PRIN Grant n.2002093437).

lumped circuits by means of the Circuit Theory approach[7], [8]. To this aim, the most suitable time-domain characterizations of a line is based on an input-state-output representation, where the traveling-wave solutions of TL equations are chosen to represent the 'state'. Such a representation provides a circuit description of the TLs in terms of resistive elements, delayed sources and dynamic elements. Here we refer, for the sake of simplicity, to two-conductor TLs. However, the method is applicable to any kind of line: multiconductor lines, lines with frequency-dependent parameters, and lines with space-varying parameters, [6].

After deriving such a characterization, the analysis of networks composed of TLs and lumped circuits is reduced to the study of networks where TLs are modeled in the same way as the lumped elements: multiports representing the TLs lines differ from multiports representing the lumped elements only in their characteristic relations. In Sect. 3.1 the problem of the well-posedness of both analytical and numerical models describing TLs connected to nonlinear and/or dynamic terminations is addressed. This problem is of a great importance both from a theoretical and from a practical point of view: even if a stable and consistent numerical scheme is adopted, the convergence of the numerical solution is assured only if the analytical and the numerical models are both well-posed: these basic requirements cannot be taken for granted. In Sect. 3.2 some case-studies are presented. The first is intended to highlight the effects of TL modeling on the integrity of propagating signals. The second case-study provides an example of a class of nonlinear circuits, where the role of TLs is crucial to provide a wide richness of nonlinear dynamics, such as multiple steady state solutions, bifurcations and chaotic dynamics.

## 2 Transmission line models

### 2.1 Physical interpretation of the TL models

Let us consider the simple interconnect of Fig. 1, made of two perfectly conducting parallel wires of length $2l$ with arbitrary cross-sections, geometrically long, embedded in a homogeneous dielectric. The electromagnetic field can be represented, in the frequency domain, through the potentials $\varphi$ and $\mathbf{A}$, as

$$\mathbf{E} = -j\omega\mathbf{A} - \nabla\varphi, \mathbf{B} = \nabla \times \mathbf{A}, \tag{1}$$

where $\omega$ is the angular frequency and the potentials $\varphi$ and $\mathbf{A}$ are expressed, assuming Lorentz gauge, in terms of the surface charge $\sigma$ and current density $\mathbf{J}_s$ by means of the integral relations

$$\mathbf{A}(\mathbf{r}_P) = \mu \int_{\Sigma_1 \cup \Sigma_2} G(r_{PQ})\mathbf{J}_s(\mathbf{r}_Q)ds, \tag{2}$$

$$\varphi(\mathbf{r}_P) = \frac{1}{\epsilon} \int_{\Sigma_1 \cup \Sigma_2} G(r_{PQ})\sigma(\mathbf{r}_Q)ds, \tag{3}$$

where $\Sigma_1, \Sigma_2$ are the conductor surfaces, $r_{PQ}$ is the distance between the field and source points, $G$ is the Green function for the homogeneous space $G(r) = \frac{\exp(-jkr)}{4\pi r}$ and $k = \omega\sqrt{\epsilon\mu}$. Here we assume that the

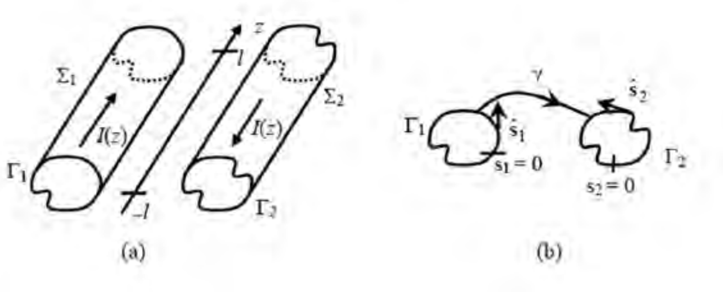


**Fig. 1.** (a) Schematic representation of the interconnect geometry; (b) cross-section

characteristic dimensions of the devices are small compared to the interconnect length, hence their effects are neglected in (2) and (3).

The unknown distributions $\sigma$ and $\mathbf{J}_s$ are determined by imposing the *boundary conditions* and the *charge conservation law*:

$$\mathbf{E}\cdot\hat{\mathbf{t}} = \mathbf{0} \;\; \textit{on } \Sigma_1 \textit{ and } \Sigma_2, \tag{4}$$

$$\nabla\cdot\mathbf{J}_s = -j\omega\sigma \;\; \textit{on } \Sigma_1 \textit{ and } \Sigma_2. \tag{5}$$

The fundamental assumptions to derive the TL models are the following:

1. the current field density has only the longitudinal component;
2. the *common mode* variables are equal to zero;
3. the dependence of $\sigma$ and $\mathbf{J}_s$ on the transverse and longitudinal coordinates is of a separable type;
4. the interconnect is transversally *electrically short*.

Hypothesis 1 depends on the cylindrical symmetry of the structure and on the way the structure is excited. In such a condition, the magnetic field is of *transverse type* (TM), hence it is possible to define uniquely at each section the voltage between the two conductors $V(z)$, which is related to the per-unit-length (p.u.l.) flux $\Phi(z)$ through:

$$-\frac{dV(z;\omega)}{dz} = j\omega\Phi(z;\omega). \tag{6}$$

Hypothesis 2 is well founded if there are no external sources of electromagnetic field. As a consequence of this assumption and of the conservation equation, the *differential* current at each section $I(z)$ is related to the p.u.l. electric charge $Q(z)$ through:

$$-\frac{dI(z;\omega)}{dz} = j\omega Q(z;\omega). \tag{7}$$

Hypothesis 3 holds if the characteristic dimensions of the conductor sections are *electrically short*, *i.e.* are small compared to the characteristic signal wavelength. The transverse problem may be solved considering the *electrostatic potentials* produced by the same conductor pair, but of *infinite length*. Hypotheses (1)-(3) allow to derive a *transmission line model*, defined by (6), (7) and by the following two integral relations:

$$\Phi(z;\omega) = \mu\int_{l}^{-l} H(z-z';\omega)I(z';\omega)dz', \tag{8}$$

$$V(z;\omega) = \frac{1}{\epsilon}\int_{l}^{-l} H(z-z';\omega)Q(z';\omega)dz', \tag{9}$$

which could be easily derived from (2), (3), as shown in [9]. The kernel of such relations $H(z)$ is expressed in terms of the Green function $G$ and become of impulsive type if hypothesis 4 holds [9]. With such an additional condition we have $\Phi(z) = LI(z)$, $V(z) = Q(z)/C$, where $L$ and $C$ are, respectively, the p.u.l. inductance and capacitance of the interconnect evaluated by solving the transverse 2D problem. By combining the above results we obtain the *standard* TL model described by the *telegrapher's equations*

$$-\frac{dV(z;\omega)}{dz} = j\omega\, LI(z;\omega),\;\; -\frac{dI(z;\omega)}{dz} = j\omega\, CV(z;\omega). \tag{10}$$

From a physical point of view, it is well-known that the TL model (10) describe the propagation of a field of transverse electromagnetic type (TEM), *e.g.*, [1]. Instead, the TL model (6), (7), (8), and (9) is a *generalized* model which could describe also the presence of continuum spectrum modes along with the fundamental one. This allows the description of high-frequency effects like *radiation losses* and *dispersion* which are not predicted by the standard TL model [9]. Table 1 summarizes the conditions when the lumped models, the standard TL model (STL) and the above *enhanced* TL model (ETL) have to be used, expressed in terms of operating frequency (through the wavenumber $k$), characteristic longitudinal ($2l$) and transverse ($h$) dimensions, and mean radius of conductor section $a$. A full-wave model is required for the analysis of all those cases not included in Table 1.

**Table 1.** Interconnect models for different cases

| model | $k \cdot 2l$ | $k \cdot h$ | $k \cdot a$ |
|---|---|---|---|
| lumped | $<< 1$ | $<< 1$ | $<< 1$ |
| STL | $\geq 1$ | $<< 1$ | $<< 1$ |
| ETL | $\geq 1$ | $\approx 1$ | $<< 1$ |

Even in hypothesis 4, when considering non-ideal structures, conductor and dielectric losses have to be taken into account: their effects destroy, in principle, the TEM structure of the field. However, in the quasi-TEM assumption (*e.g.*, [1]) the propagation may be still described by the TL model:

$$-\frac{dV(z;\omega)}{dz} = Z(z,\omega)I(z,\omega), \; -\frac{dI(z;\omega)}{dz} = Y(z,\omega)V(z). \tag{11}$$

where $Z(z,\omega)$ and $Y(z,\omega)$ are the line parameters, *i.e.* the *p.u.l. impedance* and *admittance*. The line parameters depend on the actual physical realization of the line: they can describe the simple ideal case (10) when $Z = j\omega L$ and $Y = j\omega C$. Instead, when $Z = R + j\omega L$ and $Y = G + j\omega C$, they describe the so-called RLGC lines (lossy uniform lines with negligible frequency effects). More generally they could describe non-uniform lines with strong frequency dependence, for instance due to conductor skin-effect and dielectric dispersive behavior: for most cases of practical interest, they could be conveniently described by the following *Laplace domain* model, *e.g.,* [6]:

$$Z(s) = R_\infty + sL_\infty + K\sqrt{s} + Z_r(s) \tag{12}$$

$$Y(s) = G_\infty + sC_\infty + Y_r(s) \tag{13}$$

where $(.)_\infty$ stands for the high-frequency limit, which may be evaluated from the physical model of the line or even from frequency-domain samples of the parameters provided by measurements, *e.g.,* [10]. It is important to stress that $Z_r(s)$ and $Y_r(s)$ tends to zero as $1/s$ for $s \to \infty$.

### 2.2 Equivalent circuit models

There are many possible two-port equivalent representations of TLs, both in frequency and time domain: the optimal choice strongly depends, of course, on the particular problem to be solved, *e.g.,* [11].

When dealing with high-speed circuits, usually one has to perform time-domain transient analysis of circuits made by linear TLs and non-linear lumped elements. In such cases, among all the possible two-port representations, a very convenient one is provided by the input-state-output representation obtained by assuming forward and backward waves as state-variables of the dynamic system. Such an approach would lead in the ideal-line case to the same result obtained by Branin, [12], by applying the *Method of Characteristics*. In the general case, it provides the following time domain model (*e.g.,* [6]):

$$i_1(t) = y_c(t) * i_1(t) + j_1(t), \quad i_2(t) = y_c(t) * v_2(t) + j_2(t), \tag{14}$$

where $*$ indicates the convolution product, subscripts 1,2 indicates the two line ends and $j_1$ and $j_2$ are two controlled current sources given by

$$j_1(t) = p(t) * [-2i_2(t) + j_2(t)], \quad i_2(t) = p(t) * [-2i_1(t) + j_1(t)]. \tag{15}$$

Such a dynamic model is characterized by two *impulse responses*: the *characteristic admittance* $y_c(t)$ and the propagation function $p(t)$, which can be obtained by reverse-transforming their Laplace domain expressions:

$$Y_c(s) = \sqrt{Y(s)/Z(s)}, \quad P(s) = exp\left(-2ls\sqrt{Y(s)Z(s)}\right). \tag{16}$$

The impulse responses may always be split into *irregular* and *regular* parts: the first contains *irregular* functions like Dirac pulses, and may be evaluated analytically from the asymptotic behavior of (16). After such asymptotic behavior is extracted, the regular parts may be easily evaluated numerically by

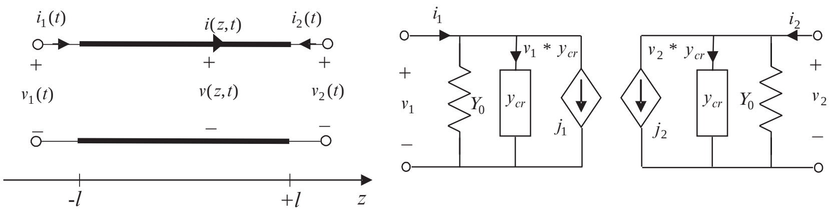


**Fig. 2.** Norton-type equivalent circuit representation of a two-conductor line

reverse transforming the Laplace domain remainders. We obtain, in the general case, the following decomposition:

$$y_c(t) = Y_0\delta(t) + y_{cr}(t), \quad p(t) = exp(-\mu T)[\delta(t-T) + p_r(t-T)], \tag{17}$$

where $Y_0 = \sqrt{C_\infty/L_\infty}$ is the ideal line characteristic admittance, $T = 2l\sqrt{C_\infty L_\infty}$ is the one-way delay time, $\mu$ is a damping factor which is known analytically, $y_{cr}(t)$ and $p_r(t)$ are the regular parts of the impulse responses, often known only numerically. Note that such properties hold for the general case of multiconductor lines with frequency-dependent parameters, with slight differences in the case of pronounced skin-effect, *e.g.* [6]. Such a line representation provides advantages both in the qualitative and numerical analysis, as shown in Sect. 3. Eqs. (14) and (15) describe each line end through the time-domain equivalent circuit of Norton type shown in Fig. 2. Apart for the effect of $Y_0$, which is always present, the solution at each line end is due to the contribution of the dynamic one-port $y_{cr}(t)$, which describes dispersion effects due to losses and frequency-dependence of line parameters, and of the controlled source $j_k(t),\ k = 1, 2$, which takes into account the reflection at the other end, the delay and dispersion introduced by the propagation along the line. Note that $j_k(t),\ k = 1, 2$ vanishes if the line is *matched* at the other end. The most important property of such a model is the fact that, at a given time instant $t$, $j_1(t)$ only depends on the solution history in the time interval $(0, t-T)$. Therefore, it could be treated as *independent* source, if the problem is solved iteratively.

## 3 Transmission lines and lumped circuits

### 3.1 Qualitative properties of the solution

Let us consider a two-conductor lossy line connecting two lumped nonlinear resistors. From the above considerations, the adopted circuit representation is a dynamic system which introduces a *state variable*, namely the current flowing into the dynamic one-port $y_{cr}(t)$. If we solve the problem recursively, at each time instant $t$ we find at the line terminations two *uncoupled networks* which may be simply represented by a resistive circuit, obtained by substituting the dynamic one-port $y_{cr}(t)$ with a constant current source $I_y$, see Fig. 3a. Note that such a procedure extends to distributed elements the concept of *associated resistive circuit*, introduced in the past for the analysis of lumped circuits, *e.g.*, [7]. Dynamic loads may be easily taken into account in a similar way: their corresponding associated resistive circuits are obtained by substituting each capacitor with a constant voltage source, and each inductor with a constant current source.

The analytical model obtained by combining (14),(15) with the characteristics of the lumped resistors is *well-posed* if it is possible to express all the *non-state circuit variables* as single-valued functions of the *state variables* and of the *source variables*. In fact, in such a case it can be proven that the model may be reduced to a well-posed system of Volterra integral equations in normal form, [6], [13]. For instance, considering two voltage-controlled nonlinear resistors

$$i_k(t) = g_k(v_k(t)), \quad k = 1, 2, \tag{18}$$

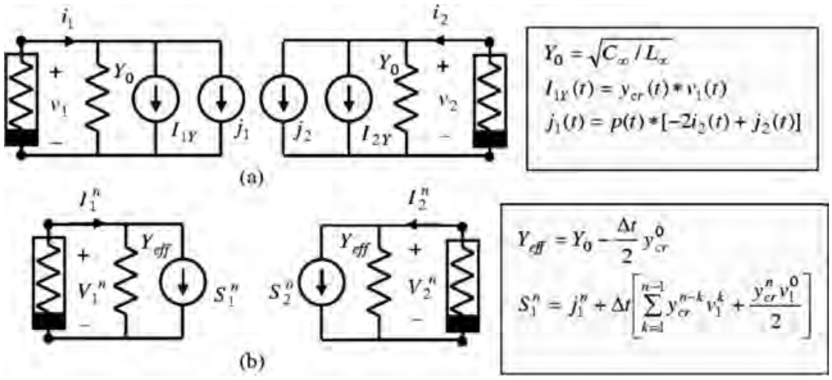

**Fig. 3.** Transmission line connecting two nonlinear resistors: (a) associated resistive circuit; (b) discrete resistive circuit

the following conditions are sufficient to obtain the well-posedness of the model

1. function $g_k$ is continuous;
2. the resistor is *weakly active*;
3. the following inequality holds:

$$dg_k/dv > -Y_0. \tag{19}$$

Inequality (19) is always satisfied if the characteristics of the resistors are *monotonically increasing*. Instead, it may be not satisfied if the characteristic has tracts with negative slopes. When this occurs, the associated resistive circuit can have more than one solution, so a normal form Volterra integral equation system may not exist and the solution may be not unique. The presence of a capacitor in parallel with the nonlinear resistor (even a parasitic one) ensures uniqueness even when the above condition of the slope of $g(v)$ is not satisfied, [6], [13]. Note that such a result may be easily generalized to multiconductor lines, [14].

Besides the associated resistive circuit, the line representation adopted here allows to introduce also the so-called *discrete resistive circuit*, [8], which describes the problem to be solved at any discrete time-step $t_n = n\Delta t$, where $\Delta t$ is the time discretization step (let $x^n$ indicate the generic variable $x(t)$ at $t = t_n$). Note that discrete circuits associated with different integration algorithms are different: Fig. 3b shows the circuit obtained by using the trapezoidal rule to integrate the convolutions (the variables at port 2 have analoguos definitions of those of port 1). The transient analysis reduces to the *dc* analysis of the resistive circuit of Fig. 3b: we can solve the associated discrete circuit step by step, by any efficient method, such as modified nodal analysis combined with Newton-Raphson method [8], through recursive updating of $S_1^n$ and $S_2^n$.

We observe that the associated discrete circuit of Fig. 3b tends to the associated resistive circuit of Fig. 3a for $\Delta t \to 0$. This implies that, if the associated resistive circuit has one and only one solution, and hence the dynamic circuit has one and only one solution, the numerical model has one and only one solution converging to the actual solution, [6], [15]. In fact, it is easy to show that the conditions ensuring the well-posedness of the associated discrete circuit are the same derived above for the associated resistive circuit, provided that $Y_0$ is replaced with $Y_{eff}$, see Fig. 3b:

$$dg_k/dv > -Y_{eff}. \tag{20}$$

If (19) is satisfied, there exists a sufficiently small $\Delta t$ to satisfy (20) also, and vice-versa, and the numerical model admits one and only one solution. If we consider non-linear resistors described through voltage-controlled non-monotone characteristics, (19) could be no longer verified, hence the original equations may admit several solutions, and condition (20) is not satisfied even if $\Delta t$ is arbitrarily small. As a consequence, the numerical model admits several solutions, and the discrete time sequence approximating the solution is no longer unique. In this case, a well-posed model is again obtained if we take into account the capacitive parasitic effects, neglected during modeling, that have a strong influence on the dynamics of the network, [6], [15].

### 3.2 Numerical analysis of practical applications

Whatever is the adopted two-port representation, the main drawback of such an approach is the high computational cost of transient analysis, mainly due to time convolution. Therefore, the literature proposes many techniques to obtain convenient reduced-order models, *e.g.*, [3]. It is known in literature that the model adopted here is the most suitable to perform transient analysis of *long* transmission lines, *i.e.* lines for which the propagation delay plays a significant role, *e.g.*, [10]. This is because such a model allows to extract analytically all the unbounded terms contained in the line impulse responses, which are then represented by simple resistive circuits and damped delayed sources. Only the regular remainders are approximated with reduced-order models, and then represented through low-order lumped networks.

Let us now refer to the case-study 1, consisting in a two-conductor microstrip of length $20cm$ analyzed in [4]. The interconnect is modeled as a TL with frequency-dependent parameters by using expressions (12) and (13), with: $C_\infty = 88.25\mu F/m, L_\infty = 0.806\mu H/m, R_\infty = 86.206\Omega/m, G_\infty = 67nS/m, K = 2.4m\Omega s^{-1/2}/m$, while $Z_r(s) = Y_r(s) = 0$. The line presents a characteristic admittance $Y_0 = 10.5mS$ and a delay time $T = 1.69ns$.

The line is synthesized by means of the equivalent circuit model of Fig. 2, by using a rational approximation for the two impulse responses remainders $y_{cr}(t)$ and $p_r(t)$. The near end is terminated on a driver, modeled as a voltage source in series with a resistor $R_1 = 50\Omega$. The voltage source supplies a rectangular pulse of amplitude $1V$, that lasts $2ns$, with rise and fall time $t_r = t_s = 50ps$. The far end is connected to a pn-junction diode, modeled by

$$i = I_s\left(\exp(v/V_T) - 1\right), \tag{21}$$

with $V_T = 1V$ and $I_s = 40\mu A$.

Figure 4 shows the far-end voltage obtained by using three different TL models: the complete one (skin), an approximated one obtained by neglecting the skin-effect (RLGC) and the lossless line limit (ideal). The simulation puts on evidence the strong effect of TL modeling on the signal shape: a correct modeling of TLs is crucial to foresee critical effects for signal integrity, like delay in the receiver switching, and false switching which may be caused by unwanted reflections, [16].

The second case-study analyzed here is intended to highlight the richness of behavior which could be observed when TLs connect nonlinear devices: multiple steady state solutions, bifurcations and chaotic dynamics. Let us consider an ideal TL connecting two nonlinear resistors: such a line is described by (17) with $y_{cr}(t) = p_r(t) = 0,\ \mu = 0$, hence the circuit state equations, obtained by combining (14) and (15) with the resistor characteristics, are nonlinear difference equations with one delay. The dynamics of the problem may be studied by analyzing the behavior of a *nonlinear one-dimensional map*, in which the time is no longer a continuous variable but a sequence of discrete values:

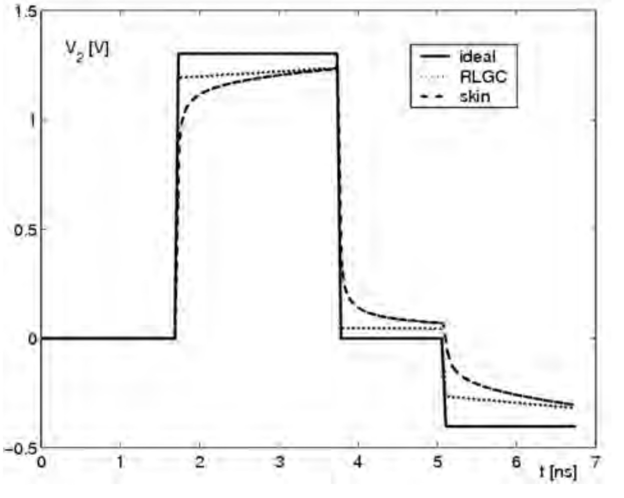


**Fig. 4.** Far-end voltage for case-study 1 predicted by different line models

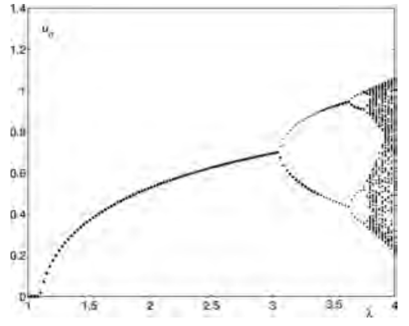


**Fig. 5.** Bifurcation diagram for case-study 2

$$u_{n+1} = f(u_n), \tag{22}$$

with a proper definition of the state variable $u_n$, [6]. By studying the main properties of these maps, bifurcations, periodic oscillations and chaos may observed when at least one terminal resistor is active and the other is nonlinear, *e.g.*, [6], [17].

Let us consider an ideal line of length $1m$, with the following parameters: $C = 3.00pF/m, L = 3.70\mu H/m$, leading to $Y_0 = 0.90mS$ and $T = 3.33ns$. Let us assume that the far-end is connected to an active linear resistor of conductance $G_1$, and the near-end is terminated on the diode of Eq.(21). A non-zero initial condition is imposed, by applying a unit voltage pulse at the far-end. In such a case, the map $f$ in (22) reduces to the *logistic map* $f = \lambda u_n(1 - u_n)$, where the parameter $\lambda$ is given by:

$$\lambda = \frac{Y_0 - G_1}{Y_0 + G_1}. \tag{23}$$

and the state variable is expressed in terms of the backward voltage wave at the far-end:

$$u_n = \lambda \frac{v_1 - i_1/Y_0}{2} \frac{1}{(1 - \beta - \ln(\beta))V_T}, \quad \beta = \frac{I_s}{V_T Y_0}. \tag{24}$$

Figure 5 shows the *bifurcation diagram* of such a map. For $0 \leq \lambda \leq 1$ the only fixed point is $u = 0$, while for increasing $\lambda$ we enter a region where a non-zero asymptotically stable fixed point may be observed. Then stable periodic orbits of period 2, 4, and so on may be observed, until $\lambda$ reaches a value such to excite chaotic dynamics. Note that the chaotic regime is interrupted by some windows where the asymptotic behavior of the orbits is again periodic.

## References

[1] Collin R.E., Foundation of Microwave Engineering. Mc Graw-Hill, New York (1992)
[2] Tripathi, V.K., Sturdivant, R., *Guest Editors*, Special Issue on: Interconnects and Packaging. IEEE Trans. on Microwave Theory and Techniques, **45** (1997)
[3] Schutt-Ainé, J. S., Kang, S. S., *Guest Editors*, Special Issues on: Interconnections - Addressing the Next Challenge of IC Technology. IEEE Proceedings, **89** (2001)
[4] Paul, C.: Analysis of Multiconductor Transmission Lines. Wiley, New-York (1994)
[5] Matick R.E.: Transmission Lines for Digital and Communication Networks. IEEE press, Piscataway, N.J. (1995)
[6] Maffucci, A., Miano, G.: Transmission Lines and Lumped Circuits. Academic Press, San Diego (2001)
[7] Hasler, M., Neirynck, J.: Nonlinear Circuits. Norwood, Artech House (1986)
[8] Chua, L.O., Lin, P.M.: Computer Aided Analysis of Electronic Circuits. Prentice-Hall, Englewood Cliffs. (1975)
[9] Maffucci, A., Miano, G., Villone, F.: An enhanced transmission line model for conductors with arbitrary cross-sections. IEEE Transactions on Advanced Packaging, **28**, 174-188 (2005)
[10] Grivet-Talocia S., *et al.*: Transient analysis of lossy transmission lines: an efficient approach based on the method of characteristics. IEEE Trans. on Advanced Packaging, **27**, 45-56 (2004)

[11] Kuznetsov D.B., Schutt-Ainé J.E.: Optimal transient simulation of transmission lines, IEEE Trans. Circuits Systems-I, **43**, 110-121 (1996)
[12] Branin F.H., Jr: Transient analysis of lossless transmission lines. Proceedings of IEEE, **55**, 2012-2013 (1967)
[13] Miano, G.: Uniqueness of solution for linear transmission lines with nonlinear terminal resistors. IEEE Trans. on Circuit and Systems-I, **44**, 569-582 (1997)
[14] Maffucci, A., Miano, G.: On the dynamic equations of linear multiconductor transmission lines with terminal nonlinear multiport resistors, IEEE Transactions on Circuit and Systems-I, **45**, 812-829 (1998)
[15] Maffucci, A., Miano, G.: On the uniqueness of the numerical solutions of non linearly loaded lossy transmission lines, Intern. Journal of Circuit Theory and Applications, **27**, 455-472 (1999)
[16] Deutsch, A., *et al.*: High-speed signal propagation on lossy transmission lines. IBM Journal Research Development. **34**, 601-616 (1990)
[17] Corti, L., De Menna, L., Miano, G., Verolino, L.: Chaotic dynamics in an infinite-dimensional electromagnetic system. IEEE Trans. Circuits Systems-I, **41**, 730-736 (1994)