

## PHYSICAL FOUNDATIONS OF COSMOLOGY

Inflationary cosmology has been developed over the last 20 years to remedy serious shortcomings in the standard hot big bang model of the universe. Taking an original approach, this textbook explains the basis of modern cosmology and shows where the theoretical results come from.

The book is divided into two parts: the first deals with the homogeneous and isotropic model of the universe, while the second part discusses how initial inhomogeneities can explain the observed structure of the universe. Analytical treatments of traditionally highly numerical topics – such as primordial nucleosynthesis, recombination and cosmic microwave background anisotropy – are provided, and inflation and quantum cosmological perturbation theory are covered in great detail. The reader is brought to the frontiers of current cosmological research by the discussion of more speculative ideas.

This is an ideal textbook both for advanced students of physics and astrophysics and for those with a particular interest in theoretical cosmology. Nearly every formula in the book is derived from basic physical principles covered in undergraduate courses. Each chapter includes all necessary background material and no prior knowledge of general relativity and quantum field theory is assumed.

VIATCHESLAV MUKHANOV is Professor of Physics and Head of the Astroparticle Physics and Cosmology Group at the Department of Physics, Ludwig-Maximilians-Universität München, Germany. Following his Ph.D. at the Moscow Physical-Technical Institute, he conducted research at the Institute for Nuclear Research, Moscow, between 1982 and 1991. From 1992, he was a lecturer at Eidgenössische Technische Hochschule (ETH) in Zürich, Switzerland, until his appointment at LMU in 1997. His current research interests include cosmic microwave background fluctuations, inflationary models, string cosmology, the cosmological constant problem, dark energy, quantum and classical black holes, and quantum cosmology. He also serves on the editorial boards of leading research journals in these areas.

In 1980–81, Professor Mukhanov and G. Chibisov discovered that quantum fluctuations could be responsible for the large-scale structure of the universe. They calculated the spectrum of fluctuations in a model with a quasi-exponential stage of expansion, later known as inflation. The predicted perturbation spectrum is in very good agreement with measurements of the cosmic microwave background fluctuations. Subsequently, Professor Mukhanov developed the quantum theory of cosmological perturbations for calculating perturbations in generic inflationary models. In 1988, he was awarded the Gold Medal of the Academy of Sciences of the USSR for his work on this theory.

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Frontmatter  
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VIATCHESLAV MUKHANOV  
*Ludwig-Maximilians-Universität München*



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## Foreword by Professor Andrei Linde

Since the beginning of the 1970s, we have witnessed spectacular progress in the development of cosmology, which started with a breakthrough in the theoretical understanding of the physical processes in the early universe and culminated in a series of observational discoveries. The time is ripe for a textbook which summarizes the new knowledge in a rigorous and yet accessible form.

The beginning of the new era in theoretical cosmology can be associated with the development of the gauge theories of weak, electromagnetic and strong interactions. Until that time, we had no idea of properties of matter at densities much greater than nuclear density  $\sim 10^{14}$  g/cm<sup>3</sup>, and everybody thought that the main thing we need to know about the early universe is the equation of state of superdense matter. In the beginning of the 1970s we learned that not only the size and the temperature of our universe, but also the properties of elementary particles in the early universe were quite different from what we see now. According to the theory of the cosmological phase transitions, during the first  $10^{-10}$  seconds after the big bang there was not much difference between weak and electromagnetic interactions. The discovery of the asymptotic freedom for the first time allowed us to investigate the properties of matter even closer to the big bang, at densities almost 80 orders of magnitude higher than the nuclear density. Development of grand unified theories demonstrated that baryon number may not be conserved, which cleared the way towards the theoretical description of the creation of matter in the universe. This in its turn opened the doors towards inflationary cosmology, which can describe our universe only if the observed excess of baryons over antibaryons can appear after inflation.

Inflationary theory allowed us to understand why our universe is so large and flat, why it is homogeneous and isotropic, why its different parts started their expansion simultaneously. According to this theory, the universe at the very early stages of its evolution rapidly expanded (inflated) in a slowly changing vacuum-like state, which is usually associated with a scalar field with a large energy density. In the simplest version of this theory, called ‘chaotic inflation,’ the whole universe could

emerge from a tiny speck of space of a Planckian size  $10^{-33}$  cm, with a total mass smaller than 1 milligram. All elementary particles surrounding us were produced as a result of the decay of this vacuum-like state at the end of inflation. Galaxies emerged due to the growth of density perturbations, which were produced from quantum fluctuations generated and amplified during inflation. In certain cases, these quantum fluctuations may accumulate and become so large that they can be responsible not only for the formation of galaxies, but also for the formation of new exponentially large parts of the universe with different laws of low-energy physics operating in each of them. Thus, instead of being spherically symmetric and uniform, our universe becomes a multiverse, an eternally growing fractal consisting of different exponentially large parts which look homogeneous only locally.

One of the most powerful tools which can be used for testing the predictions of various versions of inflationary theory is the investigation of anisotropy of the cosmic microwave background (CMB) radiation coming to us from all directions. By studying this radiation, one can use the whole sky as a giant photographic plate with the amplified image of inflationary quantum fluctuations imprinted on it. The results of this investigation, in combination with the study of supernova and of the large-scale structure of the universe, have already confirmed many of the predictions of the new cosmological theory.

From this quick sketch of the evolution of our picture of the universe during the last 30 years one can easily see how challenging it may be to write a book serving as a guide in this vast and rapidly growing area of physics. That is why it gives me a special pleasure to introduce the book *Physical Foundations of Cosmology* by Viatcheslav Mukhanov.

In the first part of the book the author considers a homogeneous universe. One can find there not only the description of the basic cosmological models, but also an excellent introduction to the theory of physical processes in the early universe, such as the theory of nucleosynthesis, the theory of cosmological phase transitions, baryogenesis and inflationary cosmology. All of the necessary concepts from the general theory of relativity and particle physics are introduced and explained in an accurate and intuitively clear way. This part alone could be considered a good textbook in modern cosmology; it may serve as a basis for a separate course of lectures on this subject.

But if you are preparing for active research in modern cosmology, you may particularly appreciate the second part of the book, where the author discusses the formation and evolution of the large-scale structure of our universe. In order to understand this process, one must learn the theory of production of metric perturbations during inflation.

In 1981 Mukhanov and Chibisov discovered, in the context of the Starobinsky model, that the accelerated expansion can amplify the initial quantum perturbations

of metric up to the values sufficient for explaining the large-scale structure of the universe. In 1982, a combined effort of many participants of the Nuffield Symposium in Cambridge allowed them to come to a similar conclusion with respect to the new inflationary universe scenario. A few years later, Mukhanov developed the general theory of inflationary perturbations of metric, valid for a broad class of inflationary models, including chaotic inflation. Since that time, his approach has become the standard method of investigation of inflationary perturbations.

A detailed description of this method is one of the most important features of this book. The theory of inflationary perturbations is quite complicated not only because it requires working knowledge of General Relativity and quantum field theory, but also because one should learn how to represent the results of the calculations in terms of variables that do not depend on the arbitrary choice of coordinates. It is very important to have a real master guiding you through this difficult subject, and Mukhanov does it brilliantly. He begins with a reminder of the simple Newtonian approach to the theory of density perturbations in an expanding universe, then extends this investigation to the general theory of relativity, and finishes with the full quantum theory of production and subsequent evolution of inflationary perturbations of metric.

The last chapter of the book provides the necessary link between this theory and the observations of the CMB anisotropy. Everyone who has studied this subject knows the famous figures of the spectrum of the CMB anisotropy, with several different peaks predicted by inflationary cosmology. The shape of the spectrum depends on various cosmological parameters, such as the total density of matter in the universe, the Hubble constant, etc. By measuring the spectrum one can determine these parameters experimentally. The standard approach is based on the numerical analysis using the CMBFAST code. Mukhanov made one further step and derived an analytic expression for the CMB spectrum, which can help the readers to obtain a much better understanding of the origin of the peaks, of their position and their height as a function of the cosmological parameters.

As in a good painting, this book consists of many layers. It can serve as an introduction to cosmology for the new generation of researchers, but it also contains a lot of information which can be very useful even for the best experts in this subject.

We live at a very unusual time. According to the observational data, the universe is approximately 14 billion years old. A hundred years ago we did not even know that it is expanding. A few decades from now we will have a detailed map of the observable part of the universe, and this map is not going to change much for the next billion years. We live at the time of the great cosmological discoveries, and I hope that this book will help us in our quest.

## Preface

This textbook is designed both for serious students of physics and astrophysics and for those with a particular interest in learning about theoretical cosmology. There are already many books that survey current observations and describe theoretical results; my goal is to complement the existing literature and to show where the theoretical results come from. Cosmology uses methods from nearly all fields of theoretical physics, among which are General Relativity, thermodynamics and statistical physics, nuclear physics, atomic physics, kinetic theory, particle physics and field theory. I wanted to make the book useful for undergraduate students and, therefore, decided not to assume preliminary knowledge in any specialized field. With very few exceptions, the derivation of every formula in the book begins with basic physical principles from undergraduate courses. Every chapter starts with a general elementary introduction. For example, I have tried to make such a geometrical topic as conformal diagrams understandable even to those who have only a vague idea about General Relativity. The derivations of the renormalization group equation, the effective potential, the non-conservation of fermion number, and quantum cosmological perturbations should also, in principle, require no prior knowledge of quantum field theory. All elements of the Standard Model of particle physics needed in cosmological applications are derived from the initial idea of gauge invariance of the electromagnetic field. Of course, some knowledge of general relativity and particle physics would be helpful, but this is not a necessary condition for understanding the book. It is my hope that a student who has not previously taken the corresponding courses will be able to follow all the derivations.

This book is meant to be neither encyclopedic nor a sourcebook for the most recent observational data. In fact, I avoid altogether the presentation of data; after all the data change very quickly and are easily accessible from numerous available monographs as well as on the Internet. Furthermore, I have intentionally restricted the discussion in this book to results that have a solid basis. I believe it is premature to present detailed mathematical consideration of controversial topics in a book on

*Preface*

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the foundations of cosmology and, therefore, such topics are covered only at a very elementary level.

Inflationary theory and the generation of primordial cosmological perturbations, which I count among the solid results, are discussed in great detail. Here, I have tried to delineate carefully the robust features of inflation which do not depend on the particular inflationary scenario. Among the other novel features of the book is the analytical treatment of some topics which are traditionally considered as highly numerical, for example, primordial nucleosynthesis, recombination and the cosmic microwave background anisotropy.

Some words must be said about my decision to imbed problems in the main text rather than gathering them at the end. I have tried to make the derivations as transparent as possible so that the reader should be able to proceed from one equation to the next without making calculations on the way. In cases where this strategy failed, I have included problems, which thus constitute an integral part of the main text. Therefore, even the casual reader who is not solving the problems is encouraged to read them.

## Acknowledgements

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## Units and conventions

*Planckian (natural) units* Gravity, quantum theory and thermodynamics play an important role in cosmology. It is not surprising, therefore, that all fundamental physical constants, such as the gravitational constant  $G$ , Planck's constant  $\hbar$ , the speed of light  $c$  and Boltzmann's constant  $k_B$ , enter the main formulae describing the universe. These formulae look much nicer if one uses (Planckian) natural units by setting  $G = \hbar = c = k_B = 1$ . In this case, all constants drop from the formulae and, after the calculations are completed, they can easily be restored in the final result if needed. For this reason, nearly all the calculations in this book are made using natural units, though the gravitational constant and Planck's constant are kept in some formulae in order to stress the relevance of gravitational and quantum physics for describing the corresponding phenomena.

After the formula for some physical quantity is derived in Planckian units, one can immediately calculate its numerical value in usual units simply by using the values of the elementary Planckian units:

$$\begin{aligned}
 l_{Pl} &= \left( \frac{G\hbar}{c^3} \right)^{1/2} = 1.616 \times 10^{-33} \text{ cm}, \\
 t_{Pl} &= \frac{l_{Pl}}{c} = 5.391 \times 10^{-44} \text{ s}, \\
 m_{Pl} &= \left( \frac{\hbar c}{G} \right)^{1/2} = 2.177 \times 10^{-5} \text{ g}, \\
 T_{Pl} &= \frac{m_{Pl}c^2}{k_B} = 1.416 \times 10^{32} \text{ K} = 1.221 \times 10^{19} \text{ GeV}.
 \end{aligned}$$

Planckian units with other dimensions can easily be built out of these quantities. For example, the Planckian density and the Planckian area are  $\varepsilon_{Pl} = m_{Pl}/l_{Pl}^3 = 5.157 \times 10^{93} \text{ g cm}^{-3}$  and  $S_{Pl} = l_{Pl}^2 = 2.611 \times 10^{-66} \text{ cm}^2$ , respectively.

Two examples below show how to make calculations using Planckian units.

**Example 1** Calculate the number density of photons in the background radiation today. In usual units, the temperature of the background radiation is  $T \simeq 2.73$  K. In dimensionless Planckian units, this temperature is equal to

$$T \simeq \frac{2.73 \text{ K}}{1.416 \times 10^{32} \text{ K}} \simeq 1.93 \times 10^{-32}.$$

The number density of photons in natural units is

$$n_\gamma = \frac{3\zeta(3)}{2\pi^2} T^3 \simeq \frac{3 \times 1.202}{2\pi^2} (1.93 \times 10^{-32})^3 \simeq 1.31 \times 10^{-96}.$$

To determine the number density of photons per cubic centimeter, we must multiply the *dimensionless* density obtained by the Planckian quantity with the corresponding dimension  $\text{cm}^{-3}$ , namely  $l_{Pl}^{-3}$ :

$$n_\gamma \simeq 1.31 \times 10^{-96} \times (1.616 \times 10^{-33} \text{ cm})^{-3} \simeq 310 \text{ cm}^{-3}.$$

**Example 2** Determine the energy density of the universe 1 s after the big bang and estimate the temperature at this time. The early universe is dominated by ultra-relativistic matter, and in natural units the energy density  $\varepsilon$  is related to the time  $t$  via

$$\varepsilon = \frac{3}{32\pi t^2}.$$

The time 1 s expressed in dimensionless units is

$$t \simeq \frac{1 \text{ s}}{5.391 \times 10^{-44} \text{ s}} \simeq 1.86 \times 10^{43};$$

hence the energy density at this time is equal to

$$\varepsilon = \frac{3}{32\pi (1.86 \times 10^{43})^2} \simeq 8.63 \times 10^{-89}$$

Planckian units. To express the energy density in usual units, we have to multiply this number by the Planckian density,  $\varepsilon_{Pl} = 5.157 \times 10^{93} \text{ g cm}^{-3}$ . Thus we obtain

$$\varepsilon \simeq (8.63 \times 10^{-89}) \varepsilon_{Pl} \simeq 4.45 \times 10^5 \text{ g cm}^{-3}.$$

To make a rough estimate of the temperature, we note that in natural units  $\varepsilon \sim T^4$ , hence  $T \sim \varepsilon^{1/4} \sim (10^{-88})^{1/4} = 10^{-22}$  Planckian units. In usual units,

$$T \sim 10^{-22} T_{Pl} \simeq 10^{10} \text{ K} \simeq 1 \text{ MeV}.$$

From this follows the useful relation between the temperature in the early Universe, measured in MeV, and the time, measured in seconds:  $T_{\text{MeV}} = O(1) t_{\text{sec}}^{-1/2}$ .



*Astronomical units* In astronomy, distances are usually measured in parsecs and megaparsecs instead of centimeters. They are related to centimeters via

$$1 \text{ pc} = 3.26 \text{ light years} = 3.086 \times 10^{18} \text{ cm}, \quad 1 \text{ Mpc} = 10^6 \text{ pc}.$$

The masses of galaxies and clusters of galaxies are expressed in terms of the mass of the Sun,

$$M_{\odot} \simeq 1.989 \times 10^{33} \text{ g}.$$

*Charge units* We use the Heaviside–Lorentz system for normalization of the elementary electric charge  $e$ . This system is adopted in most books on particle physics and in these units the Coulomb force between two electrons separated by a distance  $r$  is

$$F = \frac{e^2}{4\pi r^2}.$$

The dimensionless fine structure constant is  $\alpha \equiv e^2/4\pi \simeq 1/137$ .

*Signature* Throughout the book, we will always use the signature  $(+, -, -, -)$  for the metric, so that the Minkowski metric takes the form  $ds^2 = dt^2 - dx^2 - dy^2 - dz^2$ .