Preface

The main subject of this book is an up-to-date and in-depth survey of the theory of normal frames and coordinates in differential geometry. The existing results, as well as new ones obtained lately by the author, on the theme are presented.

The text is so organized that it can serve equally well as a reference manual, introduction to and review of the current research on the topic. Correspondingly, the possible audience ranges from graduate and post-graduate students to scientists working in differential geometry and theoretical/mathematical physics. This is reflected in the bibliography which consists mainly of standard (text)books and journal articles.

The present monograph is the first attempt for collecting the known facts concerting normal frames and coordinates into a single publication. For that reason, the considerations and most of the proofs are given in details.

Conventionally local coordinates or frames, which can be holonomic or not, are called *normal* if in them the coefficients of a linear connection vanish on some subset, usually a submanifold, of a differentiable manifold. Until recently the existence of normal frames was known (proved) only for symmetric linear connections on submanifolds of a manifold. Now the problems concerning normal frames for derivations of the tensor algebra over a differentiable manifold are well investigate; in particular they completely cover the exploration of normal frames for arbitrary linear connections on a manifold. These rigorous results are important in connection with some physical applications. They may be applied for rigorous analysis of the equivalence principle. This results in two general conclusions: the (strong) equivalence principle (in its 'conventional' formulations) is a provable theorem and the normal frames are the mathematical realization of the physical concept of 'inertial' frames. The normal frames find other important physical application in the bundle formulation of quantum mechanics. It turns out that in a normal frame the bundle Heisenberg and Schrödinger pictures of motion coincide.

Applying some freedom of language, we can state the general physical idea: the normal frames are the most suitable ones for describing free objects and events, i.e., such that on them do not act any forces. Regardless of the different realizations of that idea in general relativity and its generalizations, quantum mechanics, gauge theories etc., there is an underlying mathematical background for the general description of such situations: the existence (or non-existence) of normal frames in vector bundles. This observation fixes to a great extend the mathematical tools required for the description of some fundamental physical theories.

In the book, formally, may be distinguished three parts: The first one includes Chapters I–III and deals with a variety of mathematical problems concerning normal frames and coordinates on differentiable manifolds. The second part consists of Chapters IV and V and investigates normal frames (and possibly coordinates) in vector bundles and differentiable bundles, respectively. The last part, involving the text after Chapter V, contains inquiry material.

The requisite mathematical language required for the description of normal frames is spread over the initial sections of the chapters. In particular, Sections I.2– I.4, III.2, IV.9, IV.2, IV.14.1 and V.2–V.5 can be collected into an introductory chapter under the title "Mathematical preliminaries"[1] but this is not done by pedagogical reasons.[2] The normal coordinates and frames, in the case of linear connections on a manifold, are initially introduced in Chapter I. It contains our basic preliminary material and a review of the Riemannian coordinates. Chapter II is devoted to the existence, uniqueness, construction and other related problems concerning normal frames and coordinates in manifolds endowed with linear connection. It presents, in historical order, a detailed review of the existing literature as well as generalization of a number of results, e.g., for connections with torsion. Further, in Chapter III, problems connected with the existence, uniqueness, holonomicity etc. of normal frames for arbitrary derivations of the tensor algebra over a manifold are investigated. Next (Chapter IV), the same range of problems is explored for normal frames for linear transports in vector bundles. This material covers completely the special case of normal frames for linear connections in vector bundles or on a differentiable manifold. The main aim of Chapter V is the exploration of normal frames (and coordinates, if any) for general connections on differentiable fibre bundles which, in particular, can be vector ones.

The general approach of the book is essentially coordinate-dependent or basis-dependent. This is due to its basic subject: frames, bases or coordinates with some special properties. However, if possible and suitable, the coordinate-free notation and methods are not neglected.

The basic mathematical prerequisites vary from chapter to chapter but generically they include the grounds of vector (linear) spaces, differentiable manifolds, vector bundles, connection theory, and a firm belief in the existence and uniqueness theorems of ordinary differential equations. Some of the corresponding concepts and results are reproduced in our text but the acquaintance with adequate literature is required. Appropriate references are given in the Introductions to the chapters and directly in the main text.

[1] As (practically) any 'preliminary' knowledge requires for its understanding some other 'preliminary' to it knowledge, in the corresponding sections are cited a number of works containing this second kind of mathematical 'luggage'.

[2] The material is so organized, that the required concepts and results appear in the logical order in which they are necessary for some particular purpose(s).

The material is so organized that a successive chapter generalizes the preceding one(s) and refers to it (them).

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Any suggestions and comments are welcome. The author's postal address is

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