Chapter 2

MULTIPLE-INPUT MULTIPLE-OUTPUT SYSTEMS WITH SPACE-TIME CODES

2.1 Introduction

Digital communication using Multiple-Input Multiple-Output (MIMO) systems is one of the most significant technical breakthroughs in modern communication. MIMO systems are simply defined as the systems containing multiple transmitter antennas and multiple receiver antennas. Communication theories show that MIMO systems can provide a potentially very high capacity that, in many cases, grows approximately linear with the number of antennas. Recently, MIMO systems have already been implemented in wireless communication systems, especially in wireless LANs (Local Area Networks) [Griffith, 2004], [Group, 2003], [Jones et al., 2003]. Different structures of MIMO systems have also been proposed by industrial organizations in the Third Generation Partnership Project (3GPP) standardizations, including the structures proposed in [Electronics, 2004], [Ericsson, 2004], [Nokia, 2004], [Samsung and SNU, 2004]. The core idea under the MIMO systems is the ability to turn multi-path propagation, which is typically an obstacle in conventional wireless communication, into a benefit for users.

The main feature of MIMO systems is *space-time* processing. Space-Time Codes (STCs) are the codes designed for the use in MIMO systems. In STCs, signals are coded in both temporal and spatial domains. Among different types of STCs, *orthogonal* Space-Time Block Codes (STBCs) possess a number of advantages over other types of STCs (as mentioned in details later in this chapter) and are considered in this book.

In addition, the combination of STBCs and closed loop transmission diversity techniques using feedback loops has been investigated in the literature. When
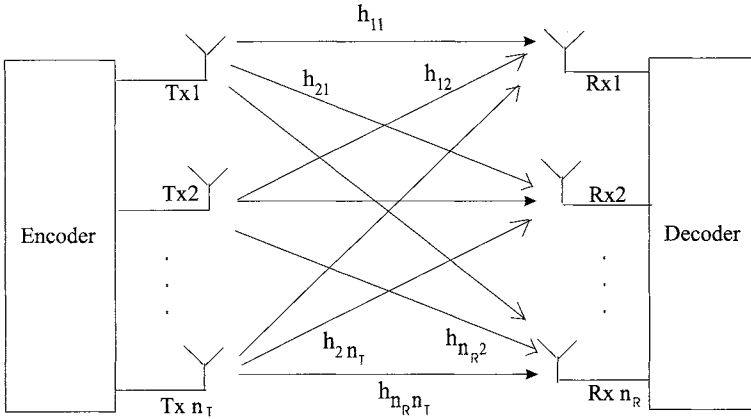
Figure 2.1. The diagram of MIMO systems.

applied, this combination improves significantly the performance of wireless systems. Several such transmission diversity techniques have been derived in the literature, such as space-time coded systems with beamforming [Nokia, 2002], antenna weighting [Electronics, 2002] or transmitter diversity antenna selection [Katz et al., 2001].

In Sections 2.2, 2.3 and 2.4 of this chapter, three main topics are respectively mentioned including:

- MIMO systems from capacity perspectives;

- Space-Time Block Codes;

- Typical transmission diversity techniques

These topics are very important for readers to have the basic knowledge related to the issues mentioned in this book. Conclusions and research problems addressed in this book are mentioned in Section 2.5.

2.2 Multiple-Input Multiple-Output Wireless Communications

2.2.1 MIMO System Model

We consider a single user MIMO system comprising n_T transmitter antennas (n_T Tx antennas) and n_R receiver antennas (n_R Rx antennas). In particular, a complex baseband system described in discrete-time domain is of interest throughout the book. The block diagram of the MIMO system is presented

in Fig. 2.1. During *each Symbol Time Slot* (STS), the transmitted signals are presented as an $n_T \times 1$ column vector **x**, whose entry x_i, for $i = 1, \ldots, n_T$, is the transmitted signal at the ith Tx antenna during the considered STS.

We consider here an additive Gaussian channel (with or without Rayleigh fading) for which the optimal distribution of the transmitted signals in **x** is also Gaussian, i.e., the transmitted signals x_i, for $i = 1, \ldots, n_T$, are zero-mean, identically independently distributed (i.i.d.) complex random variables. The covariance matrix of **x** is

$$\mathbf{R}_{XX} = E\{\mathbf{x}\mathbf{x}^H\}$$

where $E\{.\}$ denotes the expectation, and $(.)^H$ denotes the Hermitian transposition operation. The total power of transmitted signals (during each STS) is constrained to P, regardless of the number of transmitter antennas n_T. It implies that

$$P = tr(\mathbf{R}_{XX})$$

where $tr(.)$ denotes the trace operation of the argument matrix.

In all following sections, we assume that channel coefficients (or transmission coefficients) are *perfectly known at the receiver*, but they *may or may not be known* at the transmitter. The scenario where channel coefficients are *unknown at both transmitter and receiver* is mentioned in [Marzetta and Hochwald, 1999]. Readers may refer to [Marzetta and Hochwald, 1999] for more details.

In the case where channel coefficients are unknown at the transmitter (but known at the receiver), we assume that the transmitted power at each Tx antenna is the same and equal to

$$P_{tj} = \frac{P}{n_T}$$

for $j = 1, \ldots, n_T$. In the case where the channel coefficients are known at the transmitter, the transmitted power is unequally assigned to the Tx antennas following the *water-filling* rule (see Appendix 1.1 in [Vucetic and Yuan, 2003]). We will mention this case in more details later in this chapter.

The channel is presented by an $n_R \times n_T$ complex matrix **H**, whose elements h_{ij} are the channel coefficients between the jth Tx antenna ($j = 1, \ldots, n_T$) and the ith Rx antenna ($i = 1, \ldots, n_R$). Channel coefficients h_{ij} are assumed to be zero-mean, i.i.d. complex Gaussian random variables with a distribution $\mathcal{CN}(0, 1)$.

Noise at the receiver is presented by an $n_R \times 1$ column vector **n** whose elements are zero-mean, i.i.d. complex Gaussian random variables with identical variances (power) σ^2.

If we denote **r** to be the column vector of signals received at Rx antennas during each STS, then the transmission model is presented as

$$\mathbf{r} = \mathbf{Hx} + \mathbf{n}$$

If we assume that the average total power P_r received by each Rx antenna (regardless of noises) is equal to the average total transmitted power P from n_T Tx antennas, the Signal-to-Noise Ratio (SNR) at each Rx antenna is then

$$\rho = \frac{P_r}{\sigma^2} = \frac{P}{\sigma^2}$$

To guarantee the assumption that $P_r = P$, *for a channel with fixed channel coefficients* and with the *equal* transmitted power per Tx antenna P/n_T (i.e., in the case where channel coefficients are *known at the receiver*, but *unknown at the transmitter*), we must have the following constraint:

$$\sum_{j=1}^{n_T} |h_{ij}|^2 = n_T \tag{2.1}$$

for $i = 1, \ldots, n_R$. For a channel with *random* channel coefficients and with *equal* transmitted power per Tx antenna, the formula (2.1) is calculated with the expected value.

The *system capacity* $C(bits/s)$ is defined as the maximum possible transmission rate such that the error probability is arbitrarily small. In this book, we also consider the normalized capacity $C/W(bits/s/Hz)$, which is the system capacity C normalized to the channel bandwidth W.

2.2.2 Capacity of Additive White Gaussian Noise Channels with Fixed Channel Coefficients

In this section, at first, we derive the most general formula to calculate the channel capacity for both cases where channel coefficients are known as well as unknown at the transmitter. Based on this general formula, we will then derive the formulas for channel capacity in some particular cases.

The most general formula for calculating channel capacity in the case where channel coefficients are *either* known *or* unknown at the transmitter is the Shannon capacity formula (see Eq. (1.19) in [Vucetic and Yuan, 2003]):

$$C = W \sum_{i=1}^{r} \log_2 \left(1 + \frac{P_{ri}}{\sigma^2} \right) \tag{2.2}$$

where W is the bandwidth of each sub-channel, r is the rank of the channel coefficient matrix $\mathbf{H}$ (r is equal to the number of non-zero eigenvalues of $\mathbf{H}^H\mathbf{H}$), P_{ri} is the received power at each Rx antenna from the ith sub-channel, for $i = 1, \ldots, r$, during the considered symbol time slot. The term *"sub-channel"* is defined here as that mentioned in Section 1.3 of [Vucetic and Yuan, 2003]. Readers may refer to that section for more details. The rank r is at most equal to $m = \min(n_T, n_R)$.

2.2.2.1   Unknown Channel Coefficients at the Transmitter

In this case, as mentioned earlier, the transmitted power per Tx antenna is assumed to be identical and equal to $P_{tj} = P/n_T$. Let $\mathbf{Q}$ be the Wishart matrix defined as

$$\mathbf{Q} = \begin{cases} \mathbf{HH}^H & \text{if } n_R < n_T \\ \mathbf{H}^H\mathbf{H} & \text{if } n_R \geq n_T \end{cases}$$

From Eq. (2.2), it has been proved (see Eq. (1.30) in [Vucetic and Yuan, 2003]) that the channel capacity for such a scenario is

$$C = W \, \log_2 \left[\det \left(\mathbf{I}_r + \frac{P}{n_T\sigma^2}\mathbf{Q} \right) \right] = W \, \log_2 \left[\det \left(\mathbf{I}_r + \frac{\rho}{n_T}\mathbf{Q} \right) \right] \quad (2.3)$$

where $\det(.)$ denotes the determinant of the argument matrix.

We consider some particular cases as follows:

- Single antenna channel: In this case, we have $r = n_T = n_R = 1$ and $\mathbf{Q} = h = 1$ (see Eq. (2.1)). From (2.3), the channel capacity is calculated as

$$C = W \, \log_2 \left[\det \left(1 + \frac{P}{\sigma^2} \right) \right] \quad (2.4)$$

At SNR $\rho = \frac{P}{\sigma^2} = 20dB$, for instance, the normalized capacity of the single antenna channel is $C/W = 6.658$ bits/s/Hz.

- Receive diversity: In this case, $n_T = 1$, $n_R \geq 2$ and $\mathbf{H} = (h_1 \ldots h_{n_R})^T$, where $(.)^T$ denotes the transposition operation. From (2.3), the channel capacity is calculated as

$$C = W \, \log_2 \left(1 + \frac{P}{\sigma^2} \sum_{i=1}^{n_R} |h_i|^2 \right)$$

Assuming that $|h_i|^2 = 1$, for $i = 1, \ldots, n_R$, then we have

$$C = W \, \log_2 \left(1 + \frac{P n_R}{\sigma^2}\right) \tag{2.5}$$

For $n_R = 2$ and SNR $\rho = 20 dB$, we have $C/W = 7.6511$ bits/s/Hz. We can see that the normalized capacity in this case is larger than that in the case of channels with single Tx and Rx antennas.

- Transmit diversity: In this case, $n_T \geq 2$, $n_R = 1$ and $\mathbf{H} = (h_1 \ldots h_{n_T})$. From (2.3), the channel capacity is calculated as

$$C = W \, \log_2 \left(1 + \frac{P}{n_T \sigma^2} \sum_{i=1}^{n_T} |h_i|^2\right)$$

Assuming that $|h_i|^2 = 1$ for $i = 1, \ldots, n_T$, then we have

$$C = W \, \log_2 \left(1 + \frac{P}{\sigma^2}\right) \tag{2.6}$$

From (2.6), we see that the capacity of the channel where *channel coefficients are fixed* and *unknown at the transmitter* is the same as that of the single antenna channel (see Eq. (2.4)), regardless of the number n_T of Tx antennas. Hence, for $n_T = 2$, $n_R = 1$ and SNR $\rho = 20 dB$, we have $C/W = 6.658$ bits/s/Hz.

2.2.2.2 Known Channel Coefficients at the Transmitter

The channel capacity can be increased if channel coefficients are known at the transmitter. In this case, the transmitted power is assigned unequally to the Tx antennas, according to the "water-filling" rule, i.e., a larger power is assigned to a better sub-channel and visa versa (see Appendix 1.1 in [Vucetic and Yuan, 2003]). The power assigned to the ith sub-channel is

$$P_{ti} = \left(\mu - \frac{\sigma^2}{\lambda_i}\right)^+, \qquad i = 1, \ldots, r$$

where $(a)^+ = \max(a, 0)$, λ_i's are the non-zero eigenvalues of the matrix $\mathbf{H}^H \mathbf{H}$ (also $\mathbf{H}\mathbf{H}^H$) and μ is determined to satisfy the power constraint

$$\sum_{i=1}^{r} P_{ti} = P \tag{2.7}$$

For the ith sub-channel, the received power P_{ri} at the receiver antenna is calculated as (see Eq. (1.20) in [Vucetic and Yuan, 2003]):

$$P_{ri} = \lambda_i P_{ti} = \left(\lambda_i \mu - \sigma^2\right)^+$$

Then, the channel capacity is derived from (2.2) as given below (see Eq. (1.35) in [Vucetic and Yuan, 2003]):

$$C = W \sum_{i=1}^{r} \log_2 \left[1 + \frac{(\lambda_i \mu - \sigma^2)^+}{\sigma^2} \right]$$

We consider the channel with $n_T \geq 2$ and $n_R = 1$ again. We have $r = \min(n_T, n_R) = 1$ and $\mathbf{H} = (h_1 \ldots h_{n_T})$. The power constraint (2.7) becomes

$$\mu - \frac{\sigma^2}{\lambda_1} = P$$

Equivalently, we have $\mu = P + \frac{\sigma^2}{\lambda_1}$, where the only eigenvalue λ_1 of the matrix $\mathbf{H}^H \mathbf{H}$ is $\lambda_1 = \sum_{i=1}^{n_T} |h_i|^2$. Therefore, we have

$$C = W \log_2 \left(1 + \frac{P \sum_{i=1}^{n_T} |h_i|^2}{\sigma^2} \right)$$

Assuming that $|h_i|^2 = 1$ for $i = 1, \ldots, n_T$, then we have

$$C = W \log_2 \left(1 + \frac{P n_T}{\sigma^2} \right)$$

For $n_T = 2$ and $SNR \; \rho = 20dB$, we have $C/W = 7.6511$ bits/s/Hz, which is larger than the channel capacity when the channel coefficients are *unknown* at the transmitter ($C/W = 6.658$ bits/s/Hz).

2.2.3 Capacity of Flat Rayleigh Fading Channels

In this scenario, channel coefficients are random, rather than being fixed as in Gaussian channels. We assume here that channel coefficients are zero-mean, i.i.d. complex Gaussian random variables with variances of 1/2 per dimension (real and imaginary). Hence, each channel coefficient has a Rayleigh distributed magnitude, uniformly distributed phase and the expected value of the squared magnitude equal to one, i.e., $E\{|h_{ij}|^2\} = 1$. We would like to stress that, in all following sections, channel coefficients are assumed to be *known at the receiver*, but *unknown at the transmitter*. Thus, the transmitted power per Tx antenna is assumed to be identical and equal to $P_{tj} = P/n_T$, for $j = 1, \ldots, n_T$.

We will consider the following scenarios, which have been widely mentioned in the literature, such as in [Telatar, 1999], [Vucetic and Yuan, 2003]:

- The channel coefficient matrix $\mathbf{H}$ is random and its entries change randomly during every symbol time slot (STS). This scenario is referred to as the *fast, flat Rayleigh fading channel*.

- **H** is random and its entries change randomly after each block containing a fixed number of STSs. This scenario is referred to as the *block* flat Rayleigh fading channel.

- **H** is random but is selected at the beginning of transmission and its entries keep constant during the whole transmission. This scenario is referred to as the *slow* or *quasi-static*, flat Rayleigh fading channel.

We will consider the first two scenarios simultaneously in the following section.

2.2.3.1   Capacity of MIMO Systems in Fast and Block Rayleigh Fading Channels

It has been derived in the literature that the capacity of MIMO systems in fast and block Rayleigh fading channels is calculated as (see Eq. (1.56) in [Vucetic and Yuan, 2003] or Theorem 1 in [Telatar, 1999])

$$C = E\left\{W \, \log_2\left[\det\left(\mathbf{I}_r + \frac{P}{n_T\sigma^2}\mathbf{Q}\right)\right]\right\} \tag{2.8}$$

where r is the rank of the matrix **H** and the matrix **Q** is the Wishart matrix defined as

$$\mathbf{Q} = \begin{cases} \mathbf{H}\mathbf{H}^H & \text{if } n_R < n_T \\ \mathbf{H}^H\mathbf{H} & \text{if } n_R \geq n_T \end{cases} \tag{2.9}$$

Eq. (2.8) can be evaluated with the aid of Laguerre polynomials (see Section 1.6.1 in [Vucetic and Yuan, 2003] or Theorem 2 in [Telatar, 1999])

$$\begin{aligned} C &= W \int_0^\infty \log_2\left(1 + \frac{P}{n_T\sigma^2}\lambda\right) \sum_{k=0}^{m-1} \frac{k!}{(k+n-m)!} \times \\ &\quad \times \left[L_k^{n-m}(\lambda)\right]^2 \lambda^{n-m}e^{-\lambda}d\lambda \end{aligned} \tag{2.10}$$

where $n = \max(n_T, n_R)$, $m = \min(n_T, n_R)$ and

$$L_k^{n-m}(\lambda) = \frac{1}{k!}e^\lambda\lambda^{m-n}\frac{d^k}{d\lambda^k}(e^{-\lambda}\lambda^{n-m+k})$$

is the Laguerre polynomial of order k.
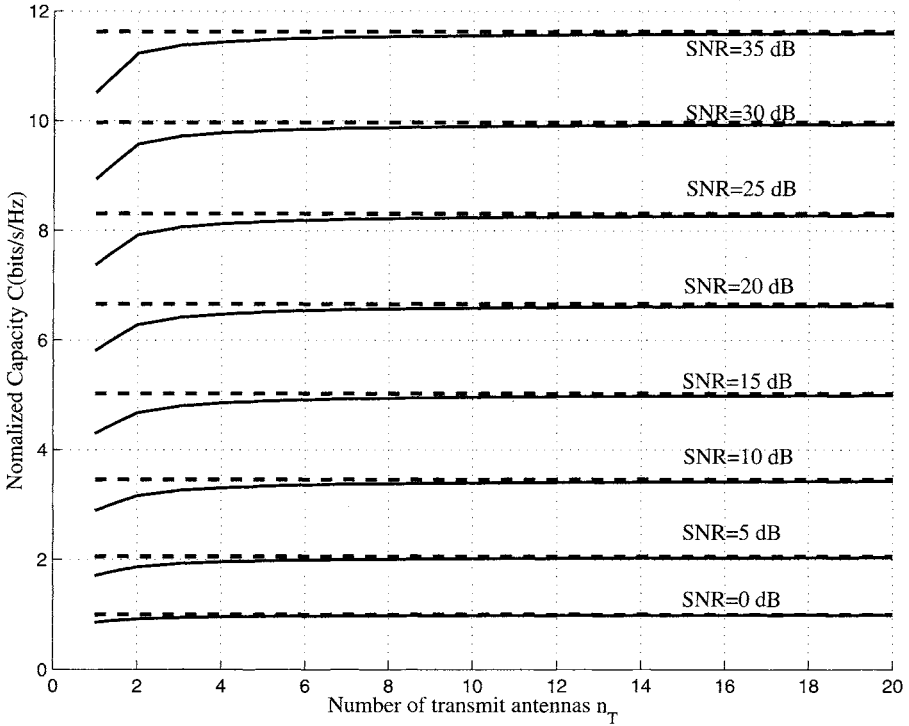
Figure 2.2. Capacity of MIMO systems with one Rx antenna in fast or block flat Rayleigh fading channels.

Furthermore, by increasing m and n, but keeping the ratio $\tau = \frac{n}{m} = const$, we have the following limit (see Eq. (1.61) in [Vucetic and Yuan, 2003] and Eq. (13) in [Telatar, 1999]):

$$\lim_{n \to \infty} \frac{C}{m} = \frac{W}{2\pi} \int_{\nu_1}^{\nu_2} \log_2 \left(1 + \frac{Pm}{n_T \sigma^2} \nu \right) \sqrt{\left(\frac{\nu_2}{\nu} - 1 \right) \left(1 - \frac{\nu_1}{\nu} \right)}$$

where

$$\nu_1 = (\sqrt{\tau} - 1)^2$$

$$\nu_2 = (\sqrt{\tau} + 1)^2$$
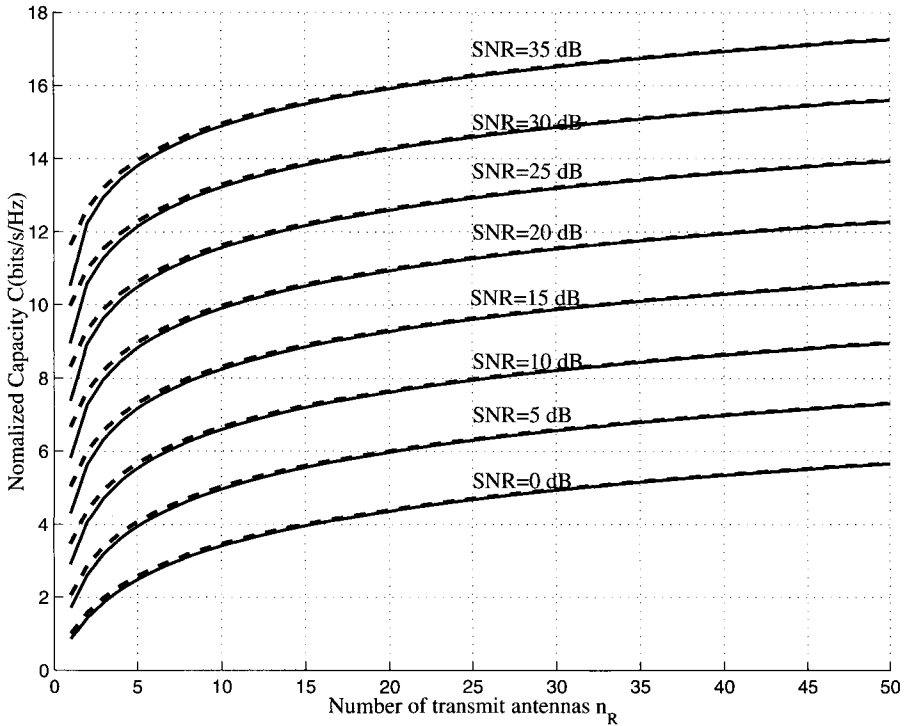
Figure 2.3. Capacity of MIMO systems with one Tx antenna in fast or block flat Rayleigh fading channels.

We consider now three scenarios:

- Transmit diversity: In this case, we have $n_T \geq 2$ and $n_R = 1$. From (2.10), we have

$$C = W \frac{1}{(n_T - 1)!} \int_0^\infty \log_2 \left(1 + \frac{P}{n_T \sigma^2} \lambda\right) \lambda^{n_T - 1} e^{-\lambda} d\lambda \qquad (2.11)$$

When n_T increases, the capacity approaches the asymptotic value

$$\lim_{n_T \to \infty} C = W \log_2 \left(1 + \frac{P}{\sigma^2}\right) \qquad (2.12)$$

We realize that Eq. (2.12) is similar to Eq. (2.6). It means that, when the number of Tx antennas is large, the capacity of the transmit diversity system in *fast or block Rayleigh* fading channels approaches the capacity of the transmit diversity system in AWGN channels where channel coefficients are

Figure 2.4.   Capacity of MIMO systems with $n_T = n_R$ in fast or block flat Rayleigh fading channels.

unknown at the transmitter. The channel capacity of the transmit diversity systems is presented in Fig. 2.2. In this figure, the solid lines present the capacity calculated by (2.11), while the dashed lines present the asymptotic capacity calculated by (2.12). The dashed lines thus also present the capacity of AWGN channels calculated following (2.6).

- Receive diversity: In this case, we have $n_T = 1$ and $n_R \geq 2$. From (2.10), we have

$$C = W \frac{1}{(n_R - 1)!} \int_0^\infty \log_2 \left(1 + \frac{P}{\sigma^2}\lambda\right) \lambda^{n_R - 1} e^{-\lambda} d\lambda \qquad (2.13)$$

When n_R increases, the capacity approaches the asymptotic value

$$\lim_{n_R \to \infty} C = W \log_2 \left(1 + \frac{P n_R}{\sigma^2}\right) \qquad (2.14)$$

We realize that (2.14) is similar to (2.5). It means that, when the number of Rx antennas is large, the capacity of the receive diversity system in *fast or block Rayleigh* fading channels approaches the capacity of the receive diversity system in AWGN channels. Channel capacity of the receiv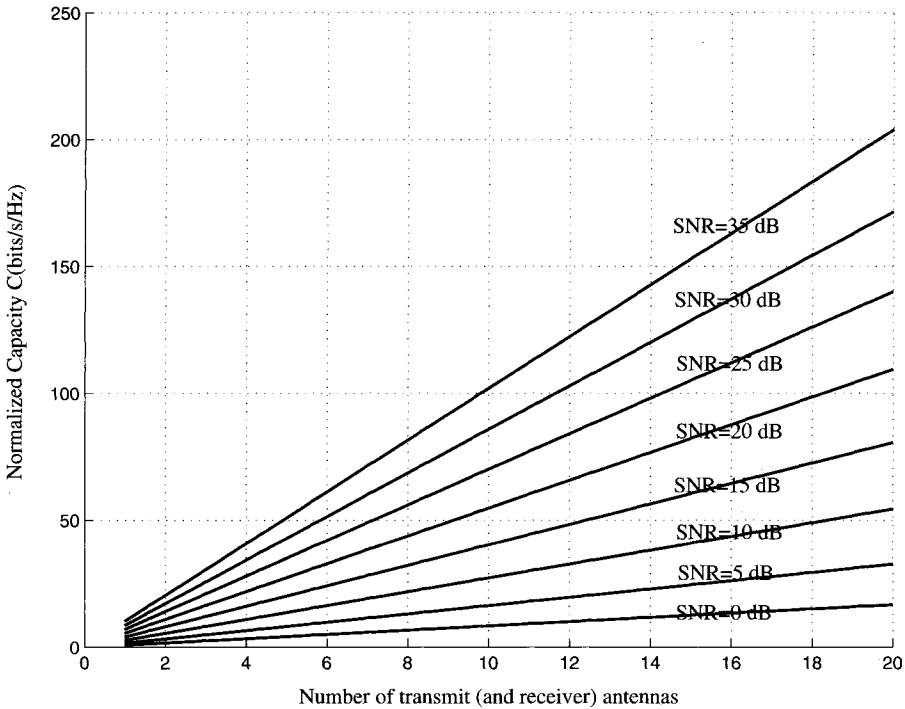e diversity systems is presented in Fig. 2.3. The solid lines present the capacity calculated by (2.13), while the dashed lines present the asymptotic capacity calculated by (2.14). Similarly, the dashed lines also present the capacity of AWGN channels calculated following (2.5).

■ Transmit and receive diversity: We assume further that $n_T = n_R$, and hence, $m = n = n_T = n_R$. Thus, from (2.10), channel capacity is calculated as

$$C = W \int_0^\infty \log_2 \left(1 + \frac{P\lambda}{n_R \sigma^2}\right) \sum_{k=0}^{n_R - 1} [L_k^0(\lambda)]^2 e^{-\lambda} d\lambda \qquad (2.15)$$

where

$$L_k^0(\lambda) = \frac{1}{k!} e^\lambda \frac{d^k}{d\lambda^k}(e^{-\lambda}\lambda^k)$$

With the note that $m = n = n_T = n_R$, the empirical bound of the capacity in (2.15) has the following closed form (see Eq. (1.76) in [Vucetic and Yuan, 2003]):

$$\lim_{n \to \infty} \frac{C}{Wn} \geq \log_2 \left(\frac{P}{\sigma^2}\right) - 1$$

From this formula, it is clear that the capacity almost increases linearly with the number of Tx (and Rx) antennas. The capacity of the channel with $n_T = n_R$ is presented in Fig. 2.4.

2.2.3.2   Capacity of MIMO Systems in Slow Rayleigh Fading Channels

The results mentioned here were originally derived by Foschini and Gans [Foschini and Gans, 1998]. We consider a MIMO system where the channel coefficient matrix **H** is chosen randomly at the start of transmission and it stays constant during the whole transmission. The entries of **H** follow the Rayleigh distribution. Wireless Local Area Networks (LANs) with high data rates and low fade rates are examples of this scenario.

Again, we consider three cases as follows:

- Receive diversity: In this case, we have $n_T = 1$ and $n_R \geq 2$. It has been shown by Eq. (10) in [Foschini and Gans, 1998] or by Eq. (1.78) in [Vucetic and Yuan, 2003] that the channel capacity is calculated as

$$C = W \log_2 \left(1 + \frac{P}{\sigma^2} \chi^2_{2n_R} \right)$$

where $\chi^2_{2n_R}$ is a chi-square random variable with $2n_R$ degrees of freedom.

- Transmit diversity: In this case, we have $n_T \geq 2$ and $n_R = 1$. It has been shown by Eq. (11) in [Foschini and Gans, 1998] or by Eq. (1.79) in [Vucetic and Yuan, 2003] that the channel capacity is calculated as

$$C = W \log_2 \left(1 + \frac{P}{n_T \sigma^2} \chi^2_{2n_T} \right)$$

where $\chi^2_{2n_T}$ is a chi-square random variable with $2n_T$ degrees of freedom.

- Transmit and receive diversity: We further assume that $n = n_T = n_R$ and n is large, then it is shown by Eq. (20) in [Foschini and Gans, 1998] or by Eq. (1.82) in [Vucetic and Yuan, 2003] that the lower bound on the capacity is

$$\frac{C}{Wn} > \left(1 + \frac{\sigma^2}{P} \right) \log_2 \left(1 + \frac{P}{\sigma^2} \right) - \log_2 e + \varepsilon_n \qquad (2.16)$$

where ε_n is a Gaussian random variable with the mean and variance as given below:

$$E\{\varepsilon_n\} = \frac{1}{n} \log_2 \left(1 + \frac{P}{\sigma^2} \right)^{-1/2}$$

$$Var\{\varepsilon_n\} = \left(\frac{1}{n \ln 2} \right)^2 \left[\ln \left(1 + \frac{P}{\sigma^2} \right) - \frac{\frac{P}{\sigma^2}}{1 + \frac{P}{\sigma^2}} \right]$$

From (2.16), we realize that when the number of Tx (and Rx) antennas is large, the channel capacity is linearly proportional to the number of antennas. For SNR $\rho = \frac{P}{\sigma^2} = 20dB$, and $n = n_T = n_R = 8$, the normalized capacity is $C/W \approx 37$ bits/s/Hz.

2.3   Space-Time Block Codes

Communication requires a very high rate with high reliability these days. Two major difficulties to obtain reliable communication via high rate wireless communication systems are bandwidth limitation of communication channels and multipath fading. To surmount these difficulties, multiple antenna systems referred to as MIMO systems, which provide a transmit and/or receive diversity, can be used. As mentioned in Section 2.2 of this book or in [Foschini, 1996], [Foschini and Gans, 1998], [Marzetta and Hochwald, 1999], [Telatar, 1999], MIMO systems can provide a potentially huge capacity gain with the same requirements for power and bandwidth as the single antenna systems. In many cases, the capacity of channels is proved to increase linearly with the lower number among the number of transmitter antennas (Tx antennas) and that of receiver antennas (Rx antennas).

Based on those information theoretical results, various schemes for the transmission of signals via MIMO systems have been proposed, including Bell Lab Layered Space-Time (BLAST) [Foschini, 1996], Space-Time Trellis Codes (STTCs) [Tarokh et al., 1998], Space-Time Block Codes (STBCs) [Alamouti, 1998], [Tarokh et al., 1999a], [Tarokh et al., 1999b] and Unitary Space-Time Codes [Hochwald and Marzetta, 2000] among many others. All designs are targeted to the transmission of signals in MIMO systems to achieve diversity and data rates as high as possible, while bandwidth expansion (if any) must be kept as small as possible.

Particularly, Tarokh et al. [Tarokh et al., 1998] proposed a few Space-Time Trellis Codes (STTCs) for 2–4 Tx antennas, which perform well in slow fading environment and have almost no loss of capacity compared to the channel capacity. However, the complexity of decoders increases exponentially with the size of signal constellations. The authors in [Tarokh et al., 1998] also derived the criteria for designing a good Space-Time Code (STC), including rank criterion and determinant criterion.

Later, Alamouti [Alamouti, 1998] discovered a very simple transmitter diversity technique for two Tx antennas, which provides a full diversity order, has no loss of capacity (if the number of Rx antennas is equal to one), and possesses a simple and fast maximum likelihood (ML) decoding. Instead of being joined, the transmitted signals are decoded separately at decoders due to the orthogonality between the columns (and rows) of the code. The Alamouti code could be considered as the *original* Space-Time Block Code (STBC) - the Space Time Codes (STCs) constructed from orthogonal designs. The discovery of the Alamouti code is presented by a milestone in Fig. 1.1 of Chapter 1.

Although some other designs were proposed for STCs, such as non-orthogonal designs based on number theory [A.-Meraim et al., 2002], [Damen et al., 2002], orthogonal STBCs are currently receiving an intensive attention due to the following reasons:

1 They posses a fast and very simple maximum likelihood decoding [Tarokh et al., 1999a], [Tarokh et al., 1998] due to their orthogonality.

2 They provide a full diversity order for a certain number of Tx antennas. Consequently, these codes have good error probability characteristics [Tarokh et al., 1999b].

Motivated by the Alamouti code, Tarokh et al. [Tarokh et al., 1999a], [Tarokh et al., 1999b] proposed STBCs for various numbers of Tx antennas. Based on signal constellations, the authors classified STBCs into two classes, namely, STBCs for real signals and STBCs for complex signals. Real STBCs can be used in the case of Pulse Amplitude Modulation (PAM) while complex STBCs are used for Phase Shift Keying (PSK) or Quadrature Amplitude Modulation (QAM) constellations. Both of them may, or may not, include linear processing (LP) at transmitters. The term "linear processing" will be explained in more details later.

Real STBCs have been well examined. There is a systematic method to construct real STBCs with the maximum rate $R_{max} = 1$ for up to 8 Tx antennas based on Huwitz-Radon theory. The background on the Huwitz-Radon theory can be found in [Ganesan and Stoica, 2000], [Geramita and Seberry, 1979]. These codes also provide a maximum Signal-to-Noise Ratio (SNR) at receivers [Ganesan and Stoica, 2001].

Unlike real STBCs, complex STBCs have not been well known in the literature yet, while they are more practical. Complex STBCs have recently received much attention. For these reasons, in this book, we will mainly focus on complex orthogonal STBCs (CO STBCs).

In this section, we present the basic theories on STBCs which are mainly based on the contributions of Alamouti [Alamouti, 1998], Liang [Liang, 2003], and Tarokh et al. [Tarokh et al., 1999a], [Tarokh et al., 1999b], [Tarokh et al., 1999c], [Tarokh et al., 1998].

A STBC representing a relationship between original transmitted symbols s_i and their replicas artificially created by the transmitter for transmission over the channel with multiple Tx antennas is defined by a $p \times n_T$ matrix, where p is the number of symbol time slots (STSs) for transmission of one code block and n_T is the number of Tx antennas. Generally, the elements of the matrix are linear
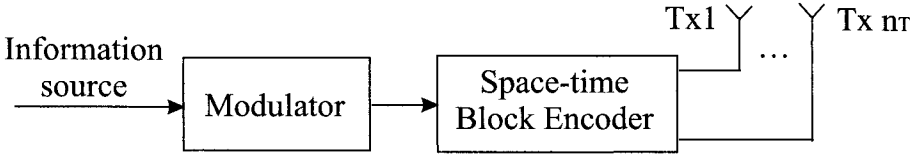
Tx1 Y   Y Tx n$_T$

Information
source   Space-time   ...
$\longrightarrow$ Modulator $\longrightarrow$ Block Encoder

Figure 2.5. Space-time block encoding.

combinations of k input symbols s_i (i=1...k), which represent the information-bearing binary bits to be transmitted. Assuming that the signal constellation consisting of 2^b points is considered. Then, b binary bits are represented by a symbol s_i. Therefore, a block of $k \times b$ binary bits is entered into the encoder at a time and the encoding process is carried out in both space and time, hence, the code block at the output of the encoder is referred to as a Space-Time Block Codes (STBC). The space-time encoding process is presented by Fig. 2.5.

This block of the transmitted symbols is mathematically represented by a matrix $\mathbf{X}$ of size $p \times n_T$ as follows:

$$\begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1n_T} \\ g_{21} & g_{22} & \cdots & g_{2n_T} \\ \cdots & \cdots & \cdots & \cdots \\ g_{p1} & g_{p2} & \cdots & g_{pn_T} \end{bmatrix}$$

where g_{jl}, for $j = 1, \ldots, p$ and $l = 1, \ldots, n_T$, represents a linear combination of the symbols s_i. The entries $g_{j1}, \ldots, g_{jn_T}$ are transmitted simultaneously from n_T Tx antennas at the jth time slot. Clearly, the length p of the STBC represents the delay for transmission (and for decoding as well). Since k symbols are transmitted during p STSs, the code rate of the STBC is defined by the ratio

$$R = k/p$$

The code rate is related to the spectral efficiency of the STBC as given below:

$$\eta = \frac{r_b}{B} = \frac{r_s b}{B} \qquad bits/s/Hz$$

where B denotes the bandwidth, r_b and r_s denote the bit rate and the symbol rate, respectively. The bandwidth B is calculated as

$$B = \frac{r_s}{R} = \frac{r_s p}{k} \qquad Hz \qquad (2.17)$$

Therefore, the spectral efficiency of the STBC is

$$\eta = \frac{r_b}{B} = bR = \frac{bk}{p} \qquad bits/s/Hz$$

Clearly, for the same signal constellation, a higher code rate results in a more efficient STBC. In other words, the code rate R represents the spectral efficiency of the STBC.

From Eq. (2.17), we also realize that if R is smaller than one, then there exists a bandwidth expansion to transmit STBCs, compared to, for instance, the transmission without using STBCs where each symbol is transmitted during each STS via one Tx antenna. It will be analyzed in more details later that the Alamouti STBC [Alamouti, 1998] achieves the full rate, i.e. $R = 1$, and consequently, does not expand transmission bandwidth.

It should be emphasized that, in the literature, STBCs usually refer to *orthogonal STBCs*. Other classes of STCs include *quasi-orthogonal* STBCs [Jafarkhani, 2001], [Su and Xia, 2002a], [Su and Xia, 2002b], Space-Time Trellis Codes (STTCs) [Tarokh et al., 1998], Layered Space-Time Codes (LSTs) [Foschini, 1996], [Vucetic and Yuan, 2003], or Linear Space-Time Codes [Hassibi and Hochwald, 2001], [Hassibi and Hochwald, 2002]. Orthogonal STBCs are of great interest as decoding processes of those codes only involve the linear processing at the receiver thanks to the orthogonality between their columns, and consequently, it is very simple to decode them. Because of this historical reason, the term "STBCs" should be understood as *orthogonal* STBCs throughout this book.

The main milestones in examining STBCs so far are presented in Fig. 1.1 in Chapter 1, which is explained in more details by the following sections.

2.3.1   Real Orthogonal Designs

DEFINITION 2.3.1 *A generalized, real orthogonal design (also called generalized, real, orthogonal STBC) $\mathcal{G}$ is defined as a $p \times n$ matrix whose nonzero entries are the indeterminates x_1, x_2, ..., x_k over the real number field $\mathbb{R}$ or their negatives $-x_1$, $-x_2$, ..., $-x_k$ such that*

$$\mathcal{G}^T \mathcal{G} = \mathcal{D} \tag{2.18}$$

where $\mathcal{G}^T$ denotes the transpose of the matrix $\mathcal{G}$, while $\mathcal{D}$ is a diagonal matrix of size $n \times n$ with diagonal entries $\mathcal{D}_{ii}$, for $i = 1, 2, \ldots, n$, of the form $(l_{i1}x_1^2 + l_{i2}x_2^2 + \cdots + l_{ik}x_k^2)$ and coefficients $l_{i1}, \ldots, l_{ik}$ are strictly positive, real numbers. The rate of $\mathcal{G}$ is $R = k/p$. The matrix $\mathcal{G}$ is said to be a $[p, n, k]$

real orthogonal design. If $p = n$, $\mathcal{G}$ is called square, real orthogonal design. If the coefficients $l_{i1}, \ldots, l_{ik}$ satisfy $l_{i1} = \cdots = l_{ik} = 1$, for $i = 1, 2, \ldots, n$, then $\mathcal{G}$ is called real, orthogonal STBC without Linear Processing (LP). Otherwise, $\mathcal{G}$ is called real, orthogonal STBC with LP.

The condition on the positive definiteness of the coefficients l_{ij}s is to guarantee that the STBC $\mathcal{G}$ provides a full diversity order.

Proof. We prove that $\mathcal{G}$ satisfies the rank criterion for designing STBCs, i.e. [Tarokh et al., 1998]:

$$
\det \left[\mathcal{G}(\acute{x}_1 - x_1, \acute{x}_2 - x_2, \ldots, \acute{x}_k - x_k)^T \times \right. \\
\left. \times \mathcal{G}(\acute{x}_1 - x_1, \acute{x}_2 - x_2, \ldots, \acute{x}_k - x_k) \right] \neq 0
$$

for any distinct pair of codewords $\acute{x} \triangleq (\acute{x}_1, \ldots, \acute{x}_k)$ and $x \triangleq (x_1, \ldots, x_k)$. Note that $\mathcal{G}(\acute{x}_1 - x_1, \acute{x}_2 - x_2, \ldots, \acute{x}_k - x_k)$ is the matrix $\mathcal{G}(x_1, x_2, \ldots, x_k)$ when we replace the set $(x_1, x_2, \ldots, x_k)$ by the set $(\acute{x}_1 - x_1, \acute{x}_2 - x_2, \ldots, \acute{x}_k - x_k)$. From (2.18), we have

$$
\det \left(\mathcal{G}^T \mathcal{G} \right) = \prod_{i=1}^{n} \left[\sum_{j=1}^{k} l_{ij} x_j^2 \right]
$$

Therefore

$$
\det \left[\mathcal{G}(\acute{x}_1 - x_1, \acute{x}_2 - x_2, \ldots, \acute{x}_k - x_k)^T \times \right. \\
\left. \times \mathcal{G}(\acute{x}_1 - x_1, \acute{x}_2 - x_2, \ldots, \acute{x}_k - x_k) \right] \\
= \prod_{i=1}^{n} \left[\sum_{j=1}^{k} l_{ij} (\acute{x}_j - x_j)^2 \right]
$$

Evidently, if l_{ij}s are positive definite, for any distinct pair of codewords $\acute{x}$ and x, we always have

$$
\det \left[\mathcal{G}(\acute{x}_1 - x_1, \acute{x}_2 - x_2, \ldots, \acute{x}_k - x_k)^T \times \right. \\
\times \left. \mathcal{G}(\acute{x}_1 - x_1, \acute{x}_2 - x_2, \ldots, \acute{x}_k - x_k) \right] > 0
$$

It means that, $\mathcal{G}$ satisfies the design criterion concerning to rank. Hence, the STBC $\mathcal{G}$ provides a full diversity order.

Real STBCs can be used for any Pulse Amplitude Modulation (PAM) or may even be used for Binary Phase Shift Keying (BPSK) and, generally, they are only used for those modulation techniques. In addition, real STBCs have been *well examined* in the literature.

It is more convenient to express $p = 2^{4c+d}.q$ where $0 \leq c, 0 \leq d < 4$, q is odd, and c, d, q are in the natural number field $\mathbb{N}$. Furthermore, let $A(R, n) = p_{min}$ be the minimum number p such that there exists a generalized orthogonal design of size $p \times n$ providing a code rate of at least $R = k/p$ (if there is no such design, then $A(R, n) = \infty$). It is proven in [Tarokh et al., 1999b] (pp.1461) that:

1 For any R, $A(R, n) < \infty$. It means that for any code rate R (and, therefore, including $R = 1$), generalized, real orthogonal designs always exist.

2 Full-rate, real STBCs exist for any number of n.

3 For any generalized orthogonal design of full rate ($R = 1$), one has $A(1, n) = p_{min} = \min(2^{4c+d})$, where the minimization is taken from the set

$$\{c, d| \ c, d \in \mathbb{N}, \ 0 \leq c, \ 0 \leq d < 4, \ \rho(p) \overset{\Delta}{=} 8c + 2^d \geq n\}$$

In the above equation, $\rho(p)$ is referred to as the Hurwitz-Radon number, which will be defined in more details later. The minimum length of any *full-rate*, real orthogonal STBCs, i.e., $A(1, n)$, can be determined via the condition on the Hurwitz-Radon number $\rho(p)$, which is stated by the following theorem (see Proposition 4 in [Liang, 2003]).

THEOREM 2.3.2 *For any number of n, a rate-1, $[p, n, p]$ real orthogonal STBC exists if and only if (iff) the Hurwitz-Radon number $\rho(p) \geq n$, i.e.:*

$$\rho(p) \overset{\Delta}{=} 8c + 2^d \geq n$$

The value of $A(1, n)$ presents the minimum length of the full-rate STBCs and also presents the minimum requirement on memory to achieve the full rate. Some values of $A(1, n)$ are presented in Table 2.1.

If we consider n as the number of Tx antennas and p as the delay of STBCs, then real orthogonal designs providing a full rate exist for any number n of Tx antennas. In addition, the optimal delay for real STBCs for 1, 2, 4 and 8 Tx antennas are 1, 2, 4 and 8 symbol time slots, respectively. In other words, the *square*, real STBCs only exist for 1, 2, 4 and 8 Tx antennas.

In general, for full-rate, real STBCs, $A(1, n)$ (or p_{min}) has the following form [Tirkkonen and Hottinen, 2002]:

$$p_{min} = A(1, n) = 16^{\lfloor (n-1)/8 \rfloor} 2^{\lceil log_2(1+(n-1)mod8) \rceil}$$

Table 2.1.   Some typical values of p_{min} (or $A(1,n)$) for the *full-rate, real* STBCs.

n	c	d	p_{min}	n	c	d	p_{min}
1	0	0	1	9	1	0	16
2	0	1	2	10	1	1	32
3	0	2	4	11	1	2	64
4	0	2	4	12	1	2	64
5	0	3	8	13	1	3	128
6	0	3	8	14	1	3	128
7	0	3	8	15	1	3	128
8	0	3	8	16	1	3	128

where $\lfloor a \rfloor$ is the smallest integer which is equal to or greater than a, and $\lceil a \rceil$ is the biggest integer which is equal to or smaller than a.

Some examples of full-rate, generalized orthogonal designs are given below:
For $n=1$, $p=1$:

$$\mathcal{G}_1 = (x_1) \tag{2.19}$$

For $n=2$, $p=2$:

$$\mathcal{G}_2 = \begin{bmatrix} x_1 & x_2 \\ -x_2 & x_1 \end{bmatrix} \tag{2.20}$$

For $n=3$, $p=4$:

$$\mathcal{G}_3^T = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 \\ x_2 & x_1 & x_4 & -x_3 \\ x_3 & -x_4 & x_1 & x_2 \end{bmatrix}$$

For $n=4$, $p=4$:

$$\mathcal{G}_4 = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \end{bmatrix} \tag{2.21}$$

For $n=5$, $p=8$:

$$
\mathcal{G}_5^T = \begin{bmatrix}
x_1 & -x_2 & -x_3 & -x_4 & -x_5 & -x_6 & -x_7 & -x_8 \\
x_2 & x_1 & -x_4 & x_3 & -x_6 & x_5 & x_8 & -x_7 \\
x_3 & x_4 & x_1 & -x_2 & -x_7 & -x_8 & x_5 & x_6 \\
x_4 & -x_3 & x_2 & x_1 & -x_8 & x_7 & -x_6 & x_5 \\
x_5 & x_6 & x_7 & x_8 & x_1 & -x_2 & -x_3 & -x_4
\end{bmatrix}
$$

For $n=8$, $p=8$:

$$
\mathcal{G}_8 = \begin{bmatrix}
x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\
-x_2 & x_1 & x_4 & -x_3 & x_6 & -x_5 & -x_8 & x_7 \\
-x_3 & -x_4 & x_1 & x_2 & x_7 & x_8 & -x_5 & -x_6 \\
-x_4 & x_3 & -x_2 & x_1 & x_8 & -x_7 & x_6 & -x_5 \\
-x_5 & -x_6 & -x_7 & -x_8 & x_1 & x_2 & x_3 & x_4 \\
-x_6 & x_5 & -x_8 & x_7 & -x_2 & x_1 & -x_4 & x_3 \\
-x_7 & x_8 & x_5 & -x_6 & -x_3 & x_4 & x_1 & -x_2 \\
-x_8 & -x_7 & x_6 & x_5 & -x_4 & -x_3 & x_2 & x_1
\end{bmatrix} \tag{2.22}
$$

2.3.1.1 Maximum Rates of *Square, Real* Orthogonal Designs

As mentioned before, there always exist the real orthogonal designs $[p, n, k]$ with a full rate for any number of n. However, a question which could be raised is what the maximum number k of variables, say k_{max}, in the *square, real* orthogonal designs is. To answer this question, we need to define the Hurwitz-Radon number $\rho(n)$ which has been intensively mentioned in the literature, such as [Geramita and Seberry, 1979], [Liang, 2003], [Tarokh et al., 1999b].

DEFINITION 2.3.3 *If $n = 2^a(2b + 1)$ and $a = 4c + d$, where a, b, c and d are integers with $0 \le d < 4$, then the Hurwitz-Radon number is defined as $\rho(n)=8c + 2^d$. $\rho(n)$ can be rewritten as follows:*

$$
\rho(n) = \rho(2^a(2b+1)) = \begin{cases}
2a + 1 & \text{if } a = 0 \ (mod) \ 4 \\
2a & \text{if } a = 1 \ (mod) \ 4 \\
2a & \text{if } a = 2 \ (mod) \ 4 \\
2a + 2 & \text{if } a = 3 \ (mod) \ 4
\end{cases} \tag{2.23}
$$

where mod denotes the modulo operation.

Table 2.2.   The maximum number of variables and the maximum rates of *square, real* STBCs.

n	k_{max}	R_{Rmax}	n	k_{max}	R_{Rmax}
1	1	1	9	1	1/9
2	2	1	10	2	1/5
3	1	1/3	11	1	1/11
4	4	1	12	4	1/3
5	1	1/5	13	1	1/13
6	2	1/3	14	2	1/7
7	1	1/7	15	1	1/15
8	8	1	16	9	9/16

The Hurwitz-Radon numbers have the following properties:

$$\rho(2^a(2b+1)) = \rho(2^a)$$
$$\rho(16n) = \rho(n) + 8$$
$$\rho(2^a) < \rho(2^{a+1})$$

It has been proved that the maximum number k_{max} of variables in a square, real orthogonal design is equal to the Hurwitz-Radon number $\rho(n)$, i.e., [Adams et al., 1965], [Geramita and Seberry, 1979], [Liang, 2003]:

$$k_{max} = \rho(n) = \rho(2^a) = \rho(2^a(2b+1))$$
$$= 8c + 2^d \qquad (2.24)$$

From Eq. (2.24), we have the following corollary:

COROLLARY 2.3.4 *The maximum rate of square, real orthogonal designs for any number of Tx antennas $n = 2^{4c+d}(2b+1)$, denoted by R_{Rmax}, is calculated as*

$$R_{Rmax} = \frac{k_{max}}{n} = \frac{\rho(n)}{n} = \frac{8c + 2^d}{n} \qquad (2.25)$$

In (2.25), the subscript "R" implies that the formula is applied to *real* designs. The maximum numbers of variables and the maximum rates of *square, real* STBCs for some typical values of n are given in Table 2.2. From Eq. (2.25) and Table 2.2, some following important notes are derived:

1 If n is odd, the maximum code rate of square, real orthogonal designs is $R_{Rmax} = 1/n$. Therefore, for $n > 3$, the code rate is very small, and hence,

there might be no point to examine *square, real* STBCs for the odd number of Tx antennas which is greater than 3.

2 *Square, real* STBCs with a full rate exist only for $n = 1$, 2, 4 and 8 Tx antennas. Note that the full-rate, real STBCs exist for any number n of Tx antennas, but for $n \neq 1$, 2, 4 or 8, those full-rate, real STBCs *must be non-square*.

2.3.1.2 Constructions of Maximum Rate, Square, Real STBCs

In this section, three methods for the construction of maximum rate, square, real STBCs are presented, including 1) the Adams-Lax-Phillips construction from octonions; 2) the Adams-Lax-Phillips construction from quaternions; and 3) the Geramita-Pullman construction. A good summary of these constructions can be found in Liang's paper [Liang, 2003].

1 *The Adams-Lax-Phillips construction from octonions [Adams et al., 1965],* *[Adams et al., 1966]:* We already have 4 square, real STBCs for $n = 2^a$ with $a = 0$, 1, 2 and 3, i.e., Eq. (2.19) for $a = 0$; Eq. (2.20) for $a = 1$; Eq. (2.21) for $a = 2$ and Eq. (2.22) for $a = 3$. Denote

$$\mathcal{G}_{2^a} = \mathcal{G}_{2^a}(x_1, \ldots, x_{\rho(n)}) \tag{2.26}$$

which has $\rho(n) = \rho(2^a)$ real variables and is of size $n \times n$.

From $\mathcal{G}_{2^a}$, one can construct a square, real STBC of order 2^{a+1} comprising $\rho(n) + 1$ real variables as follows:

$$\begin{aligned} \mathcal{G}_{2^{a+1}} &= \mathcal{G}_{2^{a+1}}(x_1, \ldots, x_{\rho(n)+1}) \\ &= \begin{bmatrix} x_{\rho(n)+1}\mathbf{I}_n & \mathcal{G}_{2^a} \\ \mathcal{G}_{2^a}^T & -x_{\rho(n)+1}\mathbf{I}_n \end{bmatrix} \end{aligned} \tag{2.27}$$

where $\mathbf{I}_n$ denotes the identity matrix of order n.

From $\mathcal{G}_{2^{a+1}}$, one can construct a square, real STBC of order $16n = 2^{a+4}$ comprising $\rho(n) + 8$ real variables as follows:

$$\begin{aligned} \mathcal{G}_{2^{a+4}} &= \mathcal{G}_{2^{a+4}}(x_1, \ldots, x_{\rho(n)+8}) \\ &= \mathcal{G}_{2^{a+1}} \bigotimes \mathbf{I}_8 + \mathbf{I}_{2n} \bigotimes \mathcal{G}_8(0, \ x_{\rho(n)+2}, \ldots, \ x_{\rho(n)+8}) \end{aligned} \tag{2.28}$$

where $\mathcal{G}_8$ is defined by (2.22) and $\otimes$ denotes the Kronecker product.

From (2.26) – (2.28), the transition from order $n = 2^a$ to order $16n$ can be expressed as

$$\mathcal{G}_{2^{a+4}} = \begin{bmatrix} \mathbf{I}_n \otimes \mathcal{G}_8(y_1, y_2, \ldots, y_8) & \mathcal{G}_{2^a} \otimes \mathbf{I}_8 \\ \mathcal{G}_{2^a}^T \otimes \mathbf{I}_8 & \mathbf{I}_n \otimes \mathcal{G}_8(-y_1, y_2, \ldots, y_8) \end{bmatrix}$$

where $y_i = x_{\rho(n)+i}$ for $i = 1, \ldots, 8$.

In order to construct the square, real STBC of order $n = 2^a(2b + 1)$, from the square, real STBC of order $n = 2^a$, say $\mathcal{G}_{2^a}$, we just need to perform the Kronecker product between $\mathbf{I}_{(2b+1)}$ and $\mathcal{G}_{2^a}$.

The above constructions can be used to generate a maximum rate, square, real orthogonal design of order n with $\rho(n)$ real variables for any number of $n \in \mathbb{N}$ from the initial four orthogonal designs of orders $n = 1, 2, 4, 8$.

2 *The Adams-Lax-Phillips construction from quaternions [Adams et al., 1965]*: Another construction method using quaternions was introduced in [Adams et al., 1965]. This construction is another method for the transition from orthogonal designs of order $n = 2^a$ to order $16n$.

Consider a square, real STBC of order $n = 2^a$

$$\mathcal{G}_{2^a} = \mathcal{G}_{2^a}(x_1, \ldots, x_{\rho(n)})$$

which has $\rho(n) = \rho(2^a)$ real variables. From $\mathcal{G}_{2^a}$, we can construct a square, real STBC of order $16n = 2^{a+4}$ with $\rho(2^a) + 8$ real variables x_i for $i = 1, \ldots, \rho(2^a) + 8$, denoted by

$$\mathcal{G}_{2^{a+4}} = \mathcal{G}_{2^{a+4}}(x_1, \ldots, x_{\rho(2^a)+8})$$

as given below

$$\mathcal{G}_{2^{a+4}} = \begin{bmatrix} \mathbf{I}_n \otimes \mathbf{L}_4(y_1, y_2, y_3, y_4) & \mathbf{O}_{4n} \\ \mathbf{O}_{4n} & \mathbf{I}_n \otimes \mathbf{L}_4(y_1, y_2, y_3, y_4) \\ \mathbf{I}_n \otimes \mathbf{R}_4(y_5, -y_6, -y_7, -y_8) & \mathcal{G}_{2^a} \otimes \mathbf{I}_4 \\ \mathcal{G}_{2^a}^T \otimes \mathbf{I}_4 & \mathbf{I}_n \otimes \mathbf{R}_4(-y_5, -y_6, -y_7, -y_8) \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{I}_n \otimes \mathbf{R}_4(y_5, y_6, y_7, y_8) & \mathcal{G}_{2^a} \otimes \mathbf{I}_4 \\ \mathcal{G}_{2^a}^T \otimes \mathbf{I}_4 & \mathbf{I}_n \otimes \mathbf{R}_4(-y_5, y_6, y_7, y_8) \\ \mathbf{I}_n \otimes \mathbf{L}_4(-y_1, y_2, y_3, y_4) & \mathbf{O}_{4n} \\ \mathbf{O}_{4n} & \mathbf{I}_n \otimes \mathbf{L}_4(-y_1, y_2, y_3, y_4) \end{bmatrix}$$

where $\mathbf{L}_4$ and $\mathbf{R}_4$ are defined as

$$\mathbf{L}_4(y_1, y_2, y_3, y_4) = \begin{bmatrix} y_1 & -y_2 & -y_3 & -y_4 \\ y_2 & y_1 & y_4 & -y_3 \\ y_3 & -y_4 & y_1 & y_2 \\ y_4 & y_3 & -y_2 & y_1 \end{bmatrix}$$

$$\mathbf{R}_4(y_5, y_6, y_7, y_8) = \begin{bmatrix} y_5 & -y_6 & -y_7 & -y_8 \\ y_6 & y_5 & -y_8 & y_7 \\ y_7 & y_8 & y_5 & -y_6 \\ y_8 & -y_7 & y_6 & y_5 \end{bmatrix}$$

while $\mathbf{O}_n$ is the zero matrix of order n and $y_i = x_{\rho(n)+i}$ for $i = 1, \ldots, 8$.

3 *The Geramita-Pullman construction [Geramita and Pullman, 1974]*: Assume that we are given a square, real STBC of order $n = 2^a$ with $\rho(n) = \rho(2^a)$ real variables, denoted by

$$\begin{aligned} \mathcal{G}_{2^a} &= \mathcal{G}_{2^a}(x_1, \ldots, x_{\rho(n)}) \\ &= x_1 \mathbf{I}_n + x_2 \mathbf{M}_2 + \cdots + x_{\rho(n)} \mathbf{M}_{\rho(n)} \end{aligned}$$

where $\mathbf{M}_i$ are the real coefficient matrices of order n for $i = 1, \ldots, \rho(n)$ ($\mathbf{M}_1 = \mathbf{I}_n$). The transition from order n to order $16n$ is presented as

$$\begin{aligned} & \mathcal{G}_{2^{a+4}}(x_1, \ldots, x_{\rho(2^a)+8}) \\ =\ & x_1 \mathbf{I}_{16n} \\ & + \sum_{i=2}^{\rho(n)} \begin{bmatrix} \mathbf{O}_{8n} & \mathbf{I}_8 \otimes x_i \mathbf{M}_i \\ \mathbf{I}_8 \otimes x_i \mathbf{M}_i & \mathbf{O}_{8n} \end{bmatrix} + \begin{bmatrix} \mathbf{O}_{8n} & x_{\rho(n)+1} \mathbf{I}_{8n} \\ -x_{\rho(n)+1} \mathbf{I}_{8n} & \mathbf{O}_{8n} \end{bmatrix} \\ & + \begin{bmatrix} \mathcal{G}_8(0, x_{\rho(n)+2}, \ldots, x_{\rho(n)+8}) \otimes \mathbf{I}_n & \mathbf{O}_{8n} \\ \mathbf{O}_{8n} & \mathcal{G}_8(0, -x_{\rho(n)+2}, \ldots, -x_{\rho(n)+8}) \otimes \mathbf{I}_n \end{bmatrix} \end{aligned}$$

In order to construct the square, real STBC of order $n = 2^a(2b + 1)$, from the square, real STBC $\mathcal{G}_{2^a}$ of order $n = 2^a$, we just need to perform the Kronecker product between $\mathbf{I}_{(2b+1)}$ and $\mathcal{G}_{2^a}$.

2.3.1.3 Constructions of Full-Rate, Non-Square, Real STBCs

The method to construct full-rate, non-square, real STBCs for any number n of Tx antennas is mentioned by Liang [Liang, 2003]. Readers may refer to the Part IV in [Liang, 2003] for more details. Again, it is noted that the *full-rate, square*, real STBCs exist only for n=1, 2, 4 and 8. For other values of n, the full-rate, real STBCs *must be non-square*.

2.3.2   Complex Orthogonal Designs - CODs

DEFINITION 2.3.5 *A generalized Complex, Orthogonal Design COD (also complex, orthogonal STBC)* $\mathbf{Z} = \mathbf{X} + i\mathbf{Y}$ *is defined as a* $p \times n$ *matrix whose nonzero entries are the indeterminates* $\pm s_1$, $\pm s_2$, ..., $\pm s_k$, *their conjugates* $\pm s_1^*$, $\pm s_2^*$, ..., $\pm s_k^*$ *or their products with* $i = \sqrt{-1}$ *over the complex number field* $\mathbb{C}$, *such that*

$$\mathbf{Z}^H \mathbf{Z} = \left(\sum_{j=1}^{k} |s_j|^2 \right) \mathbf{I}_{n \times n} \tag{2.29}$$

where $\mathbf{Z}^H$ *denotes the Hermitian transpose of* $\mathbf{Z}$ *and* $\mathbf{I}_{n \times n}$ *is the identity matrix of order* n. *The rate of* $\mathbf{Z}$ *is* $R = k/p$. *The matrix* $\mathbf{Z}$ *is said to be a* $[p, n, k]$ *complex orthogonal design. If* $p{=}n$, $\mathbf{Z}$ *is called square, complex orthogonal design (square COD). Otherwise,* $\mathbf{Z}$ *is called non-square (or rectangular) COD.*

It is important to note that the Complex Orthogonal Designs (CODs) defined as above are actually the so-called Generalized Complex Orthogonal Designs (GCODs) *without Linear Processing (LP)*, which were first used by Tarokh et al. [Tarokh et al., 1999b]. Meanwhile, the term "Complex Orthogonal Designs" (CODs) in [Tarokh et al., 1999b] was used to present a class of complex orthogonal designs which are *square*, of size $n \times n$ and comprise n indeterminates. However, nowadays, the term "CODs" is usually referred to as the one defined by the Definition 2.3.5.

DEFINITION 2.3.6 *A generalized Complex Orthogonal Design COD with Linear Processing - LP (also complex linear processing orthogonal STBC)* $\mathbf{Z} = \mathbf{X} + i\mathbf{Y}$ *is defined as a* $p \times n$ *matrix whose nonzero entries are the indeterminates* $\pm s_1$, $\pm s_2$, ..., $\pm s_k$, *their conjugates* $\pm s_1^*$, $\pm s_2^*$, ..., $\pm s_k^*$ *or their products with* $i = \sqrt{-1}$ *over the complex number field* $\mathbb{C}$, *such that*

$$\mathbf{Z}^H \mathbf{Z} = \mathcal{D}$$

where $\mathcal{D}$ *is a diagonal matrix of size* $n{\times}n$ *with diagonal entries* $\mathcal{D}_{jj}$, *for* $j = 1, 2, \ldots, n$, *of the form* $(l_{j1}|x_1|^2 + l_{j2}|x_2|^2 + \cdots + l_{jk}|x_k|^2)$. *The coefficients* $l_{j1}, \ldots, l_{jk}$ *are strictly positive, real numbers.* $\mathbf{Z}^H$ *denotes the Hermitian transpose of* $\mathbf{Z}$. *The rate of* $\mathbf{Z}$ *is* $R = k/p$. *The matrix* $\mathbf{Z}$ *is said to be a* $[p, n, k]$ *COD with LP. If* $p{=}n$, $\mathbf{Z}$ *is called square, complex orthogonal design (square COD) with LP. Otherwise,* $\mathbf{Z}$ *is called non-square (or rectangular) COD with LP.*

It is important to note that CODs with LP defined as above were originally called Generalized Complex Orthogonal Designs (GCODs) *with LP* by Tarokh et al. [Tarokh et al., 1999b].

By the similar analysis as mentioned in Section 2.3.1, it is easy to realize that the matrix $\mathbf{Z}$ defined in the Definitions 2.3.5 and 2.3.6 provides a full diversity order as $\det(\mathbf{Z}^H\mathbf{Z}) > 0$ for any distinct pair of codewords $\mathbf{s} \overset{\Delta}{=} (s_1, \dots, s_k)$ and $\acute{\mathbf{s}} \overset{\Delta}{=} (\acute{s_1}, \dots, \acute{s_k})$, provided that the coefficients l_{jl}, for $j = 1, \dots, n$ and $l = 1, \dots, k$ are definitely positive.

As opposed to real STBCs, CODs (or Complex Orthogonal STBCs - CO STBCs) can be used for PSK and QAM modulations. Also, they have not been well known unlike real STBCs.

2.3.2.1   Maximum Rate of *Square, Complex Orthogonal* STBCs

It has been proved that the maximum number k_{max} of variables in a *square* COD of size $n = 2^a(2b+1)$ is [Adams et al., 1965], [Liang, 2003], [Tirkkonen and Hottinen, 2002]:

$$k_{max} = (a+1) \tag{2.30}$$

From Eq. (2.30), we have the following corollary:

COROLLARY 2.3.7 *The maximum rate of square, complex orthogonal designs for any number* $n = 2^a(2b+1)$ *of transmitter antennas is*

$$R_{Cmax} = \frac{k_{max}}{n} = \frac{a+1}{2^a(2b+1)} \tag{2.31}$$

In (2.31), the subscript "C" implies that the formula is applied to complex designs.

The maximum number of variables, k_{max}, and the maximum rates R_{Cmax} of *square, complex, orthogonal* STBCs (CO STBCs) for some typical values of n are given in Table 2.3.

REMARK 2.3.8 *It is important to clarify that, according to Liang's paper [Liang, 2003], the maximum achievable rate for CO STBCs of orders* $n = 2m - 1$ *or* $n = 2m$ *is (see Eq. (130) in [Liang, 2003])*

$$R_{max} = (m+1)/2m \tag{2.32}$$

However, note that this maximum rate is only achievable for non-square constructions, except for the special case when $m = 1$, *i.e. when* $n = 1$ *or* $n = 2$.

Table 2.3. The maximum number of variables and the maximum rates of *square* CO STBCs.

n	k_{max}	R_{Cmax}	n	k_{max}	R_{Cmax}
1	1	1	9	1	1/9
2	2	1	10	2	1/5
3	1	1/3	11	1	1/11
4	3	3/4	12	4	1/3
5	1	1/5	13	1	1/13
6	2	1/3	14	2	1/7
7	1	1/7	15	1	1/15
8	4	1/2	16	5	5/16

For square constructions of orders $n = 2^a(2b + 1)$, the maximum achievable rate must be calculated by Eq. (2.31). When $m = 1$, (2.31) and (2.32) provide the same results. Readers should refer to Corollary 2 and Section II D in [Liang, 2003], or Section IV in [Tirkkonen and Hottinen, 2002] for more details.

Particularly, for $n = 8$, i.e., $m = 4$, $a = 3$ and $b = 0$, the maximum achievable rate of non-square CO STBCs following (2.32) is 5/8, while the maximum achievable rate of square CO STBCs according to (2.31) is only 1/2. In Liang's paper, the author made an unclear statement in the abstract that the achievable maximum rate for $n = 2m - 1$ and $n = 2m$ is $(m + 1)/2m$, but did not state if this maximum rate is achievable by square or non-square constructions, which may lead to some confusion.

From Eq. (2.31) and Table 2.3, we have some following important notes:

1 If n is odd, the maximum code rate of *square* CO STBCs is $R_{Cmax} = 1/n$. For $n > 3$, the code rate is very small, and hence, there might be no point to examine *square* CO STBC for the odd number of Tx antennas which is greater than 3.

2 *Square*, CO STBCs with a full rate exist only for $n = 1$ and $n = 2$ Tx antennas.

2.3.2.2   Maximum Possible Rate of *Non-Square* CO STBCs

If n can be presented as $n = 2m - 1$ or $n = 2m$, where m is any nonzero natural number, then the maximum possible rate of *non-square* CO STBCs is given by the following theorem, which is equivalent to Theorems 5 and 6 in [Liang, 2003]:

THEOREM 2.3.9 *For any given number of transmitter antennas $n = 2m - 1$ and $n = 2m$ with $m \in \mathbb{N}$ and $m \neq 0$, the rate of non-square, complex orthogonal STBCs satisfies*

$$R \leq \frac{m+1}{2m} \qquad (2.33)$$

From the above theorem, we can draw the following corollary:

COROLLARY 2.3.10 *For any number of Tx antennas $n = 2m - 1$ and $n = 2m$ with $m \in \mathbb{N}$ and $m \neq 0$, there exist certain values of the two parameters k (the number of variables in the code) and p (the length of the code) for which the $[p, n, k]$ non-square CO STBC achieves the maximum rate:*

$$R_{max} = \frac{m+1}{2m} \qquad (2.34)$$

EXAMPLE 2.3.1 *For $n = 8$, the maximum rate of non-square CO STBCs is $R_{max}=5/8$. As clarified earlier in Remark 2.3.8, this maximum rate does not contradict with the maximum rate $R_{Cmax}=1/2$ mentioned in Table 2.3 for the same n. It is because the maximum rate $R_{max}=5/8$ is for non-square orthogonal designs, while the maximum rate $R_{Cmax}=1/2$ is for square orthogonal designs. The maximum rate-5/8, non-square CO STBC exists for $n = 8$, $p = 112$, $k = 70$, i.e., [112,8,70] CO STBC. This construction can be found in Appendix E in [Liang, 2003].*

Some typical values of the maximum possible rates R_{max}, the maximum number of variables k_{max}, and the optimal delay p_{min} of *non-square* CO STBCs are given in Table 2.4, Table 2.5 and Table 2.6, respectively. These tables are derived from Table I and Table II in [Liang, 2003].

From these tables, some following important notes are derived:

1 If n is odd, as opposed to the case of *square* CO STBCs (see Table 2.3), the maximum code rate of *non-square* CO STBCs are still potentially high. For instance, when $n=15$, the maximum code rate in the former case is $R_{Cmax}=1/15$ while that in the later case is $R_{max}=9/16$.

2 Although having higher maximum rates, *non-square* CO STBCs require a very large decoding delay (also memory length) for $n > 6$ (see Table 2.6).

Table 2.4. The maximum possible rates of *non-square* CO STBCs.

n	R_{max}	n	R_{max}
1	1	9	3/5
2	1	10	3/5
3	3/4	11	7/12
4	3/4	12	7/12
5	2/3	13	4/7
6	2/3	14	4/7
7	5/8	15	9/16
8	5/8	16	9/16

Table 2.5. The maximum number of variables of *non-square* CO STBCs.

n	k_{max}	n	k_{max}
1	1	9	126
2	2	10	252
3	3	11	462
4	6	12	924
5	10	13	1716
6	20	14	3432
7	35	15	6435
8	70	16	12870

Table 2.6. The optimal delay of *non-square* CO STBCs with the maximum possible rates.

n	p_{min}	n	p_{min}
1	1	9	210
2	2	10	420
3	4	11	792
4	8	12	1584
5	15	13	3003
6	30	14	6006
7	56	15	11440
8	112	16	22880

2.3.2.3   Constructions of Maximum Rate, *Square* CO STBCs

In this section, three methods for the construction of *maximum rate, square CO STBCs*, namely 1) the Jozefiak construction; 2) the Adams-Lax-Phillips construction and 3) the Wolfe construction, are presented. It is noted that there exist other construction methods. For instance, from Clifford representation

theory, Tirkkonen et al. [Tirkkonen and Hottinen, 2002] proposed another method to construct *maximum rate, square* CO STBCs. Readers may refer to Section 3.1 in [Su and Xia, 2003] or Eq. (20) in [Tirkkonen and Hottinen, 2002] for more details. Other construction methods follow from the Amicable Orthogonal Designs (AODs), which are fully explained in [Geramita and Seberry, 1979] and will be mentioned in more details later in Chapter 3.

1 *The Jozefiak construction [Jozefiak, 1976]*: Assume that n is even and that $n = 2^a$. Further, assume that we already have an $n \times n$ square, complex orthogonal design $\mathbf{Z}_{2^a} = \mathbf{Z}_{2^a}(s_1, \ldots, s_{a+1})$. Then the square complex STBC of size $2n \times 2n$ is constructed as

$$
\begin{aligned}
\mathbf{Z}_{2^{a+1}} &= \mathbf{Z}_{2^{a+1}}(s_1, \ldots, s_{a+1}, s_{a+2}) \\
&= \begin{bmatrix} \mathbf{Z}_{2^a} & s_{a+2}\mathbf{I}_n \\ -s_{a+2}^*\mathbf{I}_n & \mathbf{Z}_{2^a}^H \end{bmatrix}
\end{aligned}
\tag{2.35}
$$

To construct the *maximum rate, square* CO STBC of size $n = 2^a(2b+1)$, we need to perform the Kronecker product between the identity matrix $\mathbf{I}_{(2b+1)}$ and the maximum rate, square, complex STBC of size $n = 2^a$, i.e. $\mathbf{Z}_{2^a(2b+1)} = \mathbf{I}_{(2b+1)} \bigotimes \mathbf{Z}_{2^a}$.

EXAMPLE 2.3.2
For n=1, i.e., a=0, we have $\mathbf{Z}_1 = (s_1)$.
For n=2, i.e., a=1, from (2.35), we have

$$
\mathbf{Z}_2 = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & \cdot \; s_1^* \end{bmatrix}
$$

For n=3, i.e., a=0, b=1, we have

$$
\mathbf{Z}_3 = \mathbf{I}_3 \bigotimes \mathbf{Z}_1 = \begin{bmatrix} s_1 & 0 & 0 \\ 0 & s_1 & 0 \\ 0 & 0 & s_1 \end{bmatrix}
$$

For n=4, i.e., a=2, we have

$$
\mathbf{Z}_4 = \left[\begin{array}{cc|cc} s_1 & s_2 & s_3 & 0 \\ -s_2^* & s_1^* & 0 & s_3 \\ \hline -s_3^* & 0 & s_1^* & -s_2 \\ 0 & -s_3^* & s_2^* & s_1 \end{array} \right]
$$

For n=8, i.e., a=3, we have

$$\mathbf{Z}_8 \;=\; \left[\begin{array}{c|c} \mathbf{Z}_4 & s_4\mathbf{I}_4 \\ \hline -s_4^*\mathbf{I}_4 & \mathbf{Z}_4^H \end{array}\right]$$

$$= \left[\begin{array}{cccc|cccc} s_1 & s_2 & s_3 & 0 & s_4 & 0 & 0 & 0 \\ -s_2^* & s_1^* & 0 & s_3 & 0 & s_4 & 0 & 0 \\ -s_3^* & 0 & s_1^* & -s_2 & 0 & 0 & s_4 & 0 \\ 0 & -s_3^* & s_2^* & s_1 & 0 & 0 & 0 & s_4 \\ \hline -s_4^* & 0 & 0 & 0 & s_1^* & -s_2 & -s_3 & 0 \\ 0 & -s_4^* & 0 & 0 & s_2^* & s_1 & 0 & -s_3 \\ 0 & 0 & -s_4^* & 0 & s_3^* & 0 & s_1 & s_2 \\ 0 & 0 & 0 & -s_4^* & 0 & s_3^* & -s_2^* & s_1^* \end{array}\right]$$

$$(2.36)$$

2 *The Adams-Lax-Phillips construction [Adams et al., 1965]*: This construction is similar to the Jozefiak construction except that the recursive formula (2.35) is replaced by

$$\begin{aligned} \mathbf{Z}_{2^{a+1}} &= \mathbf{Z}_{2^{a+1}}(s_1, \ldots, s_{a+1}, s_{a+2}) \\ &= \left[\begin{array}{cc} s_{a+2}\mathbf{I}_n & \mathbf{Z}_1 \\ \mathbf{Z}_1^H & -s_{a+2}^*\mathbf{I}_n \end{array}\right] \end{aligned}$$

3 *The Wolfe construction [Wolfe, 1976]*: This construction is similar to the Jozefiak construction and the Adams-Lax-Phillips construction except that the recursive formula (2.35) is replaced by

$$\begin{aligned} \mathbf{Z}_{2^{a+1}} &= \mathbf{Z}_{2^{a+1}}(s_1, \ldots, s_{a+1}, s_{a+2}) \\ &= \left[\begin{array}{cc} s_{a+2}\mathbf{I}_n & \mathbf{Z}_1 \\ -\mathbf{Z}_1^H & s_{a+2}^*\mathbf{I}_n \end{array}\right] \end{aligned}$$

2.3.2.4   Constructions of High-Rate, *Non-Square* CO STBCs

The method to construct high-rate, *non-square* CO STBCs for any number n of Tx antennas is mentioned by Liang [Liang, 2003]. Readers may refer to the Part V in [Liang, 2003] for more details. It should be emphasized that, by this method, maximum rate, *square* CO STBCs can also be constructed. However, the constructing procedures are more complicated than the methods mentioned in Section 2.3.2.3. Therefore, this construction method should only be used for constructing high-rate, *non-square* CO STBCs.

2.3.2.5 On the Maximum Rates of CODs with Linear Processing

CODs with Linear Processing (LP) are defined by Definition 2.3.6. It has been shown by the Corollary 4.1 in [Su and Xia, 2003] that, if the coefficients satisfy $l_{j1} = \cdots = l_{jk}$, for $j = 1, \ldots, n$, then relaxing the definition of CODs in order to allow LP at the transmitter fails to provide a higher code rate for *square* CODs (also *square* CO STBCs). In other words, the maximum code rate of *square* CO STBCs with LP is also calculated by Eq. (2.31).

However, it is *unknown* whether the maximum rate of *square* CO STBCs *with* LP is different to that of *square* CO STBCs *without* LP if the condition $l_{j1} = \cdots = l_{jk}$, for $j = 1, \ldots, n$, is *not* satisfied.

As opposed to *square* CO STBCs with LP, the maximum rate of *non-square* CO STBCs with LP has been *unknown* yet.

Since full-rate, *real* STBCs exist for any number of Tx antennas (see the second note of Section 2.3.1.1), we are always able to construct rate-1/2, *non-square* CO STBCs with LP for any number of Tx antennas. The construction method is mentioned by Part E in [Tarokh et al., 1999b] and by Eq. (4.4) in [Su and Xia, 2003]. Furthermore, in [Su and Xia, 2003] and [Su and Xia, 2001], W. Su et al. derived the two *non-square* CO STBCs *with LP* for 5 and 6 Tx antennas with code rates of 7/11 and 18/30 (or 0.6), respectively. At the time of their discovery, those codes were the *known maximum-rate, non-square* CO STBCs *with LP* for 5 and 6 Tx antennas.

However, up to date, these code rates were outdated already. By using the construction method proposed in [Liang, 2003] with the observation that the orthogonality is not affected by multiplying each column of a COD with a coefficient l ($l \neq 0$ and $l \neq 1$), we realize that the achievable code rate of *non-square* CO STBCs *with LP* is also calculated by Eq. (2.34). This is the *known maximum rate* of *non-square* CO STBCs *with LP* so far.

For instance, by multiplying the first column of each of the constructions (100) and (101) in [Liang, 2003] for 5 and 6 Tx antennas, i.e. the [15,5,10] and [30,6,20] CO STBCs, respectively, with a coefficient $l = 2$, we have the *non-square* CO STBCs *with LP* $\mathbf{Z}$ satisfying

$$\mathbf{Z}^H\mathbf{Z} = diag(4\sum_{i=1}^{10}|s_i|^2, \sum_{i=1}^{10}|s_i|^2, \sum_{i=1}^{10}|s_i|^2, \sum_{i=1}^{10}|s_i|^2, \sum_{i=1}^{10}|s_i|^2)$$

$$\mathbf{Z}^H\mathbf{Z} = diag(4\sum_{i=1}^{20}|s_i|^2, \sum_{i=1}^{20}|s_i|^2, \sum_{i=1}^{20}|s_i|^2, \sum_{i=1}^{20}|s_i|^2, \sum_{i=1}^{20}|s_i|^2, \sum_{i=1}^{20}|s_i|^2)$$

where *diag* denotes a diagonal matrix with the elements on the main diagonal provided in the brackets. The code rates of these *non-square* CO STBCs *with LP* are 2/3 and 2/3, respectively, which are higher than the code rates 7/11 and 18/30 in [Su and Xia, 2003], [Su and Xia, 2001].

Therefore, we can conclude that the *known maximum rate* of *non-square* CO STBCs *with LP* to date is the same as that of *non-square* CO STBCs *without LP* and is calculated by Eq. (2.34) for any number of Tx antennas. However, it is *unknown* whether or not the *true* maximum rate of *non-square* CO STBCs *with LP* is higher than that of *non-square* CO STBCs *without LP*. This requires to be further examined.

2.3.2.6   Capacity of the Channel Using STBCs

We consider here a CO STBC of size $p \times n_T$ comprising k complex variables, where n_T denotes the number of Tx antennas and p denotes the length of the CO STBC. The code rate is thus $R = k/p$. We assume that channels comprise n_R Rx antennas and they are *block*, flat Rayleigh fading channels. Channel coefficients are assumed to be *known at the receiver*, but *unknown at the transmitter*. Therefore, the transmitted power per Tx antenna is assumed to be the same and equal to P/n_T (see Section 2.2.3 for more details).

According to S. Sandhu et al. [Sandhu and Paulraj, 2000], although using STBCs can provide a high rate and a full transmission diversity order for a given number of Tx antennas with relatively simple detection, it incurs a loss of capacity in comparison with the true capacity of the channel.

The loss of capacity is a function of the rank r of the channel coefficient matrix $\mathbf{H}$ ($r \leq \min(n_T, n_R)$) and the code rate $R = k/p$. As a result, the loss of capacity depends on the number of Tx antennas n_T and Rx antennas n_R, and the code rate $R = k/p$ (see Eq. (6) in [Sandhu and Paulraj, 2000]):

$$\Delta C = W\left(1 - \frac{k}{p}\right) E\left\{\log_2\left(1 + \frac{P}{n_T\sigma^2}\parallel \mathbf{H} \parallel_F^2\right)\right\}$$
$$+ \log_2\left(1 + \frac{S}{1 + \frac{P}{n_T\sigma^2}\parallel \mathbf{H} \parallel_F^2}\right) \qquad (2.37)$$

where

$$\parallel \mathbf{H} \parallel_F^2 = \sum_{i=1}^{r} \lambda_i$$

$$S = \left(\frac{P}{n_T\sigma^2}\right)^2 \sum_{i1<i2}^{r} \lambda_{i1}\lambda_{i2} + \left(\frac{P}{n_T\sigma^2}\right)^3 \sum_{i1<i2<i3}^{r} \lambda_{i1}\lambda_{i2}\lambda_{i3} +$$
$$+ \ldots + \left(\frac{P}{n_T\sigma^2}\right)^r \prod_{i=1}^{r} \lambda_i$$

while λ_is are nonzero eigenvalues of $\mathbf{H}^H\mathbf{H}$ (or $\mathbf{H}\mathbf{H}^H$), W is the bandwidth of each sub-channel, $\frac{P}{\sigma^2}$ is the SNR at each Rx antenna. Readers may refer to Section 2.2 for more details on these notations.

From (2.37), some results of interest are derived as follows [Sandhu and Paulraj, 2000]:

- Any channel with a *full rate* CO STBC (i.e. $k = p$), such as the Alamouti code [Alamouti, 1998], used over a channel with *one Rx antenna* ($r = n_R = 1$) is always optimal with respect to capacity since $\Delta C = 0$.

- CO STBCs of *any rates*, including the full-rate codes, such as the Alamouti code, used over i.i.d. Rayleigh channel with *multiple Rx antennas*, i.e. $n_R \geq 2$, always incur a loss in capacity because ΔC is non-zero.

Therefore, although using STBCs in MIMO systems can provide a potentially high capacity and a full transmission diversity order for a given number of Tx antennas with a relatively simple decoding algorithm, there exists a loss of capacity compared to the true maximum capacity of the MIMO systems.

2.3.2.7 Examples on the Capacity of Channels and of CO STBCs

We consider a MIMO system with n_T Tx and n_R Rx antennas. The transmission model is thus

$$\mathbf{Y} = \sqrt{\frac{\rho}{n_T}}\mathbf{X}\mathbf{H} + \mathbf{N} \tag{2.38}$$

where $\mathbf{X} \in \mathbb{C}^{p \times n_T}$ denotes the matrix of complex transmitted signals during p symbol time slots (or p channel uses), $\mathbf{H} \in \mathbb{C}^{n_T \times n_R}$ denotes the channel coefficient matrix, and $\mathbf{N} \in \mathbb{C}^{p \times n_R}$ denotes the additive noise matrix. We assume that the entries of $\mathbf{H}$ and $\mathbf{X}$ are i.i.d. complex Gaussian random variables with the distribution $\mathcal{CN}(0, 1)$, which implies that

$$
\begin{aligned}
E\{tr(\mathbf{H^H H})\} &= n_T n_R \\
E\{tr(\mathbf{X^H X})\} &= n_T p
\end{aligned}
\tag{2.39}
$$

The coefficient $\sqrt{\frac{\rho}{n_T}}$ in Eq. (2.38) ensures that ρ is the SNR at each Rx antenna during each symbol time slot (channel use), independently of the number n_T of Tx antennas.

We assume further that the channel is a flat, block Rayleigh fading channel whose channel coefficients are perfectly known at the receiver, but not at the transmitter. Let $C(\rho, n_T, n_R)$ be the *true* capacity of the channel between the Tx and Rx antennas, while $C_{STBC}(\rho, n_T, n_R)$ be the capacity of the channel where the STBC is utilized.

Similarly to that mentioned earlier in Eq. (2.8), the channel capacity is calculated as

$$C = E\left\{ W \, \log_2 \left[\det \left(\mathbf{I}_r + \frac{P}{n_T \sigma^2} \mathbf{Q} \right) \right] \right\} \tag{2.40}$$

where r is the rank of the matrix $\mathbf{H}$ and $\mathbf{Q}$ is defined as

$$\mathbf{Q} = \begin{cases} \mathbf{HH}^H & \text{if } n_T < n_R \\ \mathbf{H}^H \mathbf{H} & \text{if } n_T \geq n_R \end{cases} \tag{2.41}$$

Note that the matrix $\mathbf{Q}$ in Eq. (2.41) is defined by a formula which is slightly different to that mentioned in Eq. (2.9) in Section 2.2.3.1. That is because, in Eq. (2.38), $\mathbf{H}$ has size $n_T \times n_R$, rather than having size $n_R \times n_T$ like in Section 2.2.3.1. In the case where $\mathbf{H}$ has size $n_R \times n_T$, we just need to exchange n_T and n_R in Eq. (2.41).

The channel capacity can be easily calculated (see Section 2.2.3.1 for more details). From Eq. (2.10), (2.11), (2.13) and (2.15), the channel capacity for some particular values of n_T, n_R and ρ is shown in Table 2.7.

We may have a question of how well an STBC performs from capacity perspective when comparing the maximum mutual information which can be supported to the true channel capacity. This question has been partially answered in Section 2.3.2.6. In this section, we calculate the maximum mutual information of some known STBCs in Rayleigh fading channels to expose more clearly this issue.

EXAMPLE 2.3.3 *We consider here a system with $n_T = 2$ and $n_R = 1$ using the Alamouti code*

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}$$

Table 2.7. Normalized channel capacity for several values of transmitter and receiver antenna numbers.

n_T	n_R	ρ (dB)	Formula for $\frac{C(\rho, n_T, n_R)}{W}$	$\frac{C(\rho, n_T, n_R)}{W}$
2	1	20	$\int_0^\infty \log_2(1 + \frac{\rho\lambda}{n_T})\lambda e^{-\lambda}d\lambda$	6.2810
2	2	20	$\int_0^\infty \log_2(1 + \frac{\rho\lambda}{n_T})[1 + (1-\lambda)^2]e^{-\lambda}d\lambda$	11.2898
3	1	20	$\int_0^\infty \frac{1}{2!}\log_2(1 + \frac{\rho\lambda}{n_T})\lambda^2 e^{-\lambda}d\lambda$	6.4115
		21.25		6.8213
3	2	20	$\int_0^\infty \log_2(1 + \frac{\rho\lambda}{n_T})[1 + \frac{1}{2}(2-\lambda)^2]\lambda e^{-\lambda}d\lambda$	12.1396
4	1	20	$\int_0^\infty \frac{1}{3!}\log_2(1 + \frac{\rho\lambda}{n_T})\lambda^3 e^{-\lambda}d\lambda$	6.4751
		23		7.4656
4	2	20	$\int_0^\infty \log_2(1 + \frac{\rho\lambda}{n_T})[\frac{1}{2!} + \frac{1}{6}(3-\lambda)^2]\lambda^2 e^{-\lambda}d\lambda$	12.4875
8	4	20	$\int_0^\infty \log_2(1 + \frac{\rho\lambda}{n_T})[\frac{1}{4!} + \frac{1}{5!}(5-\lambda)^2$ $+ \frac{2}{6!}(\frac{\lambda^2}{2} - 6\lambda + 15)^2$ $+ \frac{1}{7!12}(-\lambda^3 + 21\lambda^2 - 126\lambda + 210)^2]\lambda^4 e^{-\lambda}d\lambda$	24.9326

From (2.38), the transmission model becomes

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \sqrt{\frac{\rho}{2}} \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \qquad (2.42)$$

We rewrite it as

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \sqrt{\frac{\rho}{2}} \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}$$

Therefore, we have a modified transmission model as follows:

$$\acute{\mathbf{Y}} = \sqrt{\frac{\rho}{2}}\acute{\mathbf{H}}\acute{\mathbf{X}} + \acute{\mathbf{N}} \qquad (2.43)$$

It is easy to realize that the channel matrix $\mathbf{H}$ *in (2.42) is changed into the channel matrix* $\acute{\mathbf{H}}$*, which is orthogonal and has a rank* $r = 2$*. From (2.40) and (2.43), the maximum mutual information of the Alamouti code per symbol time*

slot (or per channel user - PCU) is calculated as

$$
\begin{aligned}
C_{STBC}(\rho, 2, 1) &= \frac{1}{p} E\left\{ W \log_2 \left[\det\left(\mathbf{I}_r + \frac{\rho}{n_T} \acute{\mathbf{H}}^H \acute{\mathbf{H}} \right) \right] \right\} \\
&= \frac{1}{2} E\left\{ W \log_2 \left[\det\left(\mathbf{I}_2 + \frac{\rho}{2} \acute{\mathbf{H}}^H \acute{\mathbf{H}} \right) \right] \right\} \\
&= \frac{1}{2} E\left\{ W \log_2 \left[1 + \frac{\rho}{2}(|h_1|^2 + |h_2|^2) \right]^2 \right\} \\
&= E\left\{ W \log_2 \left[1 + \frac{\rho}{2}\left(|h_1|^2 + |h_2|^2 \right) \right] \right\}
\end{aligned}
$$

We see that $C_{STBC}(\rho, 2, 1)/W \equiv C(\rho, 2, 1)/W$, where $C(\rho, 2, 1)$ is the capacity of a MIMO system comprising two Tx and one Rx antennas with the SNR ρ at the Rx antenna (see (2.8)). Therefore, $C_{STBC}(\rho, 2, 1)/W = 6.2810$ bits/s/Hz per channel use (PCU) for $\rho = 20dB$ (see the Table 2.7). In other words, the Alamouti code in the channel with only one Rx antenna does not incur a loss of capacity in comparison with the true capacity of the channel. This agrees with the note in Section 2.3.2.6.

EXAMPLE 2.3.4 *We now show that the Alamouti code used in the channel with more than one Rx antenna does incur a loss of capacity. Assume that the channel now comprises $n_R = 2$ Rx antennas. The transmission model now becomes*

$$
\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \sqrt{\frac{\rho}{2}} \begin{bmatrix} x_1 & x_2 \\ x_2^* & x_1^* \end{bmatrix} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} + \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix}
$$

It is rewritten as

$$
\begin{bmatrix} y_{11} \\ y_{21}^* \\ y_{12} \\ y_{22}^* \end{bmatrix} = \sqrt{\frac{\rho}{2}} \begin{bmatrix} h_{11} & h_{21} \\ h_{21}^* & -h_{11}^* \\ h_{12} & h_{22} \\ h_{22}^* & -h_{12}^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_{11} \\ n_{21}^* \\ n_{12} \\ n_{22}^* \end{bmatrix}
$$

Similarly, the matrix $\acute{\mathbf{H}}$ *is orthogonal and has a rank* $r = 2$. *The maximum mutual information per channel use of the code is*

$$
\begin{aligned}
&C_{STBC}(\rho, 2, 2) \\
=\ &\frac{1}{p} E\left\{ W \log_2 \left[\det\left(\mathbf{I}_r + \frac{\rho}{n_T} \acute{\mathbf{H}}^H \acute{\mathbf{H}} \right) \right] \right\} \\
=\ &\frac{1}{2} E\left\{ W \log_2 \left[\det\left(\mathbf{I}_2 + \frac{\rho}{2}\left(|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2 \right) \mathbf{I}_2 \right) \right] \right\} \\
=\ &E\left\{ W \log_2 \left[1 + \frac{2\rho}{4}\left(|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2 \right) \right] \right\} \qquad (2.44)
\end{aligned}
$$

Therefore $C_{STBC}(\rho, 2, 2) \equiv C(2\rho, 4, 1)$, *where* $C(2\rho, 4, 1)$ *is the capacity of a MIMO system comprising 4 Tx antennas and 1 Rx antenna with the SNR* 2ρ *at the Rx antenna. Therefore,* $C_{STBC}(\rho, 2, 2) = 7.4656$ *bits/s/Hz PCU for* $\rho = 20dB$ *(see Table 2.7 for* $2\rho = 23$ *dB). The channel capacity in this case, meanwhile, is*

$$
\begin{aligned}
&C(\rho, 2, 2) \\
=\ &E\left\{ W \log_2 \left[\det\left(\mathbf{I}_r + \frac{\rho}{n_T} \acute{\mathbf{H}} \acute{\mathbf{H}}^H \right) \right] \right\} \\
=\ &E\left\{ W \log_2 \left[\left(1 + \frac{\rho}{2}\left(|h_{11}|^2 + |h_{12}|^2 \right) \right)\left(1 + \frac{\rho}{2}\left(|h_{21}|^2 + |h_{22}|^2 \right) \right) \right] \right\}
\end{aligned}
$$
$$(2.45)$$

From (2.44) and (2.45), it is easy to prove that $C(\rho, 2, 2) > C_{STBC}(\rho, 2, 2)$. *In fact,* $C_{STBC}(\rho, 2, 2)/W = 7.4656$ *bits/s/Hz PCU while the normalized channel capacity* $C(\rho, 2, 2)/W = 11.2898$ *bits/s/Hz (see Table 2.7). Therefore, the Alamouti code in this case incurs a crucial loss of capacity. This also agrees with the note in Section 2.3.2.6.*

EXAMPLE 2.3.5 *We consider the scenario where* $n_T = 3$, $n_R = 1$, $p = 4$ *and the following non-square STBC (see [Hassibi and Hochwald, 2002], pp.1807):*

$$
\mathbf{X} = \sqrt{\frac{4}{3}} \begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2^* & x_1^* & 0 \\ -x_3^* & 0 & x_1^* \\ 0 & -x_3^* & x_2^* \end{bmatrix}
$$

The coefficient $\sqrt{\frac{4}{3}}$ is to ensure the constraint (2.39). We have the equivalent channel coefficient matrix as given below:

$$\acute{\mathbf{H}} = \sqrt{\frac{4}{3}} \begin{bmatrix} h_1 & h_2 & h_3 \\ h_2^* & -h_1^* & 0 \\ h_3^* & 0 & -h_1^* \\ 0 & h_3^* & -h_2^* \end{bmatrix}$$

Therefore, the maximum mutual information per channel use of this STBC is

$$\begin{aligned} C_{STBC}(\rho, 3, 1) &= \frac{1}{p} E\left\{ W \log_2 \left[\det \left(\mathbf{I}_r + \frac{\rho}{n_T} \acute{\mathbf{H}}^H \acute{\mathbf{H}} \right) \right] \right\} \\ &= \frac{1}{4} E\left\{ W \log_2 \left[\det \left(\mathbf{I}_3 + \frac{\rho}{3} \frac{4}{3} \sum_{i=1}^{3} |h_i|^2 \mathbf{I}_3 \right) \right] \right\} \\ &= \frac{3}{4} E\left\{ W \log_2 \left(1 + \frac{\frac{4}{3}\rho}{3} \sum_{i=1}^{3} |h_i|^2 \right) \right\} \\ &= \frac{3}{4} C\left(\frac{4}{3}\rho, 3, 1 \right) \end{aligned}$$

Following Table 2.7, $C_{STBC}(\rho, 3, 1)/W = \frac{3}{4}C(\frac{4}{3}\rho, 3, 1)/W = 0.75 \times 6.8213 = 5.1160$ bits/s/Hz PCU for $\rho = 20dB$. Hence, the maximum mutual information $C_{STBC}(\rho, 3, 1)$ is smaller than the true channel capacity $C(\rho, 3, 1) = 6.4115$ bits/s/Hz.

Note that, it is not always possible to calculate the maximum mutual information of STBCs following the above method, since we cannot always find the equivalent channel coefficient matrix $\acute{\mathbf{H}}$ following the above method. The general method to find the equivalent channel coefficient matrix $\acute{\mathbf{H}}$ for calculating the maximum mutual information is mentioned in [Hassibi and Hochwald, 2002] (Section III). This method was originally derived for linear space-time codes, but is also applicable to STBCs.

2.4   Transmission Diversity Techniques

2.4.1   Classification of Transmission Diversity Techniques

Transmission diversity techniques have been widely used to enhance the performance of wireless channels. Various techniques have been proposed in the literature as well as applied in practice. These techniques can be classified as follows:

Depending on the domain in which the transmission redundancy is provided, diversity techniques are divided into *time diversity*, *frequency diversity* and *space diversity*.

Depending on where diversity techniques are used, transmission diversity techniques can be classified into *transmit diversity* and *receive diversity*.

2.4.1.1   Time Diversity

Time diversity is a diversity technique where identical signals are transmitted during different time slots. These time slots are uncorrelated, i.e., the temporal separation between those slots is greater than the *coherence time* of the wireless channel, which is in turn calculated as $1/F_m = c/(vF_s)$, where F_m is the maximum Doppler frequency, c the speed of light, v the speed of mobile and F_s the frequency of the transmitted signals [Proakis, 2001]. In fact, interleavers and error control coding, such as Forward Error Correction (FEC) codes, are employed to provide time diversity for the receiver. Another example of the modern implementation of time diversity is the RAKE receiver in CDMA (Code Division Multiple Access) systems [Rappaport, 2002] (pp. 391).

The main shortcoming of this technique is that the redundancy is provided in the time domain with a penalty of a loss in bandwidth efficiency. The loss in bandwidth is due to the guard time existing between the time slots.

2.4.1.2   Frequency Diversity

In this diversity technique, several frequencies are used to transmit the same signals. The frequency separation between these carrier frequencies is an order of several times of the *coherence bandwidth* of the channel [Proakis, 2001]. Consequently, the carrier frequencies are uncorrelated, i.e., they do not experience the same fades.

In practice, frequency diversity is often used in Line-Of-Sight (LOS) microwave channels. Some examples of systems employing frequency diversity include spread spectrum systems, such as Direct Sequence Spread Spectrum (DS-SS), Frequency Hoping Spread Spectrum (FH-SS) or Multi-Carrier Spread Spectrum (MC-SS) systems.

Similarly to the case of time diversity, in frequency diversity, the redundancy is provided in the frequency domain with the penalty of a loss in spectral efficiency. The loss in spectral efficiency is due to the guard bands existing between the carrier frequencies. Additionally, the structure of the receiver is complicated as it must be able to work with a number of frequencies.

2.4.1.3   Space Diversity

Space diversity, which is also named *antenna diversity*, has been frequently implemented in practice. This diversity technique can be further classified into various schemes, such as *polarization diversity* [Rappaport, 2002] (pp. 387), *beamforming diversity* [Blogh and Hanzo, 2002], [Godara, 1997], [Katz and Ylitalo, 2000], [Larsson et al., 2003], *antenna switching* [Barrett and Arnott, 1994]. Depending on whether it is applied to the transmitter or to the receiver, it can be classified into *transmit diversity* and *receive diversity*. Depending on how the replicas of the transmitted signals are combined at the receiver, *space diversity* techniques are classified into *selection combining* technique, *switched combining* technique, *equal-gain combining* technique and *maximum ratio combining* (MRC) technique [Rappaport, 2002].

The concept of space diversity is using multiple Tx and/or Rx antennas to transmit and/or receive signals. These antennas are spatially separated from one another by some halves of the wavelength, and consequently, they may be considered to be independent of one another [Jakes, 1974], [Salz and Winters, 1994].

Unlike time diversity and frequency diversity, in space diversity, the redundancy is provided for the receiver in the spatial domain, and consequently, this technique has no loss in spectral efficiency.

In practical wireless communication, a combined diversity technique of the aforementioned techniques is employed to provide multi-dimensional diversity. For instance, in GSM cellular systems, a combination between multiple antennas at the base station (space diversity) and interleaving as well as error control coding (time diversity) is utilized to provide the 2-dimensional diversity for the receivers (mobile users).

In this book, the authors examine the combination between multiple antennas (space diversity), antenna switching techniques (space diversity), Space-Time Block Codes STBCs (space and time diversity), and maximum ratio combining (MRC) technique (space diversity) to provide more diversity for wireless communication channels utilizing STBCs.

2.4.2   Spatial Diversity Combining Methods

As mentioned earlier, spatial diversity combining methods can be divided into the following categories: selection combining, scanning combining, maximum ratio combining and equal gain combining (see [Vucetic and Yuan, 2003] (pp. 55) and [Rappaport, 2002] (pp. 385)).
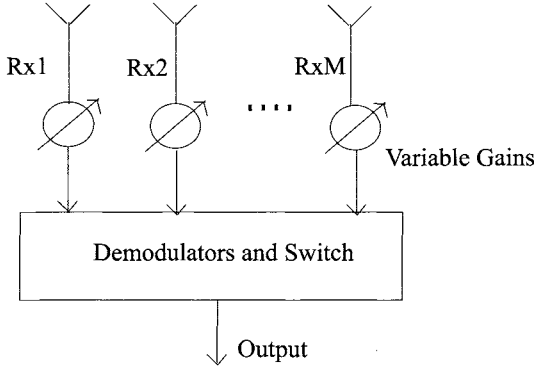
Figure 2.6. Selection combining method.

2.4.2.1 Selection Combining

This is the simplest spatial diversity combining method which requires only a SNR monitoring action and an antenna switch at the receiver.

In this technique, the receiver comprises M Rx antennas associated with M individual demodulators to provide M branches of which the gains are weighted to provide the same *average* SNR for every branch. The receiver selects the incoming signal with the highest *instantaneous* SNR to demodulate. In reality, as the measurement of the *instantaneous* SNR is difficult, the term $(S + N_0)/N_0$, where $(S + N_0)$ is the instantaneous power of the received signal (including noise), is normally measured. The diagram of the selection combining method is presented in Fig. 2.6.

The average SNR of the received signal $\bar{\gamma}$ (when selection combining is used) compared to the average SNR Γ of each branch (when no diversity is used) is calculated as follows (Eq. (7.62) in [Rappaport, 2002]):

$$\frac{\bar{\gamma}}{\Gamma} = \sum_{k=1}^{M} \frac{1}{k} \tag{2.46}$$

Clearly, for $M \geq 2$, we have $\bar{\gamma} > \Gamma$. However, this technique is not optimal as it does not use all incoming signals simultaneously to provide the best received signals.

2.4.2.2 Scanning Combining - SC

In this technique, the receiver scans all branches following a certain order and selects a particular branch which has an SNR above the predetermined SNR threshold. The signal of this branch is selected as the output until it drops
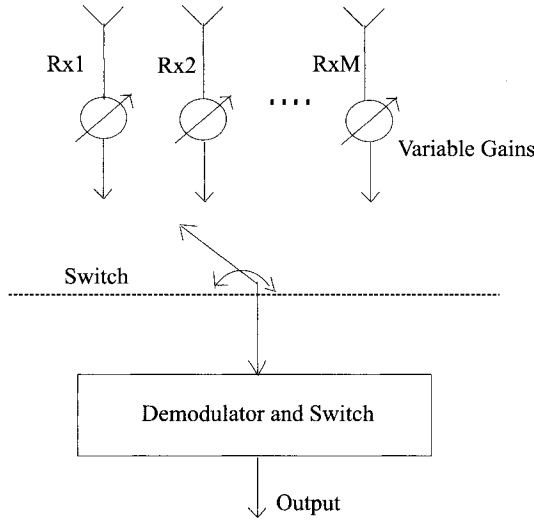
Figure 2.7. Scanning combining method.

under the predetermined threshold. The receiver then starts searching again. The diagram of the scanning combining method is presented in Fig. 2.7.

The advantage of this technique over the selection combining method is that the receiver does not need to monitor continuous and instantaneous SNRs of all branches at all times. However, it is inferior compared to selection combining method as the best incoming signal is not always selected, and consequently, the average SNR of the output is smaller than that mentioned in (2.46).

2.4.2.3   Maximum Ratio Combining - MRC

In this technique, the signals from M branches are weighted by the corresponding weighting factors $G_k = \frac{r_k}{N_0}$ of those branches ($k = 1, \ldots, M$), where r_k and N_0 are the envelopes of received signals and the noise power, and then summed. Generally speaking, the weighting factors of the branches are *proportional* to the ratios $\frac{r_k}{N_0}$ of those branches themselves. Before summing, the signals must be co-phased to provide the coherence voltage addition. The average SNR $\bar{\gamma}_M$ of the output signal is simply the sum of individual SNRs Γ of all branches (Eq. (7.70) in [Rappaport, 2002]):

$$\bar{\gamma}_M = M\Gamma \tag{2.47}$$

Clearly, this technique can provide an acceptable output signal with the expected SNR even when no incoming signal is acceptable. Certainly, the structure and the cost of the MRC are higher than those of other combining methods. The
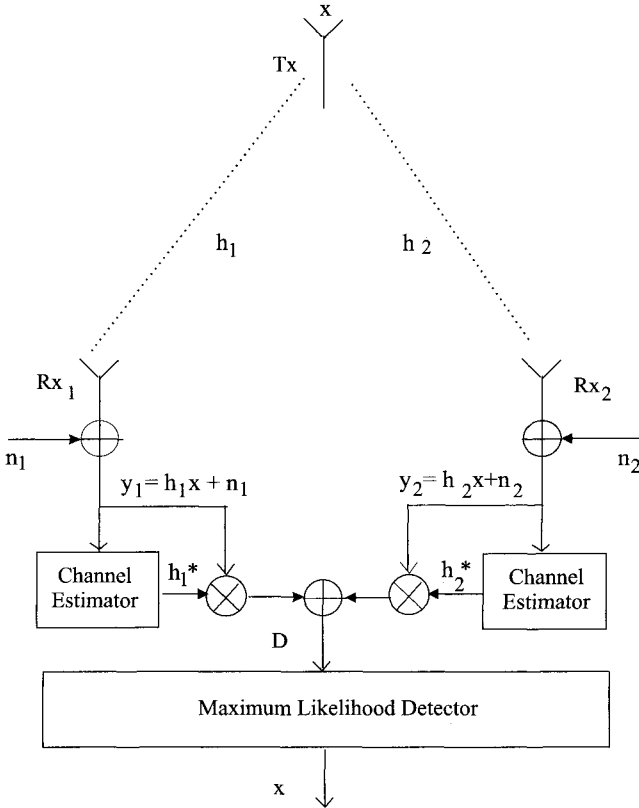
Figure 2.8. Conventional baseband MRC technique using two receiver antennas.

diagram of the conventional baseband MRC technique using 2 Rx antennas is presented in Fig. 2.8. Readers may refer to [Alamouti, 1998] or [Liew and Hanzo, 2002] (pp. 192) for more details.

2.4.2.4 Equal Gain Combining - EGC

This diversity technique is similar to the maximum ratio combining method, except that all weighting factors are set to one. The performance is marginally inferior compared to that of the maximum ratio combining method.

2.4.3 Transmit Diversity Techniques

Transmit diversity techniques [Fragouli et al., 2002], [Winters, 1994], [Winters, 1998] can be categorized to transmit diversity *with* or *without* feedback. In the uplinks (from Mobile Stations (MSs) to Base Stations (BSs)) of mobile communication systems, *receive diversity* is usually used by employing

multiple receiver antennas at BSs. However, in the downlinks (from BSs to MSs), it is more practical to utilize *transmit diversity* techniques than using receive diversity techniques due to the following reasons:

■ Multiple Rx antennas at MSs require a more complicated structure and more processing procedures at MSs, and consequently, require more power. Therefore, the lifetime of batteries in MSs is shortened.

■ Due to the tiny size of MSs, it is impractical to install more than two Rx antennas at MSs.

Various transmit diversity techniques have been proposed in the literature, such as delay diversity schemes [Seshadri and Winters, 1993], [Wittneben, 1991], [Wittneben, 1993], beamforming [Larsson et al., 2003], antenna switching [Barrett and Arnott, 1994]. In order to improve further the performance of systems, transmit diversity is combined with *modulation schemes* and/or *coding schemes* with the consequence that both *diversity gain* and *coding gain* can be improved. The combination between transmit diversity techniques and Space-Time Codes (STCs) having coding gains, such as Space-Time Trellis Codes (STTCs) [Tarokh et al., 1998], is one of such combined transmit diversity techniques.

Although STBCs do not provide coding gains, STBCs possess a simple decoding method. They provide full diversity in both space and time domains *without* [1] or with a *small bandwidth expansion* [2]. The small bandwidth extension is due to the fact that the full-rate, CO STBCs do not exist for more than two Tx antennas (see Section 2.3.5 or [Liang, 2003], [Liang and Xia, 2003], [Wang and Xia, 2003]).

In this book, we propose some diversity Antenna Selection Techniques (ASTs) for channels using either STBCs or *differential* STBCs (DSTBCs). When STBCs or DSTBCs are used, it is possible to use the MRC technique at the receiver. The association between the proposed ASTs and the MRC technique significantly improves the performance of wireless channels using either STBCs or DSTBCs. Certainly, the performance of systems can be further improved by associating *transmit diversity* and *receive diversity*.

---

[1]For instance, when the Alamouti code [Alamouti, 1998] is used in systems with one receiver antenna. Readers may refer to Section 2.3.2.6 and 2.3.2.7 for more details.
[2]For the Alamouti code used in systems with more than one receiver antenna and for any CO STBC of order greater than 2. Readers may refer to Section 2.3.2.6 and 2.3.2.7 for more details.

2.5   Issues Addressed in the Book

As analyzed before, MIMO systems potentially possess a high capacity, which is a desired property for the current communication needs requiring a very high data rate and high reliability, such as multimedia communication services, cellular mobile, and the Internet. In many cases, as pointed out previously, the capacity of MIMO systems is approximately linearly proportional to the number of antennas.

STBCs are among various practical, advanced coding techniques designed for the use of multiple transmission antennas, which potentially approach the high capacity of MIMO systems (although there exists a loss of capacity compared to the true capacity of MIMO channels). As mentioned earlier, this coding technique possesses the following properties:

1 It completely or highly takes advantage of the channel bandwidth, i.e., the maximum mutual information of STBCs is equal (in the case of the Alamouti code used in systems with 1 Rx antenna) or approaches the high capacity of MIMO channels.

2 It decreases the sensitivity to multi-path fading, and facilitates utilization of higher level modulation schemes resulting in an increase of the data rate.

3 It also facilitates the increase of the coverage area of wireless systems, and consequently, improves the reuse of frequencies in mobile communication systems.

4 Consequently, it improves the data rate, the error performance and capacity of wireless communication channels with small or even without any expansion of bandwidth. An example is given in Fig. 2.9. This figure illustrates the bit error performance improvement of the Alamouti code compared to the transmission without space-time coding.

5 It requires a relatively simple decoding technique, i.e., Maximum Likelihood (ML) decoding, due to the orthogonality between the columns of the code matrices. Therefore, Space-Time Block coding is a simple and cost-effective coding scheme to meet the requirement on quality and spectral efficiency for the next generation wireless systems without changing much the structure of the existing systems. In fact, the Alamouti code has been adopted in the third (3G) generation wireless standards, such as WCDMA and CDMA 2000, by the Third Generation Partnership Project (3GPP) [3GPP, 2002], [Al-Dhahir, 2002], [Rappaport et al., 2002].
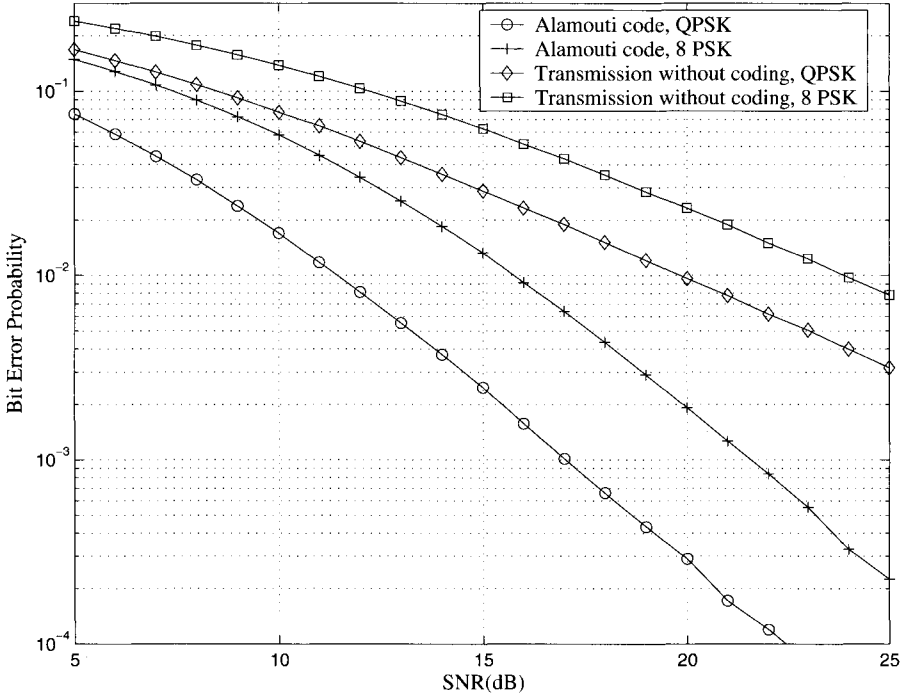
Figure 2.9. Alamouti code vs. transmission without coding with QPSK modulation and 8PSK modulation.

In this book, we consider the following research problems:

1 Although constructions of *square, maximum rate* CO STBCs, such as the Adams-Lax-Phillips construction, Jozefiak construction, and Wolfe construction, are well known (see Section 2.3.2.3 or [Liang, 2003]), the codes resulting from these constructions have numerous zeros, since these construction methods always involve identity matrices. This shortcoming impedes the practical implementation, especially in high data rate systems.

   So far, the general constructions of *square, maximum rate* CO STBCs with fewer or even with no zeros such that the transmitted symbols equally disperse through Tx antennas have not been well examined. Those codes, referred to as the *improved square CO STBCs*, have the advantages that the power tends to be equally transmitted via each Tx antenna during every symbol time slot and that a lower peak power per Tx antenna is required

to achieve the same bit error rates as for the conventional CO STBCs with zeros. *This book proposes such constructions of improved CO STBCs.*

2 To increase the bit rates of CO STBCs, Multi-Modulation Schemes (MMSs) can be applied to CO STBCs. The CO STBCs resulted from our aforementioned, discovered constructions, where some variables appear more often than the others, are especially good constructions for the use of MMSs. With the same MMSs and the same peak power per Tx antenna, such constructions may provide a *higher* data transmission rate and possess *better* error performance than the conventional STBCs where numerous zeros are present.

In addition, in MMSs, the optimal inter-symbol power allocation to achieve the best error performance is an important property. Although MMSs and the method examining the optimal inter-symbol power allocation have been somewhat mentioned in [Tirkkonen and Hottinen, 2001], these issues have not been well examined yet.

Therefore, this book proposes MMSs applied to CO STBCs to increase the transmission rate and a general method for examining the optimal inter-symbol power allocation in such MMSs to achieve the optimal error properties.

3 As analyzed earlier, the benefit of diversity techniques is evident. The combination between CO STBCs and a closed loop transmission diversity technique (using a feedback loop) to improve further the performance of wireless channels has been intensively examined in the case of *coherent detection.* However, it will be better if the time required to process the feedback information at the transmitter is shortened, and consequently, the system is quickly updated to the change of channels. *This book derives an improved transmitter antenna selection technique, which reduces the time required to process feedback information and enhances further the performance of space-time coded wireless channels in the case of coherent detection.*

4 So far, the closed loop transmit diversity techniques with the aid of a feedback loop and with *a limited number of training symbols (or pilot symbols)* to select transmitter and/or receiver antennas in the channels using Differential Space-Time Block Codes (DSTBCs) have not been intensively investigated yet. *This book proposes two transmission antenna selection techniques for the channels using DSTBCs, improving significantly the performance of*

wireless channels and being very robust, even in correlated, flat Rayleigh fading channels or in the channels with imperfect carrier recovery.

5 Although the methods to generate correlated Rayleigh fading envelopes in wireless channels have been intensely examined in the literature, those methods have their own shortcomings, which seriously limit their applicability. A more generalized algorithm to generate Rayleigh fading envelopes, which are correlated in *spatial*, *temporal* and/or *spectral* domains, and can be applied to the case of either *discrete-time instants* or *a real-time scenario*, is essential for researchers in simulating and modelling the channels. *This book proposes such a more generalized algorithm to generate correlated Rayleigh fading envelopes that overcomes all shortcomings of the conventional methods.*