Preface

Geometry is one of the oldest branches of mathematics, nearly as old as human culture. Its beauty has always fascinated mathematicians, among others. In writing this book we had the purpose of sharing with readers the pleasure derived from studying geometry, as well as giving a taste of its importance, its deep connections with other branches of mathematics and the highly diverse viewpoints that may be taken by someone entering this field.

We also want to propose a specific way to introduce concepts that have arisen from the heyday of the Greek school of geometry to the present day. We work with coordinate models, since this facilitates the use of algebraic and analytic results, and we follow the viewpoint proposed by Felix Klein in the 19th century, of studying geometry via groups of symmetries of the space in question.

We intend this book to be both an introduction to the subject addressed to undergraduate students in mathematics and physics, and a useful text-book for mathematicians and scientists in general who want to learn the basics of classical geometry: Euclidean, affine, elliptic, hyperbolic and projective geometry. These are all presented in a unified way and the essential content of this book may be covered in a single semester, though a longer period of study would allow the student to grasp and assimilate better the material in it.

We essentially restrict the whole discussion to "plane geometry", that is dimension 2, since this is already rich enough. We include some aspects of the 3- or n-dimensional extension of these plane geometries whenever it is simple to do so. Once plane geometry is well understood, it is much easier to go into higher dimensions, and we give guidelines for further reading.

We assume that the reader is familiar with a few facts of plane Euclidean geometry (especially those concerning triangles and circles), and also with the elements of analytic geometry (the intersection of straight lines and planes, their equations and the main properties of conics and quadrics) and of linear algebra (up to eigenvalues and eigenvectors of a linear transformation of the plane or 3-space).

Also, we suppose the student is familiar with the notions and results of differential and integral calculus for real functions of a single real variable, and that he or she is taking at least a first course in calculus of several variables; basic knowledge of the complex numbers will be useful in the last chapter.

With this background it is possible to present formally, applying the analytic method to coordinate models, the geometries that “come after” Euclidean geometry: affine, projective, elliptic and hyperbolic.

The study we make of these geometries allows us to understand the fundamental role played by the groups of “symmetries” and shows how algebra, analysis and geometry combine to give a better understanding of a concept, a result or a problem’s solution.

We have also paid attention to the exercises, choosing them so that their solution deepens or widens our understanding. For exercises of greater than average difficulty we give references as support. The best measure of the degree of understanding is the percentage of exercises solved: all of them should be attempted!

After assimilating the material in this text the reader will be prepared to step forward, if that is his or her will, into more advanced areas of geometry, such as differential geometry, foliations and group actions on differentiable manifolds, among others.