

# 2

## Two-Dimensional Wave Equations and Wave Characteristics

A practicing coastal engineer must have a basic and relatively easy to use theory that defines the important characteristics of two-dimensional waves. This theory is required in order to analyze changes in the characteristics of a wave as it propagates from the deep sea to the shore. Also, this theory will be used as a building block to describe more complex sea wave spectra. Such a theory—the small amplitude wave theory—is presented in this chapter along with related material needed to adequately describe the characteristics and behavior of two-dimensional waves.

### 2.1 Surface Gravity Waves

When the surface of a body of water is disturbed in the vertical direction, the force of gravity will act to return the surface to its equilibrium position. The returning surface water has inertia that causes it to pass its equilibrium position and establish a surface oscillation. This oscillation disturbs the adjacent water surface, causing the forward propagation of a wave.

A wave on the water surface is thus generated by some disturbing force which may typically be caused by the wind, a moving vessel, a seismic disturbance of the shallow sea floor, or the gravitational attraction of the sun and moon. These forces impart energy to the wave which, in turn, transmits the energy across the water surface until it reaches some obstacle such as a structure or the shoreline which causes the energy to be reflected and dissipated. The wave also transmits a signal in the form of the oscillating surface time history at a point.

As a wave propagates, the oscillatory water motion in the wave continues because of the interaction of gravity and inertia. Since water particles in the wave are continuously accelerating and decelerating as the wave propagates, dynamic pressure gradients develop in the water column. These dynamic pressure gradients are superimposed on the vertical hydrostatic pressure gradient. As the wave

propagates energy is dissipated, primarily at the air–water boundary and, in shallower water, at the boundary between the water and the sea floor.

The different wave generating forces produce waves with different periods. Wind-generated waves have a range of periods from about 1 to 30 s with the dominant periods for ocean storm waves being between 5 and 15 s. Vessel-generated waves have shorter periods, typically between 1 and 3 s. Seismically generated waves (tsunamis) have longer periods from about 5 min to an hour and the dominant periods of the tide are around 12 and 24 hours.

Wind waves in the ocean have a height (vertical distance crest to trough) that is typically less than 10 ft, but it can exceed 20 ft during significant storms. Vessel waves rarely exceed 3 ft in height. At sea, tsunami waves are believed to have a height of 2 ft or less, but as the tsunami approaches the coast heights often increase to greater than 10 ft, depending on the nature of the nearshore topography. Similarly, tide wave heights (tide ranges) in the deep ocean are relatively low, but along the coast tide ranges in excess of 20 ft occur at a number of locations.

Wind-generated waves are complex, consisting of a superimposed multitude of components having different heights and periods. In this chapter we consider the simplest theory for the characteristics and behavior of a two-dimensional monochromatic wave propagating in water of constant depth. This will be useful in later chapters as a component of the spectrum of waves found at sea. It is also useful for first-order design calculations where the height and period of this monochromatic wave are selected to be representative of a more complex wave spectrum. Also, much laboratory research has used, and will continue to use, monochromatic waves for basic studies of wave characteristics and behavior such as the wave-induced force on a structure or the nature of breaking waves.

The simplest and often most useful theory (considering the effort required in its use) is the two-dimensional small-amplitude or linear wave theory first presented by Airy (1845). This theory provides equations that define most of the kinematic and dynamic properties of surface gravity waves and predicts these properties within useful limits for most practical circumstances. The assumptions required to derive the small-amplitude theory, an outline of its derivation, the pertinent equations that result, and the important characteristics of waves described by these equations are presented in this chapter. More detail on the small-amplitude wave theory can be found in Wiegel (1964), Ippen (1966), Dean and Dalrymple (1984), U.S. Army Coastal Engineering Research Center (1984), and Sorensen (1993).

## 2.2 Small-Amplitude Wave Theory

The small-amplitude theory for two-dimensional, freely propagating, periodic gravity waves is developed by linearizing the equations that define the free surface boundary conditions. With these and the bottom boundary condition,

a periodic velocity potential is sought that satisfies the requirements for irrotational flow. This velocity potential, which is essentially valid throughout the water column except at the thin boundary layers at the air–water interface and at the bottom, is then used to derive the equations that define the various wave characteristics (e.g., surface profile, wave celerity, pressure field, and particle kinematics). Specifically, the required assumptions are:

1. The water is homogeneous and incompressible, and surface tension forces are negligible. Thus, there are no internal pressure or gravity waves affecting the flow, and the surface waves are longer than the length where surface tension effects are important (i.e., wave lengths are greater than about 3 cm).
2. Flow is irrotational. Thus there is no shear stress at the air–sea interface or at the bottom. Waves under the effects of wind (being generated or diminished) are not considered and the fluid slips freely at the bottom and other solid fixed surfaces. Thus the velocity potential  $\phi$  must satisfy the Laplace equation for two-dimensional flow:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (2.1)$$

where  $x$  and  $z$  are the horizontal and vertical coordinates, respectively.

3. The bottom is stationary, impermeable, and horizontal. Thus, the bottom is not adding or removing energy from the flow or reflecting wave energy. Waves propagating over a sloping bottom, as for example when waves propagate toward the shore in the nearshore region, can generally be accommodated by the assumption of a horizontal bottom if the slope is not too steep.
4. The pressure along the air–sea interface is constant. Thus, no pressure is exerted by the wind and the aerostatic pressure difference between the wave crest and trough is negligible.
5. The wave height is small compared to the wave length and water depth. Since particle velocities are proportional to the wave height, and wave celerity (phase velocity) is related to the water depth and the wave length, this requires that particle velocities be small compared to the wave celerity. This assumption allows one to linearize the higher order free surface boundary conditions and to apply these boundary conditions at the still water line rather than at the water surface, to obtain an easier solution. This assumption means that the small-amplitude wave theory is most limited for high waves in deep water and in shallow water and near wave breaking where the waves peak and wave crest particle velocities approach the wave phase celerity. Given this, the small-amplitude theory is still remarkably useful and extensively used for wave analysis.

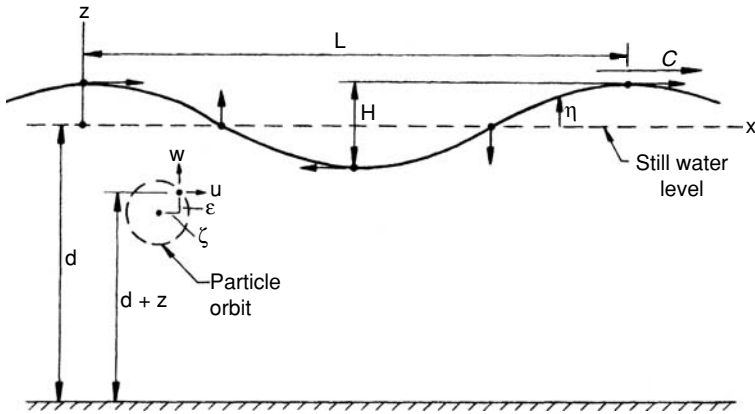


Figure 2.1. Definition of progressive surface wave parameters.

Figure 2.1 depicts a monochromatic wave traveling at a phase celerity  $C$  on water of depth  $d$  in an  $x, z$  coordinate system. The  $x$  axis is the still water position and the bottom is at  $z = -d$ . The wave surface profile is defined by  $z = \eta$ , where  $\eta$  is a function of  $x$  and time  $t$ . The wave length  $L$  and height  $H$  are as shown in the figure. Since the wave travels a distance  $L$  in one period  $T$ ,

$$C = L/T \quad (2.2)$$

The arrows at the wave crest, trough, and still water positions indicate the directions of water particle motion at the surface. As the wave propagates from left to right these motions cause a water particle to move in a clockwise orbit. The water particle velocities and orbit dimensions decrease in size with increasing depth below the still water line. Particle orbits are circular only under certain conditions as defined in Section 2.4.

The horizontal and vertical components of the water particle velocity at any instant are  $u$  and  $w$ , respectively. The horizontal and vertical coordinates of a water particle at any instant are given by  $\zeta$  and  $\epsilon$ , respectively. The coordinates are referenced to the center of the orbital path that the particle follows. At any instant, the water particle is located a distance  $d - (-z) = d + z$  above the bottom.

The following dimensionless parameters are often used:

$$k = 2\pi/L(\text{wave number})$$

$$\sigma = 2\pi/T(\text{wave angular frequency})$$

We also use the terms “wave steepness” defined as the wave height divided by the wave length (i.e.,  $H/L$ ) and “relative depth” defined as the water depth divided by the wave length (i.e.,  $d/L$ ) in discussions of wave conditions.

The small-amplitude wave theory is developed by solving Eq. (2.1) for the domain depicted in Figure 2.1, with the appropriate boundary conditions for the free surface (2) and the bottom (1).

At the bottom there is no flow perpendicular to the bottom which yields the bottom boundary condition (BBC):

$$w = \frac{\partial \phi}{\partial z} = 0 \text{ at } z = -d \quad (2.3)$$

At the free surface there is a kinematic boundary condition (KSBC) that relates the vertical component of the water particle velocity at the surface to the surface position:

$$w = \frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x} \text{ at } z = \eta \quad (2.4)$$

The Bernoulli equation for unsteady irrotational flow may be written

$$\frac{1}{2}(u^2 + w^2) + \frac{p}{\rho} + gz + \frac{\partial \phi}{\partial t} = 0 \quad (2.5)$$

where  $g$  is the acceleration of gravity,  $p$  is the pressure, and  $\rho$  is the fluid density. At the surface where the pressure is zero the dynamic boundary condition (DSBC) becomes

$$\frac{1}{2}(u^2 + w^2) + gz + \frac{\partial \phi}{\partial t} = 0 \text{ at } z = \eta \quad (2.6)$$

The KSBC and the DSBC have to be linearized and applied at the still water line rather than at the a priori unknown water surface. This yields for the KSBC

$$w = \frac{\partial \eta}{\partial t} \text{ at } z = 0 \quad (2.7)$$

and for the DSBC

$$g\eta + \frac{\partial \phi}{\partial t} = 0 \text{ at } z = 0 \quad (2.8)$$

Employing the Laplace equation, the BBC, and the linearized DSBC, we can derive the velocity potential for the small-amplitude wave theory (see Ippen, 1966; Sorensen, 1978; or Dean and Dalrymple, 1984). The most useful form of this velocity potential is

$$\phi = \frac{gH}{2\sigma} \frac{\cosh k(d+z)}{\cosh kd} \sin(kx - \sigma t) \quad (2.9)$$

The velocity potential demonstrates an important point. Since the wave length or wave number ( $k = 2\pi/L$ ) depends on the wave period and water depth [see Eq. (2.14)], when the wave height and period plus the water depth are known the wave is fully defined and all of its characteristics can be calculated.

We can insert the velocity potential into the linearized DSBC with  $z = 0$  to directly determine the equation for the wave surface profile:

$$\eta = \frac{H}{2} \cos(kx - \sigma t) \quad (2.10)$$

which can also be written

$$\eta = \frac{H}{2} \cos 2\pi \left( \frac{x}{L} - \frac{t}{T} \right) \quad (2.11)$$

by inserting the wave number and wave angular frequency. Thus, the small-amplitude wave theory yields a cosine surface profile. This is reasonable for low-amplitude waves, but with increasing wave amplitude the surface profile becomes vertically asymmetric with a more peaked wave crest and a flatter wave trough (as will be shown in Chapter 3).

Combining the KSBC and the DSBC by eliminating the water surface elevation yields

$$\frac{\partial^2 \phi}{\partial t^2} + g \frac{\partial \phi}{\partial z} = 0 \text{ at } z = 0$$

Then, inserting the velocity potential, differentiating, and rearranging we have

$$\sigma^2 = gk \tanh kd$$

or

$$C = \frac{\sigma}{k} = \sqrt{\frac{g}{k} \tanh kd}$$

and

$$C = \sqrt{\frac{gL}{2\pi} \tanh \frac{2\pi d}{L}} \quad (2.12)$$

Equation (2.12) indicates that for small-amplitude waves, the wave celerity is independent of the wave height. As the wave height increases there is a small but growing dependence of the wave celerity on the wave height (see Chapter 3). Equation (2.12) can also be written [by inserting Eq. (2.2)]

$$C = \frac{gT}{2\pi} \tanh \frac{2\pi d}{L} \quad (2.13)$$

$$L = \frac{gT^2}{2\pi} \tanh \frac{2\pi d}{L} \quad (2.14)$$

From Eq. (2.14), if the water depth and the wave period are known, the wave length can be calculated by trial and error. Then the celerity can be determined from  $C = L/T$ . Tables are available (U.S. Army Coastal Engineering Research Center, 1984) for the direct determination of  $L$  given the water depth and wave period.

Equations (2.12) to (2.14) collectively are commonly known as the dispersion equation. For a spectrum of waves having different periods (or lengths), the longer waves will propagate at a higher celerity and move ahead while the shorter waves will lag behind.

It can be demonstrated (see Ippen, 1966) that as a wave propagates from deep water in to the shore, the wave period will remain constant because the number of waves passing sequential points in a given interval of time must be constant. Other wave characteristics including the celerity, length, height, surface profile, particle velocity and acceleration, pressure field, and energy will all vary during passage from deep water to the nearshore area.

### 2.3 Wave Classification

An important classification of surface waves is based on the relative depth ( $d/L$ ). When a wave propagates from deep water offshore in to shallower water nearshore the wave length decreases [see Eq. (2.14)], but at a slower rate than that at which the depth decreases. Thus, the relative depth decreases as a wave approaches the shore. When  $d/L$  is greater than approximately 0.5,  $\tanh (2\pi d/L)$  is essentially unity and Eqs. (2.12) to (2.14) reduce to

$$C_o = \sqrt{\frac{gL_o}{2\pi}} \quad (2.15)$$

$$C_o = \frac{gT}{2\pi} \quad (2.16)$$

and

$$L_o = \frac{gT^2}{2\pi} \quad (2.17)$$

respectively. Waves in this region are called deep water waves and this condition is commonly denoted by the subscript zero (except for the wave period which is

not depth dependent and thus does not change as the relative depth decreases). Wave particle velocities and orbit dimensions decrease with increasing distance below the free surface. In deep water at a depth of  $-z/L > 0.5$  the particle velocities and orbit dimensions are close to zero. Since for  $d/L > 0.5$  the waves do not interact with the bottom, wave characteristics are thus independent of the water depth [e.g., see Eqs. (2.15) to (2.17)].

### Example 2.3-1

A wave in water 100 m deep has a period of 10 s and a height of 2 m. Determine the wave celerity, length, and steepness. What is the water particle speed at the wave crest?

#### Solution:

Assume that this is a deep water wave. Then, from Eq. (2.17)

$$L_o = \frac{9.81(10)^2}{2\pi} = 156 \text{ m}$$

Since the depth is greater than half of the calculated wave length, the wave is in deep water and the wave length is 156 m. [Otherwise, Eq. (2.14) would have to be used to calculate the wave length.] The wave celerity is from Eq. (2.2)

$$C_o = \frac{156}{10} = 15.6 \text{ m/s}$$

and the steepness is

$$\frac{H_o}{L_o} = \frac{2}{156} = 0.013$$

For deep water the particle orbits are circular having a diameter at the surface equal to the wave height. Since a particle completes one orbit in one wave period, the particle speed at the crest would be the orbit circumference divided by the period or

$$u_c = \frac{\pi H_o}{T} = \frac{3.14(2)}{10} = 0.63 \text{ m/s}$$

Note that this is much less than  $C_o$ .

When the relative depth is less than 0.5 the waves interact with the bottom. Wave characteristics depend on both the water depth and the wave period, and



continually change as the depth decreases. The full dispersion equations must be used to calculate wave celerity or length for any given water depth and wave period. Dividing Eq. (2.13) by Eq. (2.16) or Eq. (2.14) by Eq. (2.17) yields

$$\frac{C}{C_o} = \frac{L}{L_o} = \tanh \frac{2\pi d}{L} \quad (2.18)$$

which is a useful relationship that will be employed in a later chapter. Waves propagating in the range of relative depths from 0.5 to 0.05 are called intermediate or transitional water waves.

When the relative depth is less than approximately 0.05,  $\tanh (2\pi d/L)$  approximately equals  $2\pi d/L$  and the dispersion equation yields

$$C = \sqrt{gd} \quad (2.19)$$

or

$$L = \sqrt{gd}T \quad (2.20)$$

Waves in this region of relative depths are called shallow water waves. In shallow water the small-amplitude wave theory gives a wave celerity that is independent of wave period and dependent only on the water depth (i.e., the waves are not period dispersive). The finite-amplitude wave theories presented in the next chapter show that the shallow water wave celerity is a function of the water depth and the wave height so that in shallow water waves are amplitude dispersive. Remember that it is the relative depth, not the actual depth alone, that defines deep, intermediate, and shallow water conditions. For example, the tide is a very long wave that behaves as a shallow water wave in the deepest parts of the ocean.

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### Example 2.3-2

Consider the wave from Example 2.3-1 when it has propagated in to a nearshore depth of 2.3 m. Calculate the wave celerity and length.

Solution:

Assuming this is a shallow water wave, Eq. (2.19) yields

$$C = \sqrt{9.81(2.3)} = 4.75 \text{ m/s}$$

and Eq. (2.2) yields

$$L = 4.75(10) = 47.5 \text{ m}$$

So  $d/L = 2.3/47.5 = 0.048 < 0.05$  and the assumption of shallow water was correct. Compare these values to the results from Example 2.3-1

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## 2.4 Wave Kinematics and Pressure

Calculation of the wave conditions that will cause the initiation of bottom sediment motion, for example, requires a method for calculating water particle velocities in a wave. The water particle velocity and acceleration as well as the pressure field in a wave are all needed to determine wave-induced forces on various types of coastal structures.

### Wave Kinematics

The horizontal and vertical components of water particle velocity ( $u$  and  $w$ , respectively) can be determined from the velocity potential where

$$u = \frac{\partial\phi}{\partial x}, \quad w = \frac{\partial\phi}{\partial z}$$

This yields, after inserting the dispersion relationship and some algebraic manipulation

$$u = \frac{\pi H}{T} \left[ \frac{\cosh k(d+z)}{\sinh kd} \right] \cos(kx - \sigma t) \quad (2.21)$$

and

$$w = \frac{\pi H}{T} \left[ \frac{\sinh k(d+z)}{\sinh kd} \right] \sin(kx - \sigma t) \quad (2.22)$$

Equations (2.21) and (2.22) give the velocity components at the point  $(x, -z)$  as a function of time as different water particles pass through this point.

Note that each velocity component consists of three parts: (1) the surface deep water particle speed  $\pi H/T$ , (2) the term in brackets which accounts for particle velocity variation over the vertical water column at a given location and for particle velocity variation caused by the wave moving from deep to shallow water, and (3) a phasing term dependent on position in the wave and time. Note that  $d+z$  is the distance measured up from the bottom as demonstrated in Figure 2.1. Also, as would be expected, the horizontal and vertical velocity components are  $90^\circ$  out of phase.

The horizontal component of particle acceleration  $a_x$  may be written

$$a_x = u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

where the first two terms on the righthand side are the convective acceleration and the third term is the local acceleration. The magnitude of the convective

acceleration for a small-amplitude wave is of the order of the wave steepness ( $H/L$ ) squared while the magnitude of the local acceleration is of the order of the wave steepness. Since the wave steepness is much smaller than unity, we can usually neglect the higher order convective acceleration term in determining the particle acceleration. This yields

$$a_x = \frac{2\pi^2 H}{T^2} \left[ \frac{\cosh k(d+z)}{\sinh kd} \right] \sin(kx - \sigma t) \quad (2.23)$$

for the horizontal component and

$$a_z = -\frac{2\pi^2 H}{T^2} \left[ \frac{\sinh k(d+z)}{\sinh kd} \right] \cos(kx - \sigma t) \quad (2.24)$$

for the vertical component of acceleration. The terms in brackets are the same for both the particle velocity and acceleration components. The cosine/sine terms indicate that the particle velocity components are  $90^\circ$  out of phase with the acceleration components. This is easily seen by considering a particle following a circular orbit. The velocity is tangent to the circle and the acceleration is toward the center of the circle or normal to the velocity.

As water particles orbit around a mean position (see Figure 2.1) the horizontal and vertical coordinates of the particle position relative to the mean position are given by  $\zeta$  and  $\varepsilon$ , respectively. These components can be found by integrating the particle velocity components with time. This yields

$$\zeta = \frac{-H}{2} \left[ \frac{\cosh k(d+z)}{\sinh kd} \right] \sin(kx - \sigma t) \quad (2.25)$$

and

$$\varepsilon = \frac{H}{2} \left[ \frac{\sinh k(d+z)}{\sinh kd} \right] \cos(kx - \sigma t) \quad (2.26)$$

where  $H/2$  is the orbit radius for a particle at the surface of a deep water wave. The position coordinates are evaluated for the orbit of the particle that is passing through the point  $x, -z$  at that instant, but the small-amplitude assumptions allow us to assume that these coordinates [given by Eqs. (2.25) and (2.26)] apply to the orbit mean position.

As a wave propagates from deep water into shallow water, the particle orbit geometries undergo the transformation depicted in Figure 2.2. In deep water the orbits are circular throughout the water column but decrease in diameter with increasing distance below the water surface, to approximately die out at a distance

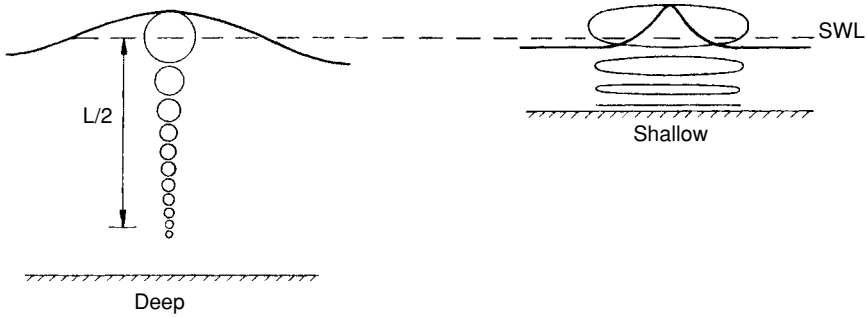


Figure 2.2. Deep and shallow water surface profiles and particle orbits.

of  $L/2$ . In transitional to shallow water, the orbits reach the bottom and become elliptical—with the ellipses becoming flatter near the bottom. At the bottom the particles follow a reversing horizontal path. (This is for the assumed irrotational motion—for real conditions a bottom boundary layer develops and the horizontal dimension of the particle orbit reduces to zero at the bottom.) Since the terms in brackets are the same for the respective velocity, acceleration, and displacement equations, the particle velocity and acceleration component magnitudes demonstrate the same spatial change as do the displacement coordinates.

According to the small-amplitude theory surface waves have a sinusoidal surface profile. This is reasonable for low steepness waves in deep water. But, for steeper deep water waves or as waves propagate into transitional and shallow water the surface profile becomes trochoidal, having long flat troughs and shorter peaked crests (see Figure 2.2). The amplitude of the crest increases while the amplitude of the trough decreases. In transitional and shallow water, particles still move in essentially closed orbits. Since they must travel the same distance forward under the crest in less time (owing to the trochoidal profile) as they travel back under the trough in more time, peak velocities under the wave crest will exceed those under the trough. As with the profile asymmetry, this velocity asymmetry is not predicted by the small amplitude wave theory.

It is useful to consider the deep and shallow water limits for the term in brackets in the particle velocity, acceleration, and orbit displacement equations. At these limits we have:

$$\text{Deep water: } \frac{\cosh k(d+z)}{\sinh kd} = \frac{\sinh k(d+z)}{\sinh kd} = e^{kz} \quad (2.27)$$

$$\text{Shallow water: } \frac{\cosh k(d+z)}{\sinh(kd)} = \frac{1}{kd} \quad (2.28)$$

$$\frac{\sinh k(d+z)}{\sinh kd} = 1 + \frac{z}{d} \quad (2.29)$$

Substitution of Eq. (2.27) into Eqs. (2.21) to (2.26) indicates that, in deep water, the particle velocity, acceleration, and orbit displacement decay exponentially with increasing distance below the still water line. At  $z = -L/2$  they are reduced to 4.3% of their value at the surface.

Substitution of Eqs. (2.28) and (2.29) into Eqs. (2.21) and (2.22) respectively yields (after some algebraic manipulation) the following equations for water particle velocity in shallow water:

$$u = \frac{H}{2} \sqrt{\frac{g}{d}} \cos(kx - \sigma t) \quad (2.30)$$

$$w = \frac{\pi H}{T} \left(1 + \frac{z}{d}\right) \sin(kx - \sigma t) \quad (2.31)$$

Equation (2.30) indicates that, in shallow water, the horizontal component of water particle velocity is constant from the water surface to the bottom. The vertical component of particle velocity can be seen from Eq. (2.31) to decrease linearly from a maximum at the water surface to zero at the bottom. Similar statements can be made for the particle acceleration and orbit dimensions.

#### *Pressure Field*

Substitution of the velocity potential into the linearized form of the equation of motion [Eq. (2.5) without the velocity squared terms] yields the following equation for the pressure field in a wave:

$$p = -\rho g z + \frac{\rho g H}{2} \left[ \frac{\cosh k(d+z)}{\cosh kd} \right] \cos(kx - \sigma t) \quad (2.32)$$

The first term on the right gives the normal hydrostatic pressure variation and the second term is the dynamic pressure variation owing to the wave-induced particle acceleration. These components are plotted in Figure 2.3 for vertical sections through the wave crest and trough. Since particles under the crest are accelerating downward, a downward dynamic pressure gradient is required. The reverse is true under a wave trough. Halfway between the crest and trough the acceleration is horizontal so the vertical pressure distribution is hydrostatic. Equation (2.32) is not valid above the still water line owing to the linearization of the DSBC and its application at the still water line. Above the still water line the pressure must regularly decrease to zero at the water surface.

In deep water, the dynamic pressure reduces to near zero at  $z = -L/2$ . A pressure gage at this depth would essentially measure the static pressure for the given depth below the still water line. A pressure gage (located above  $-L/2$ ) can be used as a wave gage. The period of the pressure fluctuation is the wave period which can be used to calculate the wave length from the dispersion equation. The

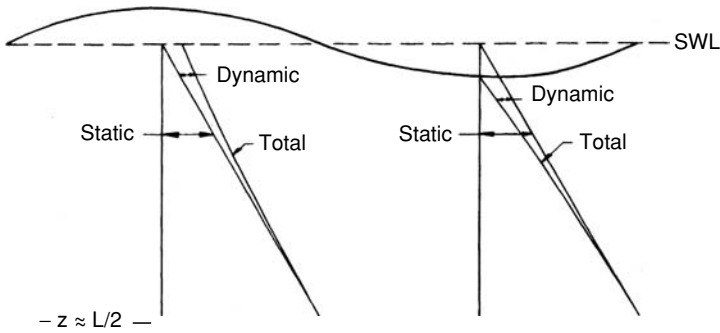


Figure 2.3. Deep water wave vertical pressure distributions.

wave height can then be calculated from Eq. (2.32), assuming the position of the gage, the wave period and length, and the water depth are known.

Note that the term in brackets differs from the terms in brackets for the particle velocity, acceleration, and orbit displacement equations. At the deep and shallow water limits we have,

$$\frac{\cosh k(d+z)}{\cosh kd} = e^{kz} \text{ (deep water)} \quad (2.33)$$

$$= 1 \text{ (shallow water)}$$

Thus, from the small-amplitude wave theory, in deep water there is also an exponential decay in the dynamic pressure with distance below the still water line. In shallow water the total pressure distribution is given by

$$p = \rho g(\eta - z) \quad (2.34)$$

## 2.5 Energy, Power, and Group Celerity

An important characteristic of gravity waves is that they have mechanical energy and that this energy is transmitted forward as they propagate. It is important to be able to quantify this energy level and the rate of energy transmission (energy flux or power) for a given wave height and period and water depth.

### Wave Energy

The total mechanical energy in a surface gravity wave is the sum of the kinetic and potential energies. Equations for each may be derived by considering Figure 2.4. The kinetic energy for a unit width of wave crest and for one wave length  $E_k$

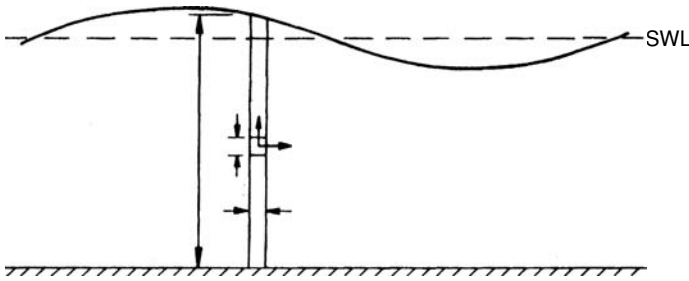


Figure 2.4. Definition sketch for wave energy derivation.

is equal to the integral over one wave length and the water depth of one-half times the mass of a differential element times the velocity of that element squared. Thus

$$E_k = \int_0^L \int_{-d}^0 \frac{1}{2} \rho dx dz (u^2 + w^2)$$

where the upper limit of the vertical integral is taken as zero in accord with the assumptions of the small-amplitude wave theory. Inserting the velocity terms [Eqs. (2.21) and (2.22)], integrating, and performing the required algebraic manipulation yields the kinetic energy

$$E_k = \frac{\rho g H^2 L}{16}$$

If we subtract the potential energy of a mass of still water (with respect to the bottom) from the potential energy of the wave form shown in Figure 2.4 we will have the potential energy due solely to the wave form. This gives the potential energy per unit wave crest width and for one wave length  $E_p$  as

$$E_p = \int_0^L \rho g (d + \eta) \left( \frac{d + \eta}{2} \right) dx - \rho g L d \left( \frac{d}{2} \right)$$

The surface elevation as a function of  $x$  is given by Eq. (2.10) with  $t = 0$ . Performing the integration and simplifying yields

$$E_p = \frac{\rho g H^2 L}{16}$$

Thus, the kinetic and potential energies are equal and the total energy in a wave per unit crest width  $E$  is

$$E = E_k + E_p = \frac{\rho g H^2 L}{8} \tag{2.35}$$

A wave propagating through a porous structure, for example, where the water depth is the same on both sides of the structure, will have the same period and wave length on both sides. Thus, a reduction of wave energy because of reflection from the structure and viscous dissipation within the structure will result in a decrease in the wave height. A 50% reduction in wave energy would result in only a 29% decrease in the wave height because the wave energy is proportional to the wave height squared.

Both the kinetic and potential energies are variable from point to point along a wave length. However, a useful concept is the average energy per unit surface area given by

$$\bar{E} = \frac{E}{L(1)} = \frac{\rho g H^2}{8} \quad (2.36)$$

This is usually known as the energy density or specific energy of a wave. Equations (2.35) and (2.36) apply for deep to shallow water within the limits of the small-amplitude wave theory.

#### Wave Power

Wave power  $P$  is the wave energy per unit time transmitted in the direction of wave propagation. Wave power can be written as the product of the force acting on a vertical plane normal to the direction of wave propagation times the particle flow velocity across this plane. The wave-induced force is provided by the dynamic pressure (total pressure minus hydrostatic pressure) and the flow velocity is the horizontal component of the particle velocity. Thus

$$P = \frac{1}{T} \int_0^T \int_{-d}^0 (p + \rho g z) u dz dt$$

where the term in parentheses is the dynamic pressure. Inserting the dynamic pressure from Eq. (2.32) and the horizontal component of velocity from Eq. (2.21) and integrating leads to

$$P = \frac{\rho g H^2 L}{16T} \left( 1 + \frac{2kd}{\sinh 2kd} \right)$$

or

$$P = \frac{E}{2T} \left( 1 + \frac{2kd}{\sinh 2kd} \right) \quad (2.37)$$

Letting

$$n = \frac{1}{2} \left( 1 + \frac{2kd}{\sinh 2kd} \right) \quad (2.38)$$



Equation (2.37) becomes

$$P = \frac{nE}{T} \quad (2.39)$$

The value of  $n$  increases as a wave propagates toward the shore from 0.5 in deep water to 1.0 in shallow water. Equation (2.39) indicates that  $n$  can be interpreted as the fraction of the mechanical energy in a wave that is transmitted forward each wave period.

As a train of waves propagates forward the power at one point must equal the power at a subsequent point minus the energy added, and plus the energy dissipated and reflected per unit time between the two points. For first-order engineering analysis of waves propagating over reasonably short distances it is common to neglect the energy added, dissipated, or reflected, giving

$$P = \left(\frac{nE}{T}\right)_1 = \left(\frac{nE}{T}\right)_2 = \text{constant} \quad (2.40)$$

Equation (2.40) indicates that, for the assumptions made, as a two-dimensional wave travels from deep water to the nearshore the energy in the wave train decreases at a rate inversely proportional to the increase in  $n$  since the wave period is constant.

As waves approach the shore at an angle and propagate over irregular hydrography they vary three-dimensionally owing to refraction. (See Chapter 4 for further discussion and analysis of wave refraction.) If we construct lines that are normal or orthogonal to the wave crests as a wave advances and assume that no energy propagates along the wave crest (i.e., across orthogonal lines) the energy flux between orthogonals can be assumed to be constant. If the orthogonal spacing is denoted by  $B$ , Eq. (2.40) can be written

$$\left(\frac{BnE}{T}\right)_1 = \left(\frac{BnE}{T}\right)_2 = \text{constant}$$

Inserting the wave energy from Eq. (2.35) yields

$$\frac{H_1}{H_2} = \sqrt{\frac{n_2 L_2}{n_1 L_1}} \sqrt{\frac{B_2}{B_1}} \quad (2.41)$$

The first term on the right represents the effects of shoaling and the second term represents the effects of orthogonal line convergence or divergence owing to refraction. These are commonly called the coefficient of shoaling  $K_s$  and the coefficient of refraction  $K_r$ , respectively.

Equation (2.41) allows us to calculate the change in wave height as a wave propagates from one water depth to another depth. Commonly, waves are

predicted for some deep water location and then must be transformed to some intermediate or shallow water depth nearshore using Eq. (2.41). For this, Eq. (2.41) becomes

$$\frac{H}{H_o} = \sqrt{\frac{L_o}{2nL}} \sqrt{\frac{B_o}{B}} \tag{2.42}$$

or

$$\frac{H}{H_o} = \frac{H'}{H'_o} \sqrt{\frac{B_o}{B}}$$

where the prime denotes the change in wave height from deep water to the point of interest considering only two-dimensional shoaling effects.

Figure 2.5 is a plot of  $H/H_o$  versus  $d/L$  and  $d/L_o$  from deep to shallow water. Initially, as a wave enters intermediate water depths the wave height decreases because  $n$  increases at a faster rate than  $L$  decreases [see Eq. (2.42)].  $H/H_o$  reaches a minimum value of 0.913 at  $d/L = 0.189$  ( $d/L_o = 0.157$ ). Shoreward of this point the wave height grows at an ever-increasing rate until the wave becomes unstable and breaks.

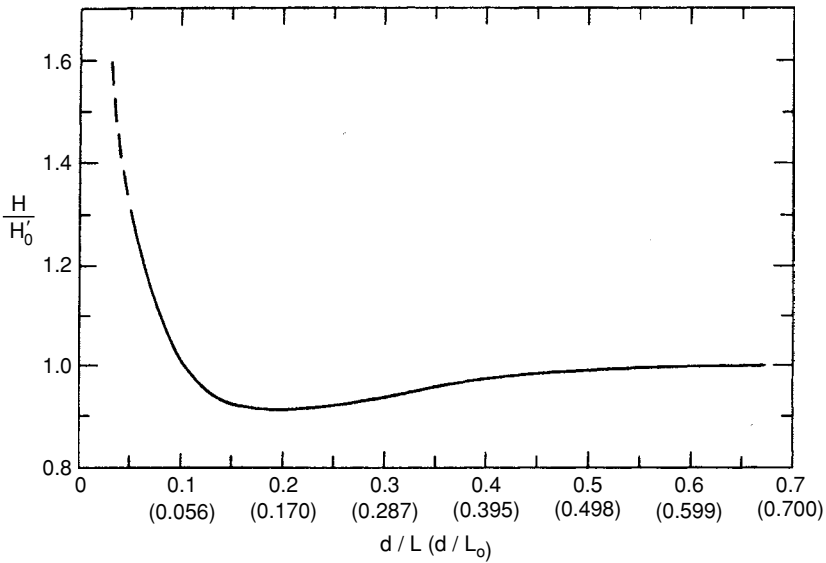


Figure 2.5. Dimensionless wave height versus relative depth for two-dimensional wave transformation.

**Example 2.5-1**

Consider the wave from Example 2.3–1 when it has propagated into a water depth of 10 m without refracting and assuming energy gains and losses can be ignored. Determine the wave height and the water particle velocity and pressure at a point 1 m below the still water level under the wave crest. (Assume fresh water.)

Solution:

From Example 2.3–1 we have  $L_o = 156$  m and Eq. (2.14) gives

$$L = \frac{9.81(10)^2}{2\pi} \tanh \frac{2\pi(10)}{L}$$

which can be solved by trial to yield  $L = 93.3$  m. Then,  $k = 2\pi/93.3 = 0.0673 \text{ m}^{-1}$  and from Eq. (2.38)

$$n = \frac{1}{2} \left( 1 + \frac{2(0.0673)(10)}{\sinh(2(0.0673)(10))} \right) = 0.874$$

With  $K_r = 1$ , Eq. (2.42) yields

$$H = 2 \sqrt{\frac{156}{2(0.874)(93.3)}} = 1.97 \text{ m}$$

At the crest of the wave  $\cos(kx - \sigma t) = 1$ , and  $z = -1$ , so Eq. (2.21) gives

$$u = \frac{\pi(1.97)}{10} \left[ \frac{\cosh(0.0673)(9)}{\sinh(0.0673)(10)} \right] = 1.01 \text{ m/s}$$

which is the total particle velocity since  $w = 0$  under the wave crest. Equation (2.32) gives

$$\begin{aligned} P &= -1000(9.81)(-1) + \frac{1000(9.81)(1.97)}{2} \left[ \frac{\cosh(0.0673)9}{\cosh(0.0673)10} \right] \\ &= 19,113 \text{ N/m}^2 \end{aligned}$$

Remember, Eqs. (2.40) to (2.42) neglect energy transfer to and from waves by surface and bottom effects. The nature of these effects is discussed briefly below. Bottom effects, of course, require that the water depth be sufficiently shallow for a strong interaction between the wave train and the bottom.

### *Wave Reflection*

If the bottom is other than horizontal, a portion of the incident wave energy will be reflected seaward. This reflection is generally negligible for wind wave periods on typical nearshore slopes. However, for longer period waves and steeper bottom slopes wave reflection would not be negligible. Any sharp bottom irregularity such as a submerged structure of sufficient size will also reflect a significant portion of the incident wave energy.

### *Wind Effects*

Nominally, if the wind has a velocity component in the direction of wave propagation that exceeds the wave celerity the wind will add energy to the waves. If the velocity component is less than the wave celerity or the wind blows opposite to the direction of wave propagation the wind will remove energy from the waves. For typical nonstormy wind conditions and the distances from deep water to the nearshore zone found in most coastal locations, the wind effect can be neglected in the analysis of wave conditions nearshore.

### *Bottom Friction*

As the water particle motion in a wave interacts with a still bottom, an unsteady oscillatory boundary layer develops near the bottom. For long period waves in relatively shallow water this boundary layer can extend up through much of the water column. But, for typical wind waves the boundary layer is quite thin relative to the water depth, and if propagation distances are not too long and the bottom is not too rough, bottom friction energy losses can be neglected.

### *Bottom Percolation*

If the bottom is permeable to a sufficient depth, the wave-induced fluctuating pressure distribution on the bottom will cause water to percolate in and out of the bottom and thus dissipate wave energy.

### *Bottom Movement*

When a wave train propagates over a bottom consisting of soft viscous material (such as the mud deposited at the Mississippi River Delta) the fluctuating pressure on the bottom can set the bottom in motion. Viscous stresses in the soft bottom dissipate energy provided by the waves.

### *Wave Group Celerity*

Consider a long constant-depth wave tank in which a small group of deep water waves is generated. As the waves travel along the tank, waves in the front of the group will gradually decrease in height and, if the tank is long enough, disappear

in sequence starting with the first wave in the group. As the waves in the front diminish in height, new waves will appear at the rear of the group and commence to grow. One new wave will appear each wave period so the total number of waves in the group will continually increase. This phenomenon causes the wave group to have a celerity that is less than the celerity of the individual waves in the group. Since the total energy in the group is constant (neglecting dissipation) the average height of the waves in the group will continually decrease.

An explanation for this phenomenon can be found in the fact that only a fraction  $[n; \text{ see Eq. (2.39)}]$  of the wave energy goes forward with the wave as it advances each wave length. Thus, the first wave in the group is diminished in height by the square root of  $n$  during the advance of one wave length. Waves in the group lose energy to the wave immediately behind and gain energy from the wave in front. The last wave in the group leaves energy behind so, relative to the group, a new wave appears each  $T$  seconds and gains additional energy as time passes.

A practical consequence of the deep water group celerity being less than the phase celerity of individual waves is that when waves are generated by a storm, prediction of their arrival time at a point of interest must be based on the group celerity.

To develop an equation for calculating the group celerity  $C_g$  consider two trains of monochromatic waves having slightly different periods and propagating in the same direction. Figure 2.6 shows the wave trains separately (above) and superimposed (below) when propagating in the same area. The superimposition of the two wave trains results in a beating effect in which the waves are alternately in and out of phase. This produces the highest waves when the two components are in phase, with heights diminishing in the forward and backward directions to zero height where the waves are exactly out of phase. The result is a group of waves advancing at a celerity  $C_g$ . If you follow an individual wave in the wave group its amplitude increases to a peak and then diminishes as it passes through the group and disappears at the front of the group.

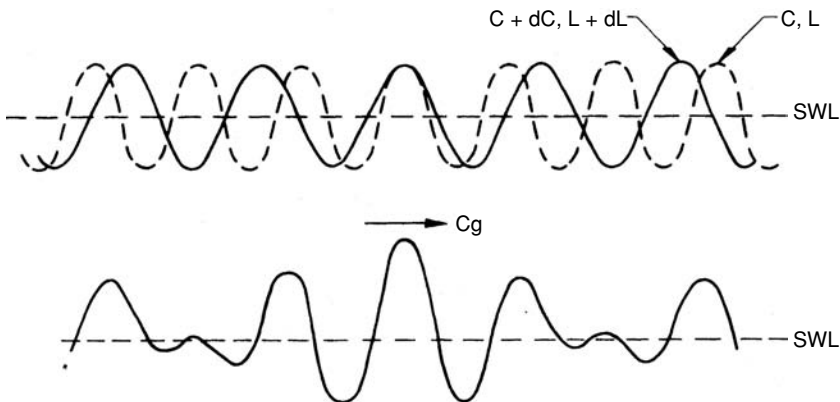


Figure 2.6. Two wave trains shown separately and superimposed.

Referring to Figure 2.6, the time required for the lag between the two components  $dL$  to be made up is  $dt$ , where  $dt$  equals the difference in component lengths divided by  $dC$ , the difference in component celerities, i.e.,  $dt = dL/dC$ . The group advances a distance  $dx$  in the time  $dt$ , where  $dx$  is the distance traveled by the group in the time interval  $dt$  minus the one wave length that the peak wave dropped back (as the in-phase wave drops back one wave length each period). This can be written

$$dx = \left[ \frac{(C + dC) + C}{2} \right] dt - \frac{(L + dL) + L}{2} \approx Cdt - L.$$

if  $dL$  and  $dC$  are very small compared to  $L$  and  $C$ . Then,

$$C_g = \frac{dx}{dt} = \frac{Cdt - L}{dt} = C - \frac{L}{dt}$$

since  $dt = dL/dC$  this leads to

$$C_g = C - L \left( \frac{dC}{dL} \right) \quad (2.43)$$

In shallow water, small-amplitude waves are not dispersive ( $dC/dL = 0$ ) so  $C_g = C$ . In deep water  $dC/dL = C/2L$  [from Eq. (2.15)] so the group celerity is half of the phase celerity. For a general relationship for the group celerity, employing the dispersion relationship with Eq. (2.43) yields

$$C_g = \frac{C}{2} \left( 1 + \frac{2kd}{\sinh 2kd} \right) \quad (2.44)$$

Thus, with  $n$  as defined in Eq. (2.38)

$$C_g = nC \quad (2.45)$$

So  $n$  is also the ratio of the wave group celerity to the phase celerity. Another way to look at this is that the wave energy is propagated forward at the group celerity.

## 2.6 Radiation Stress and Wave Setup

In fluid flow problems, some analyses are best carried out by energy considerations (e.g., head loss along a length of pipe) and some by momentum considerations (e.g., force exerted by a water jet hitting a wall). Similarly, for waves it is

better to consider the flux of momentum for some problem analyses. For wave analyses, the flux of momentum is commonly referred to as the wave “radiation stress” which may be defined as “the excess flow of momentum due to the presence of waves” (Longuet-Higgins and Stewart, 1964). Problems commonly addressed by the application of radiation stress include the lowering (setdown) and raising (setup) of the mean water level that is induced by waves as they propagate into the nearshore zone, the interaction of waves and currents, and the alongshore current in the surf zone induced by waves obliquely approaching the shore.

### Radiation Stress

The horizontal flux of momentum at a given location consists of the pressure force acting on a vertical plane normal to the flow plus the transfer of momentum through that vertical plane. The latter is the product of the momentum in the flow and the flow rate across the plane. From classical fluid mechanics, the momentum flux from one location to another will remain constant unless there is a force acting on the fluid in the flow direction to change the flux of momentum.

If we divide the momentum flux by the area of the vertical plane through which flow passes, we have for the  $x$  direction

$$p + \rho u^2$$

For a wave, we want the excess momentum flux owing to the wave, so the radiation stress  $S_{xx}$  for a wave propagating in the  $x$  direction becomes

$$S_{xx} = \int_{-d}^{\eta} \overline{(p + \rho u^2)} dz - \int_{-d}^0 \rho g dz \quad (2.46)$$

where the subscript  $xx$  denotes the  $x$ -directed momentum flux across a plane defined by  $x = \text{constant}$ . In Eq. (2.46)  $p$  is the total pressure given by Eq. (2.32) so the static pressure must be subtracted to obtain the radiation stress for only the wave. The overbar denotes that the first term on the right must be averaged over the wave period. Inserting the pressure and the particle velocity from Eq. (2.21) leads to (Longuet-Higgins and Stewart, 1964)

$$S_{xx} = \frac{\rho g H^2}{8} \left( \frac{1}{2} + \frac{2kd}{\sinh 2kd} \right) = \bar{E} \left( 2n - \frac{1}{2} \right) \quad (2.47)$$

For a wave traveling in the  $x$ -direction there also is a  $y$ -directed momentum flux across a plane defined by  $y = \text{constant}$ . This is

$$S_{yy} = \frac{\rho g H^2}{8} \left( \frac{kd}{\sinh kd} \right) = \bar{E} (n - 1/2) \quad (2.48)$$

The radiation stress components  $S_{xy}$  and  $S_{yx}$  are both zero. Note that in deep water Eqs. (2.47) and (2.48) become

$$S_{xx} = \frac{\bar{E}}{2}, S_{yy} = 0 \quad (2.49)$$

And in shallow water they become

$$S_{xx} = \frac{3\bar{E}}{2}, S_{yy} = \frac{\bar{E}}{2} \quad (2.50)$$

so, like wave energy, the radiation stress changes as a wave propagates through water of changing depth (as well as when a force is applied).

If a wave is propagating in a direction that is situated at an angle to the specified  $x$  direction, the radiation stress components become

$$\begin{aligned} S_{xx} &= \bar{E}[n(\cos^2 \theta + 1) - 1/2] \\ S_{yy} &= \bar{E}[n(\sin^2 \theta + 1) - 1/2] \\ S_{xy} &= \frac{\bar{E}}{2} n \sin^2 \theta = \bar{E} n \sin \theta \cos \theta \end{aligned} \quad (2.51)$$

where  $\theta$  is the angle between the direction of wave propagation and the specified  $x$  direction.

### *Wave Setup*

When a train of waves propagates toward the shore, at some point, depending on the wave characteristics and nearshore bottom slope, the waves will break. Landward of the point of wave breaking a surf zone will form where the waves dissipate their energy as they decay across the surf zone.

As the waves approach the breaking point there will be a small progressive set down of the mean water level below the still water level. This setdown is caused by an increase in the radiation stress owing to the decreasing water depth as the waves propagate toward the shore. The setdown is maximum just seaward of the breaking point. In the surf zone, there is a decrease in radiation stress as wave energy is dissipated. This effect is stronger than the radiation stress increase owing to continued decrease in the water depth. The result is a progressive increase or setup of the mean water level above the still water level in the direction of the shore. This surf zone setup typically is significantly larger than the setdown that occurs seaward of the breaking point.

The equations that predict the wave-induced nearshore setdown and setup can be developed by considering the horizontal momentum balance for two-dimensional waves approaching the shore (Longuet-Higgins and Stewart, 1964). The



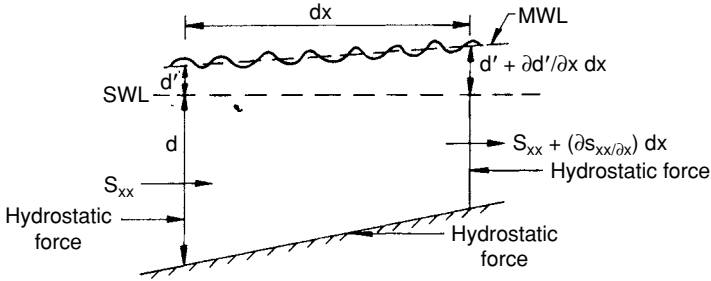


Figure 2.7. Force balance for wave-induced setup analysis.

net force caused by the cyclic bottom shear stress is reasonably neglected. Consider Figure 2.7 which shows a shore-normal segment of length  $dx$  with a setup  $d'$ . The forces and related change in the radiation stress at the boundaries are as shown. Writing the force-momentum flux balance for a segment of unit width parallel to the shore yields

$$\frac{\rho g}{2}(d + d')^2 - \frac{\rho g}{2}\left(d + d' + \frac{\partial d'}{\partial x} dx\right)^2 = \frac{\partial S_{xx}}{\partial x} dx$$

where the two terms on the left are the fore and aft hydrostatic forces and the term on the right is the resulting change in radiation stress. Assuming  $d \gg d'$  and neglecting higher order terms this leads to

$$\frac{dS_{xx}}{dx} + \rho g d \frac{dd'}{dx} = 0 \tag{2.52}$$

Equation (2.52) basically relates the change in radiation stress (caused either by a depth change and/or wave energy dissipation) to the resulting slope of the mean water level. This equation applies to the regions before and after the breaking point.

For the region just seaward of the breaking point assume that the wave power is constant and employ Eq. (2.47) to integrate Eq. (2.52). This leads to the setdown of the mean water level given by

$$d' = -\frac{1}{8} \frac{H^2 k}{\sinh 2kd} \tag{2.53}$$

For deep water, Eq. (2.53) shows that the setdown is zero irrespective of the wave height because the  $\sinh$  term is very large. In shallow water, which may be used as an estimate of the conditions just prior to breaking,  $d' = -H^2/16d$ .

In the surf zone, the rate of energy dissipation by wave breaking will depend on the type of breaker that occurs. This rate of energy dissipation is complex and typically nonuniform. However, to reasonably develop an equation for wave setup, we will assume that the wave height across the surf zone is proportional to the depth below the local mean water level, i.e.,  $H = \gamma(d + d')$ . A reasonable value for  $\gamma$  is 0.9 (see Section 2.8). Also, we will assume that shallow water wave conditions exist so  $S_{xx} = 3E/2$ . These assumptions lead to a solution to Eq. (2.52) given by

$$\frac{dd'}{dx} = \left(1 + \frac{8}{3\gamma^2}\right)^{-1} \frac{dd}{dx} \quad (2.54)$$

which gives the slope of the mean water level as a function of the bottom slope in the surf zone.

### Example 2.6-1

Consider a wave that has a height of 2 m in water 2.2 m deep (below the mean water level) as it is about to break. The nearshore bottom slope through the surf zone is 0.02. Find the setdown at the breaker point and the setup (above the still water line) at the still water line contour of the shore. Assume shallow water wave conditions throughout.

#### Solution:

The setdown at the breaker line is

$$d' = -\frac{(2)^2}{16(2.2)} = -0.11 \text{ m}$$

The slope of the rising mean water level through the surf zone is

$$\frac{dd'}{dx} = \left(1 + \frac{8}{3(0.9)^2}\right)^{-1} (0.02) = 0.0047$$

For a bottom slope of 0.02 the still water line at the beach will be  $(2.2 + 0.11)/(0.02) = 115.5$  m shoreward of the breaker line. At this point the mean water level will be  $-0.11 + (115.5)(0.0047) = 0.43$  m above the still water level.

Equations (2.53) and (2.54) indicate that the setdown is a function of the incident wave height but the slope of the mean water level through the surf zone is not. However, higher incident waves will break further seaward so the same mean water level slope will yield a higher mean water level throughout the surf zone.

It should be kept in mind that the development of Eqs. (2.53) and (2.54) employed the small-amplitude wave theory, which is less accurate in the near-shore zone. However, experiments conducted by Saville (1961) in a large two-dimensional wave tank yielded results that favorably agree with predictions from these equations. Also, the equations apply to waves approaching normal to the shore. If the waves approach obliquely to the shore, only the shore normal component of radiation stress will induce setdown and setup.

## 2.7 Standing Waves, Wave Reflection

A solid structure such as a vertical wall will reflect an incident wave, the amplitude of the reflected wave depending on the wave and wall characteristics. When the reflected wave passes through the incident wave a standing wave will develop. It is worthwhile to investigate the nature of wave reflection and standing waves, particularly the resulting surface profile and particle kinematics of the resulting wave motion as well as the dependence of the reflected wave characteristics on the reflecting structure makeup.

### *Standing Waves*

Consider two waves having the same height and period but propagating in opposite (+/−) directions along the  $x$  axis. When these two waves are superimposed the resulting motion is a standing wave as depicted in Figure 2.8a. The water surface oscillates from one position to the other and back to the original position in one wave period. The arrows indicate the paths of water particle oscillation. Under a nodal point particles oscillate in a horizontal plane while under an antinodal point they oscillate in a vertical plane. When the surface is at one of the two envelope positions shown, water particles instantaneously come to rest and all of the wave energy is potential. Halfway between the envelope positions the water surface is horizontal and all wave energy is kinetic. The net energy flux (if the two component waves are identical) is zero.

The velocity potential for a standing wave can be obtained by adding the velocity potentials for the two component waves that move in opposite directions. This yields

$$\phi = \frac{gH}{\sigma} \left[ \frac{\cosh k(d+z)}{\cosh kd} \right] \cos kx \sin \sigma t \quad (2.55)$$

With the velocity potential given by Eq. (2.55), we can derive the various standing wave characteristics in the same way as for a progressive wave. This yields a surface profile given by

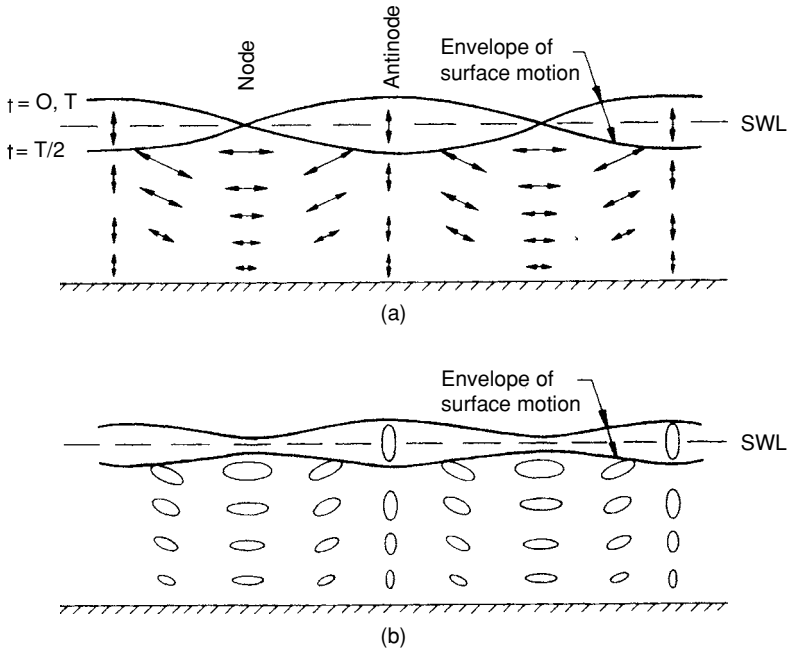


Figure 2.8. Standing wave particle motion and surface profile envelope. (a)  $C_r = 1.0$ , (b)  $C_r < 1.0$ .

$$\eta = H \cos kx \sin \sigma t; \quad (2.56)$$

horizontal and vertical velocity components given by

$$u = \frac{\pi H}{T} \left[ \frac{\cosh k(d+z)}{\sinh kd} \right] \sin kx \sin \sigma t \quad (2.57)$$

and

$$w = \frac{\pi H}{T} \left[ \frac{\sinh k(d+z)}{\sinh kd} \right] \cos kx \sin \sigma t; \quad (2.58)$$

a pressure field given by

$$p = -\rho g z + \rho g H \left[ \frac{\cosh k(d+z)}{\cosh kd} \right] \cos kx \cos \sigma t; \quad (2.59)$$

and horizontal and vertical particle displacements given by

$$\zeta = -H \left[ \frac{\cosh k(d+z)}{\sinh kd} \right] \sin kx \cos \sigma t \quad (2.60)$$

and

$$\varepsilon = H \left[ \frac{\sinh k(d+z)}{\sinh kd} \right] \cos kx \cos \sigma t \quad (2.61)$$

Equations (2.56) through (2.61) demonstrate some interesting features of a standing wave. If the component progressive wave heights are  $H$ , the standing wave height is  $2H$ . The terms in brackets that define wave decay/shoaling effects are the same as for the equivalent progressive wave characteristic. However, at a given point  $(x, -z)$  the horizontal and vertical velocity and displacement components are in phase, rather than being  $90^\circ$  out of phase as is the case for progressive waves. The pressure is hydrostatic under a node where particle acceleration is horizontal; but under an antinode there is a fluctuating vertical component of dynamic pressure.

The energy in a standing wave per unit crest width and for one wave length is

$$E = \frac{\rho g H^2 L}{4} \quad (2.62)$$

where, again,  $H$  is the height of a component progressive wave. This consists of potential and kinetic energy components given by

$$E_p = \frac{\rho g H^2 L}{4} \cos^2 \sigma t \quad (2.63)$$

and

$$E_k = \frac{\rho g H^2 L}{4} \sin^2 \sigma t \quad (2.64)$$

Equations (2.63) and (2.64) demonstrate, as discussed above, that at  $t = 0, T/2, \dots E = E_p$  and at  $T = T/4, 3T/4, \dots E = E_k$ .

### Wave Reflection

In a standing wave, the particle velocity under an antinode is always vertical. If a frictionless, rigid, vertical, impermeable wall were placed at the antinode the water particle motion would be unaffected. Thus, we would have a standing wave caused by the reflection of a progressive wave from the wall. The particle velocity and the pressure distribution along the wall would be given by Eqs. (2.58) and (2.59), respectively with  $\cos kx = 1$ .

As the wall slope decreases, the wall becomes elastic and/or the wall surface becomes rough and permeable, the reflected wave height becomes less than the incident height. The surface profile and the particle motion in this standing wave are depicted in Figure 2.8b. We can define a reflection coefficient  $C_r$  as

$$C_r = \frac{H_r}{H_i} \quad (2.65)$$

where  $H_r$  is the reflected wave height and  $H_i$  is the incident wave height (i.e., the reflection coefficient will be equal to or less than unity). Considering Figure 2.8, as the reflection coefficient decreases from unity to zero the particle trajectories transition from those for a pure standing wave to those of the orbital pattern for a pure progressive wave.

The envelope height at the antinode for a standing wave is  $H_i + H_r$  and the nodal envelope height is  $H_i - H_r$  (Ippen, 1966). It can also be shown the reflection coefficient equals the difference between the two envelope heights divided by the sum of the two envelope heights. When wave tank tests are being run with monochromatic waves and a reflecting structure, the wet mark on the side of the tank displays the upper envelope shown in Figure 2.8b and is an indicator of the amount of wave reflection from the structure. A wave gage mounted on a carriage and slowly moved at least one wave length along the wave tank will measure the node and antinode envelope heights which can be used to calculate the reflection coefficient for a monochromatic wave.

## 2.8 Wave Profile Asymmetry and Breaking

As a wave propagates into intermediate and shallow water an initial profile asymmetry develops around the horizontal axis as the wave crest steepens and the wave trough flattens. Further on an asymmetry also develops around a vertical axis through the wave crest (neither asymmetry is defined by the small amplitude wave theory). These asymmetries ultimately lead to wave instability and breaking.

### *Profile Asymmetry*

Figure 2.9 shows a typical asymmetric wave profile as a wave propagates through relatively shallow water prior to breaking. Besides the vertical asymmetry resulting in a crest amplitude that exceeds half the wave height, the front face of the wave becomes steeper than the back face and the distance (in the direction of wave propagation) from crest to trough is less than the distance from trough to crest. These asymmetries increase as the wave moves into shallower and shallower water. They also contribute to increased particle velocities at the wave crest and ultimately to crest instability and wave breaking.

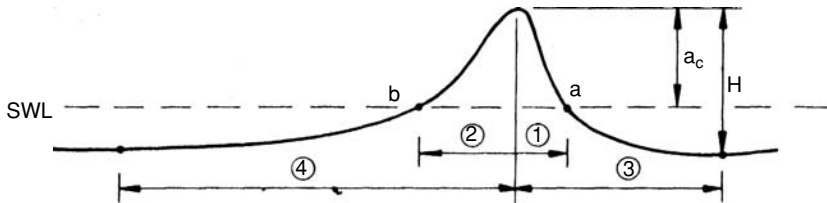


Figure 2.9. Definition of profile asymmetry terms.

Wave tank experiments were conducted by Adeyemo (1968) for intermediate depth waves shoaling on slopes from 1:18 to 1:4. These slopes are somewhat steeper than found in most nearshore areas. He presented his data in terms of four values defined as follows (see Figure 2.9):

$$\text{Vertical asymmetry} = a_c/H$$

$$\text{Slope asymmetry} = 0.5(\text{slope } a + \text{slope } b)$$

$$\text{Horizontal asymmetry (1)} = \text{distance 1}/\text{distance 2}$$

$$\text{Horizontal asymmetry (2)} = \text{distance 3}/\text{distance 4}$$

The slopes were stated in radians with slope  $b$  being a positive value and slope  $a$  being a negative value.

The experiments showed the vertical asymmetry continuously increased as the wave shoaled, reaching a maximum of between 0.62 and 0.74 at breaking. In shallower depths ( $d/L < 0.10$ ) wave vertical asymmetry was greater for flatter slopes. Flatter slopes mean that the wave has more travel time for the asymmetry to develop. Thus, for natural beach slopes that are flatter than the experimental slopes one might expect vertical asymmetries greater than the 0.62 to 0.74 values reported. The slope and horizontal asymmetries also continuously increased as the wave shoaled; but, as opposed to vertical asymmetries steeper bottom slopes caused greater slope and horizontal asymmetries.

### Wave Breaking

If a wave has sufficient height in any water depth it will break. In deep water, for a given wave period, the crest particle velocity is proportional to the wave height. From the small-amplitude wave theory, the wave celerity is independent of the wave height. So, as the wave height increases the crest particle velocity will eventually equal the wave celerity and the wave will break. In shallow water, as the water depth decreases the crest particle velocity increases and the wave celerity decreases, leading to instability and breaking.

Miche (1944) developed a simple equation for wave breaking in any water depth given by

$$\left(\frac{H}{L}\right)_{\max} = \frac{1}{7} \tanh \frac{2\pi d}{L} \quad (2.66)$$

This equation ignores the bottom slope which, as discussed above, affects development of wave asymmetry and breaking as a wave shoals. As a consequence, Eq. (2.66) gives a good indication of deep water breaking limits on wave height but only an approximate rule of thumb for shallow water breaking conditions. For deep water Eq. (2.66) reduces to

$$\left(\frac{H_o}{L_o}\right)_{\max} = \frac{1}{7} \quad (2.67)$$

indicating that the maximum wave height in deep water is limited to one-seventh of the wave length. In shallow water we have

$$\left(\frac{H}{L}\right)_{\max} = \frac{1}{7} \left(\frac{2\pi d}{L}\right)$$

or

$$\left(\frac{H}{d}\right)_{\max} = 0.9 \quad (2.68)$$

Thus, in shallow water wave heights are limited by the water depth. This is often an important consideration in the design of structures built seaward of the water's edge. No matter how high the deep water wind generated waves are, the highest wave that can reach the structure is dependent primarily on the water depth in front of the structure. Thus, as structures are extended further seaward they tend to be exposed to higher, more damaging waves.

Waves breaking on a beach are commonly classified into three categories (U.S. Army Coastal Engineering Research Center, 1984) depicted in Figure 2.10. These three breaker classes are:

*Spilling.* As breaking commences, turbulence and foam appear at the wave crest and then spread down the front face of the wave as it propagates toward the shore. The turbulence is steadily dissipating energy, resulting in a relatively uniform decrease in wave height as the wave propagates forward across the surf zone.

*Plunging.* The wave crest develops a tongue that curls forward over the front face and plunges at the base of the wave face. The breaking action and



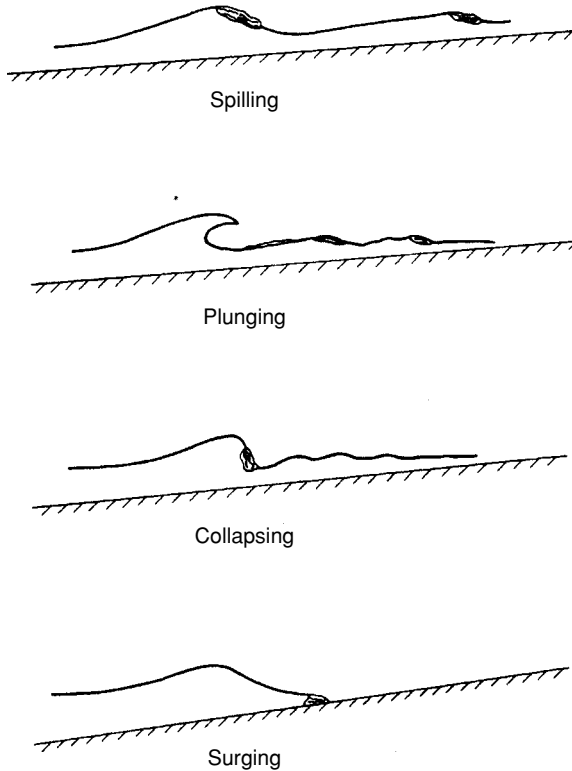


Figure 2.10. Wave breaker classification.

energy dissipation are more confined to the point of breaking than is the case for a spilling wave. The plunging tongue of water may regenerate lower more irregular waves that propagate forward and break close to the shore.

*Surging.* The crest and front face of the wave approximately keep their asymmetric shape as they surge across the beach slope. This form of breaking is a progression toward a standing or reflecting wave form.

While the above three classes are relatively distinct, for gradually changing incident wave steepnesses and bottom slopes there is a gradual transition from one form to the next. (Some investigators add a transitional class—collapsing breakers—between plunging and surging.) Only spilling and plunging breakers occur in deep water and they are the most common types of breakers in shallow water. Spilling breakers, accompanied by “whitecapping” if there is a strong wind, are most common in deep water. The type of breaker is important, for example, to the stability of a stone mound structure exposed to breaking waves.

It will also affect the amount of energy reflected from a slope and the elevation of wave runoff on a slope.

As discussed above, Eq. (2.68) only gives an approximate rule of thumb for wave breaking in shallow water. A number of experimenters have investigated nearshore breaking conditions in the laboratory and presented procedures for predicting the breaking height  $H_b$  and water depth at breaking  $d_b$  as a function of incident wave characteristics and bottom slope  $m$ . Figures 2.11 and 2.12, modified slightly from the U.S. Army Coastal Engineering Research Center (1984) and based on studies by Goda (1970) and Weggel (1972), are commonly used for estimating breaking conditions.

Given the beach slope, the unrefracted deep water wave height, and the wave period one can calculate the deep water wave steepness and then determine the breaker height from Figure 2.11. The regions for the three classes of wave breaker types are also denoted on this figure. With the breaker height one can then determine the water depth at breaking from Figure 2.12. Note the range of  $d_b/H_b$  values in Figure 2.12 versus the guidance given by Eq. (2.68). If a wave refracts as it propagates toward the shore, the equivalent unrefracted wave height given by

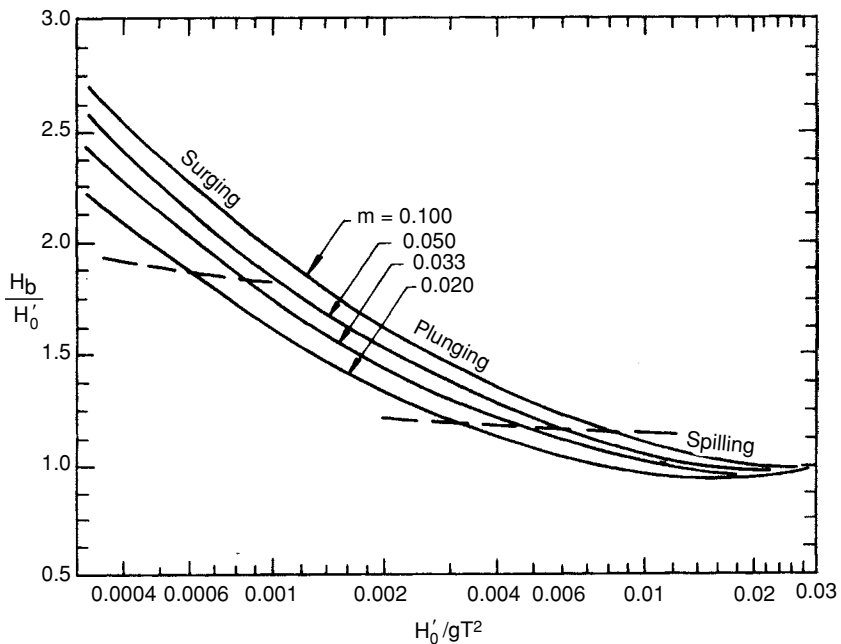


Figure 2.11. Dimensionless breaker height and class versus bottom slope and deep water steepness. (Modified from U.S. Army Coastal Engineering Research Center, 1984.)

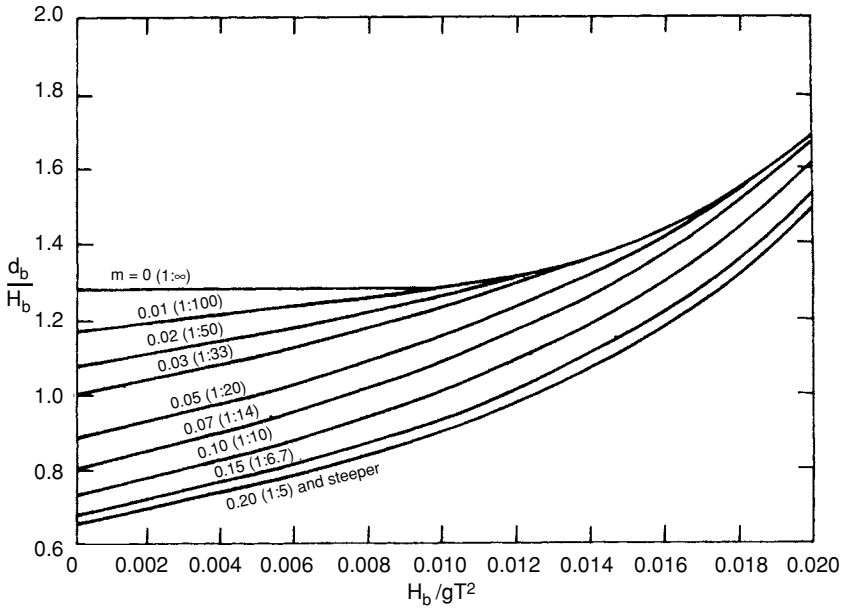


Figure 2.12. Dimensionless breaker depth versus bottom slope and breaker steepness. (Modified from U.S. Army Coastal Engineering Research Center, 1984.)

$$H'_o = K_r H_o \quad (2.69)$$

should be used in Figure 2.11.

Figures 2.11 and 2.12 do not consider the effects of wind on wave breaking. Douglass (1990) conducted limited laboratory tests on the effect of inline following and opposing winds on nearshore wave breaking. He found that offshore directed winds retarded the growth of wave height toward the shore and consequently caused the waves to break in shallower water than for the no wind condition. Onshore winds had the opposite effect but to a lesser extent. However,  $H_b/d_b$  was greater for offshore winds than for onshore winds, given the same incident wave conditions and beach slope. For the same incident waves, offshore winds caused plunging breakers when onshore winds caused waves to spill.

The design of some coastal structures is dependent on the higher wave that breaks somewhat seaward of the structure and plunges forward to hit the structure. Thus, when designing a structure for breaking wave conditions, the critical breaking depth is some point seaward of the structure that is related to the breaker plunge distance  $X_p$ , as depicted in Figure 2.13. Smith and Kraus (1991), based on experiments with plane slopes and slopes having a submerged

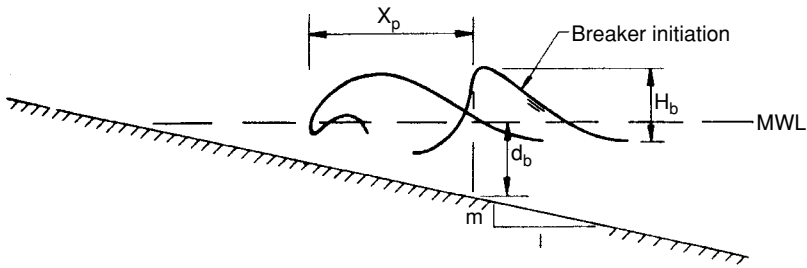


Figure 2.13. Definition sketch for breaker plunge distance.

bar that trips wave breaking, found the following relationships for the plunge distance. For plane slopes,

$$\frac{X_p}{H_b} = 3.95 \left( \frac{\sqrt{H_o/L_o}}{m} \right)^{0.25} \quad (2.70)$$

and, for slopes with a submerged bar

$$\frac{X_p}{H_b} = 0.63 \left( \frac{\sqrt{H_o/L_o}}{m} \right) + 1.81 \quad (2.71)$$

For structure design one might typically use the wave that breaks at  $0.5X_p$  seaward of the structure.

## 2.9 Wave Runup

After a wave breaks, a portion of the remaining energy will energize a bore that will run up the face of a beach or sloped shore structure. Figure 2.14 depicts this

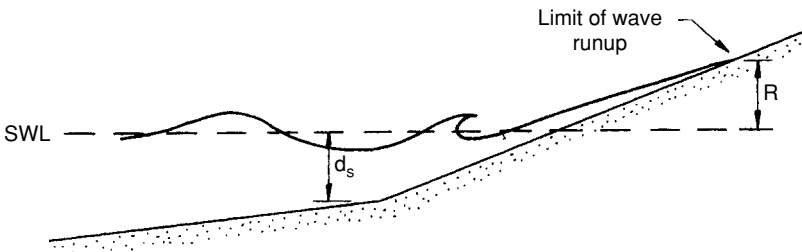


Figure 2.14. Definition sketch for wave runup.

process where the runup  $R$  is the maximum vertical elevation above the still water level to which the water rises on the beach or structure. Prediction of the wave runup is important, for example, for the determination of the required crest elevation for a sloping coastal structure or to establish a beach setback line for limiting coastal construction.

The runup depends on the incident deep water wave height and period, the surface slope and profile form if not planar, the depth  $d_s$  fronting the slope (see Figure 2.14), and the roughness and permeability of the slope face. Dimensional analysis leads to

$$\frac{R}{H'_o} = fcn\left(\alpha, \frac{H'_o}{gT^2}, \frac{d_s}{H'_o}\right) \quad (2.72)$$

for a given surface shape and condition (where  $\cot \alpha = 1/m$ ).

Figure 2.15 is a typical plot of experimental data from a laboratory wave runup study with monochromatic waves. These data are for a smooth, planar, impermeable slope with  $d_s/H'_o$  between 1 and 3. (See U.S. Army Coastal Engineering Research Center, 1984 for similar plots for other slope conditions.) Figure 2.15 indicates that, for a given structure slope, steeper waves (higher  $H'_o/gT^2$ ) have a lower relative runup ( $R/H'_o$ ). Also, for most beach and revetment slopes (which are flatter than 1 on 2), the wave runup increases as the slope becomes steeper.

Table 2.1, developed from a number of laboratory experiments, gives an indication of the effect of slope surface condition on wave runup. The factor  $r$  is the ratio of the runup on the given surface to that on a smooth impermeable surface and may be multiplied by the runup determined from figures such as Figure 2.15 to predict the wave runup.

### Example 2.9-1

Consider the deep water wave in Example 2.3-1 propagating toward the shore without refracting. The wave breaks and runs up on a 1:10 grass covered slope having a toe depth of 4 m. Determine the breaking wave height and the wave runup elevation on the grass-covered slope.

Solution:

For a deep water unrefracted wave height of 2 m and a period of 10 s we have

$$\frac{H'_o}{gT^2} = \frac{2}{(9.81)(10)^2} = 0.002$$

From Figure 2.11 for  $m = 0.1$

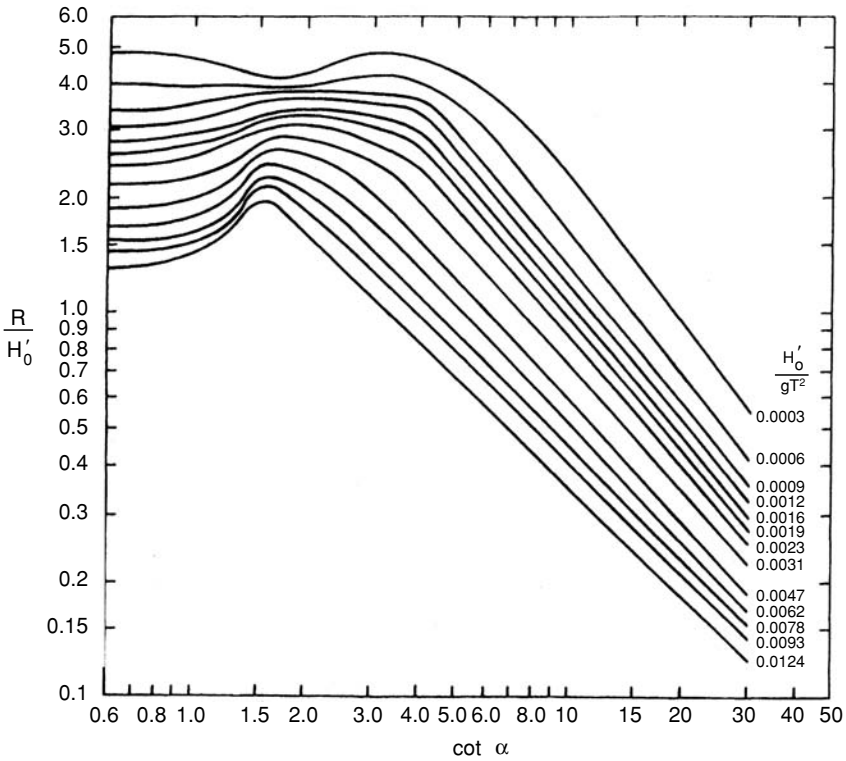


Figure 2.15. Dimensionless runup on smooth impermeable slopes versus bottom slope and incident deep water wave steepness;  $1 < d_s/H'_0 < 3$ . (Modified from U.S. Army Coastal Engineering Research Center, 1984.)

Table 2.1. Runup Factors for Various Slope Conditions

Slope facing	r
Concrete slabs	0.9
Placed basalt blocks	0.85–0.9
Grass	0.85–0.9
One layer of riprap on an impermeable base	0.8
Placed stones	0.75–0.8
Round stones	0.6–0.65
Dumped stones	0.5–0.6
Two or more layers of riprap	0.5
Tetrapods, etc.	0.5

From Battjes, 1970.

$$\frac{H_b}{H'_o} = 1.6$$

or

$$H_b = 1.6(2) = 3.2 \text{ m}$$

The wave would form a plunging breaker. From Figure 2.15, since  $d_s/H'_o = 4/2 = 2$ , at  $\cot \alpha = 10$

$$\frac{R}{H'_o} = 0.85$$

or the uncorrected smooth slope runup is

$$R = 0.85(2) = 1.7 \text{ m}$$

Using  $r = 0.875$  from Table 2.1 gives a runup of

$$R = 0.875(1.7) = 1.5 \text{ m}$$

on the grass-covered slope.

---

## 2.10 Summary

Experiments conducted in wave tanks (Wiegel, 1950; Eagleson, 1956; LeMehaute et al., 1968) give some indication of the accuracy of small-amplitude wave theory in predicting the transformation of monochromatic two-dimensional waves as they travel into intermediate and shallow water depths, and of the accuracy in predicting particle kinematics given the wave height and period and the water depth. A summary follows:

1. For most typical bottom slopes the dispersion equation is satisfactory for predicting the wave celerity and length up to the breaker zone.
2. For increasing beach slopes and wave steepnesses, the wave height predictions given by Eq. (2.42) will be lower than the real wave heights. This discrepancy increases as the relative depth decreases. As an example, on a 1:10 slope, for a relative depth of 0.1 and a deep water wave steepness of 0.02, the experimental wave height exceeded the calculated wave height by 15%.
3. For waves on a relatively flat slope and having a relative depth greater than about 0.1, the small-amplitude theory is satisfactory for predicting horizontal

water particle velocities. At lesser relative depths the small-amplitude theory still predicts reasonably good values for horizontal velocity near the bottom, but results are poorer (up to 50% errors on the low side) near the surface.

Limitations of the small-amplitude theory in shallow water and for high waves in deep water suggest a need to consider nonlinear or finite-amplitude wave theories for some engineering applications. The next chapter presents an overview of selected aspects of the more useful finite-amplitude wave theories, as well as their application and the improved understanding of wave characteristics that they provide.

## 2.11 References

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## **2.12 Problems**

1. A two-dimensional wave tank has a still water depth of 1.9 m and a 1:20 plane slope installed with its toe at the tank midpoint. The tank is 1 m wide. A wave generator produces monochromatic waves that, when measured at a wave gage installed before the toe of the slope, have a height of 0.5 m and a period of 2.8 s.
  - (a) Determine the wave length, celerity, group celerity, energy, and energy density at the wave gage.
  - (b) At the instant that a wave crest passes the wave gage, determine the water particle velocity and acceleration below the gage at mid depth.
  - (c) Is the wave passing the gage a deep water wave? If not, what would the equivalent deep water length, celerity, group celerity, energy, and energy density be? Compare these values to those in part a.
  - (d) Calculate the wave height as a function of distance along the slope from the toe to the point at which the wave breaks.
2. An ocean bottom-mounted pressure sensor measures a reversing pressure as a train of swells propagates past the sensor toward the shore. The pressure fluctuations have a 5.5 s period and vary from a maximum of 54.3 kN/m<sup>2</sup> to a minimum of 41.2 kN/m<sup>2</sup>.
  - (a) How deep is the pressure sensor (and bottom) below the still water level?
  - (b) Determine the wave height, celerity, group celerity, energy, and power as it passes the sensor.
  - (c) As a wave crest is passing the sensor determine the water particle velocity and acceleration at a point 1.5 m above the bottom.

- (d) Calculate the deep water wave celerity, length, group celerity, energy, and power if the wave propagates along a line perpendicular to the shore without refracting.
  - (e) The nearshore bottom slope is 1:30. Calculate and plot the wave height as a function of position from deep water into the point at which the wave breaks.
3. Offshore, in deep water, a wave gage measures the height and period of a train of waves to be 2 m and 7.5 s, respectively. The wave train propagates toward the shore in a normal direction without refracting and the nearshore bottom slope is 1:40. It passes the outer end of a pier located in water 4.5 m deep.
- (a) Determine the wave length, celerity, group celerity, energy density, and power in deep water.
  - (b) Determine the wave length, height, celerity, group celerity, energy density, and power at the end of the pier. Is this a deep, transitional or shallow water wave at the end of the pier?
  - (c) At the instant that a wave crest passes the end of the pier, what is the pressure at a point 2 m below the still water level?
  - (d) Calculate the horizontal components of the water particle velocity and acceleration at this point 2 m below the still water level 1 s before the wave crest passes the end of the pier.
  - (e) At what water depth will the wave break? What will the wave height be as the wave breaks? What type of breaker will it be?
4. A wave gage mounted on the seaward end of a pier where the water depth is 6 m, measures a wave having  $H = 2.3$  m and  $T = 7.1$  s. This wave is one of a train of waves that is traveling normal to the shore without refracting. The bottom slope is 1:30.
- (a) Determine the deep water wave height and energy.
  - (b) Determine the wave height and water depth where the wave breaks.
  - (c) What are the water particle pressure, velocity and acceleration 1.7 m above the bottom 1.3 seconds after the wave crest passes the gage?
5. A wave has a height of 1.5 m in water 5 m deep and a wave period of 6 s. Plot the horizontal component of velocity, the vertical component of acceleration, and the dynamic pressure at a point 2 m below the still water level versus time for a 6-s interval. Plot the three values on the same diagram and comment on the results.
6. Estimate the maximum height wave that can be generated in a wave tank having water 1.8 m deep if the wave period is 1 s and if the period is 3 s.
7. A wave has a measured height of 1.4 m in water 5.6 m deep. If it shoals on a 1:50 slope how wide will the surf zone be? Assume the wave propagates normal to the shore without refracting.
8. A pressure gage located 1 m off the bottom in water 10 m deep measures an average maximum pressure of  $100 \text{ kN/m}^2$  having an average fluctuation period of 12 s. Determine the height and period of the wave causing the measured pressure fluctuation.

9. Derive an equation for the horizontal component of particle convective acceleration in a wave. Compare the horizontal components of convective and local acceleration versus time for a time interval of one wave period, at a distance of 2 m below the still water level and for a 1 m high 6 s wave in water 5 m deep.

10. Demonstrate, using Eq. (2.43), that  $C_g = C/2$  in deep water and  $C_g = C$  in shallow water.

11. Calculate and plot  $n$  and  $L$  nondimensionalized by dividing by the deep water values for  $n$  and  $L$ , as a function of  $d/L$  for  $d/L$  from 0.5 to 0.05.

12. Derive the equations for the horizontal and vertical components of particle acceleration in a standing wave, starting from the velocity potential [Eq. (2.55)].

13. As the tide enters a river and propagates upstream, the water depth at a given location is 3.7 m. At this location the tide range is 1 m. If the dominant tidal component has a period of 12.4 hours, estimate the peak flood tidal flow velocity at this location in the river.

14. Consider the conditions given in Problem 13. At a location in the river where the water depth is 5.1 m estimate the tide range and peak flood tidal flow velocity.

15. The first wave of a group of waves advancing into still water is 0.30 m high. The water depth is 4.5 m and the wave period is 2 s. How high is this wave 8 s later?

16. Consider a 1 m high, 4 s wave in water 5 m deep. Plot sufficient velocity potential lines to define their pattern and then sketch in orthogonal streamlines.

17. Consider a deep water wave having a height of 2.1 m and a period of 9 s shoaling on a 1:50 slope without refraction. Calculate, for comparison, the crest particle velocity in deep water, at  $d = 20$  m, and just prior to breaking. Calculate the wave celerity just prior to breaking and compare it to the crest particle velocity. Comment on the reason for any discrepancies.

18. A 12 s, 2 m high wave in deep water shoals without refracting. Calculate the maximum horizontal velocity component and the maximum horizontal displacement from the mean position for a particle 5 m below the still water level in deep water and where the water depth is 6 m.

19. As the wave given in Problem 4 propagates toward the shore determine the mean water level setdown at the breaker line and the setup 40 m landward of the breaker line.

20. Equation (2.56) defines the surface profile as a function of time for a standing wave. From this, derive the potential energy per wave length and the potential energy density, both as a function of time. Realizing that at the instant a standing wave has zero particle velocity throughout, all energy is potential energy, determine the total and kinetic energies per wave length and the total and kinetic energy densities.

21. For the conditions in Example 2.6-1, calculate and plot (to a 10:1 vertical scale distortion) the bottom, still water line, and the mean water line from a point 20 m seaward of the breaker point to the shoreline.

22. A wave having a height of 2.4 m and a period of 8 s in deep water is propagating toward the shore without refracting. A water particle velocity of 0.25 m/s on the bottom is required to initiate movement of the sand on the sea floor. At what water depth will sand movement commence as the wave shoals?

23. Using shallow water wave equations for celerity and water particle velocity and the criteria that at incipient breaking the crest particle velocity equals the wave celerity, derive a criterion for wave breaking. Comment on the result of this derivation.

24. To an observer moving in the direction of a monochromatic wave train at the wave celerity, the wave motion appears to be steady. The surface particle velocity at the wave crest  $U_c = C_o - \pi H/T$  and at the trough  $U_t = C_o + \pi H/T$  for a deep water wave. Apply the Bernoulli equation between these two points to derive Eq. (2.16).

25. For a given wave height and period and water depth which of the following wave parameters depend on the water density: celerity, length, energy density, particle pressure, and particle velocity at a given depth? Explain each answer.

26. How does increased water viscosity affect a wave train as it propagates toward the shore?

27. Waves with a period of 10 s and a deep water height of 1 m arrive normal to the shore without refracting. A 100-m long device that converts wave motion to electric power is installed parallel to the shore in water 6 m deep. If the device is 45% efficient what power is generated?

28. Demonstrate that the velocity potential defined by Eq. (2.9) does represent irrotational flow.

29. A wave with a period of 7 s propagates toward the shore from deep water. Using the limit presented in this chapter, at what water depth does it become a shallow water wave? If the deep water wave height is 1 m would this wave break before reaching the shallow water depth? Assume that no refraction occurs and that the nearshore slope is 1:30.