2

Two. Else there is danger of. Solitude.

Channel models for digital transmission

The aim of this chapter is to examine in some detail the problem of constructing models for the channels to be used for digital transmission. The emphasis here is on wireless channels, and special attention will be paid to modeling of fading. The impact of the model on the performance assessment of a coded transmission system is also discussed.

2.1 Time- and frequency-selectivity

We work with baseband-equivalent channel models, both continuous time and discrete time. In this Chapter we use the following notations: in continuous time, $s(t)$, $y(t)$, and $w(t)$ denote the transmitted signal, the received signal, and the additive noise, respectively. In discrete time we use the notations $s(n)$, $y(n)$, and $w(n)$, with n the discrete time. We consider only *linear* channels here. The most general model is

$$y(t) = \int h(t;\tau)s(t-\tau)\,d\tau + w(t) \qquad y(n) = \sum_k h(n;k)s(n-k) + w(n) \quad (2.1)$$

where $h(t;\tau)$ is the channel response at time t to a unit impulse $\delta(\,\cdot\,)$ transmitted at time $t-\tau$. Similarly, $h(n;k)$ is the channel impulse response at time n to a unit impulse $\delta(n)$ transmitted at time $n-k$. This channel is said to be *time selective and frequency selective,* where time selectivity refers to the presence of a time-invariant impulse response and frequency selectivity to an input–output relationship described by a convolution between input and impulse response. By assuming that the sum in (2.1) includes $L+1$ terms, we can represent the discrete channel by using the convenient block diagram of Figure 2.1, where z^{-1} denotes unit delay.

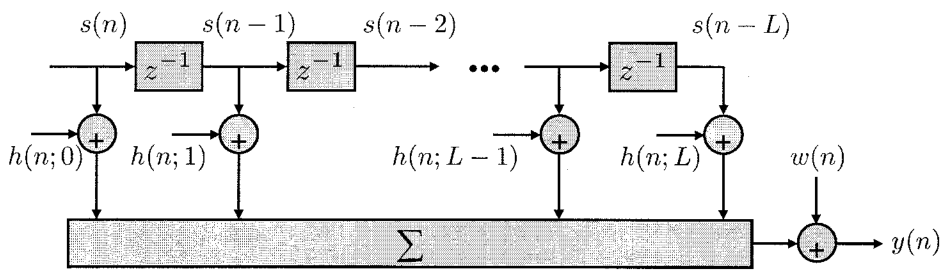

Figure 2.1: *Block diagram of a discrete time-selective, frequency-selective channel.*

If the channel is time invariant, then $h(t;\tau)$ is a constant function of t. We write $h(\tau) \triangleq h(0;\tau)$ for the (time-invariant) response of the channel to a unit impulse transmitted at time 0, and we have the following model of a *non-time-selective, frequency-selective* channel:

$$y(t) = \int h(\tau)s(t-\tau)\,d\tau + w(t) \qquad y(n) = \sum_k h(k)s(n-k) + w(n) \quad (2.2)$$

The block diagram of Figure 2.1 is still valid for this channel, provided that we write $h(k)$ in lieu of $h(n; k)$.

The model of a *time-selective, non-frequency-selective* channel is obtained by assuming that $h(t; \tau) = h(t)\delta(\tau)$ (or, for discrete channels, $h(n; k) = h(n)\delta(k)$). Then we have

$$\begin{aligned} y(t) &= \int h(t;\tau)s(t-\tau)\,d\tau + w(t) \\ &= \int h(t)\delta(\tau)s(t-\tau)\,d\tau + w(t) \\ &= h(t)s(t) + w(t) \end{aligned} \tag{2.3}$$

and

$$\begin{aligned} y(n) &= \sum_k h(n;k)s(n-k) + w(n) \\ &= \sum_k h(n)\delta(k)s(n-k) + w(n) \\ &= h(n)s(n) + w(n) \end{aligned} \tag{2.4}$$

We observe that in (2.3) and (2.4) the channel impulse response affects the transmitted signal multiplicatively, rather than through a convolution.

Finally, a *non-time-selective, non-frequency-selective* channel model is obtained by assuming that, in (2.3), $h(t; \tau)$ does not depend on t; if it has the form $h(t; \tau) = h\delta(\tau)$ (or, for discrete channels, $h(n; k) = h\delta(n)$), we obtain

$$y(t) = hs(t) + w(t) \qquad y(n) = hs(n) + w(n) \tag{2.5}$$

The simplest situation here occurs when h is a deterministic constant (later on we shall examine the case of h being a random variable). If in addition $w(t)$ is white Gaussian noise, the resulting channel model is called an *additive white Gaussian noise* (AWGN) channel. Typically, it is assumed that $h = 1$ so that the only parameter needed to characterize this channel is the power spectral density of $w(t)$.

2.2 Multipath propagation and Doppler effect

The received power in a radio channel is affected by attenuations that are conveniently characterized as a combination of three effects, as follows:

(a) The *path loss* is the signal attenuation due to the fact that the power received by an antenna at distance D from the transmitter decreases as D increases.

Empirically, the power attenuation is proportional to D^{α}, with α an exponent whose typical values range from 2 to 4. In a mobile environment, D varies with time, and consequently so does the path loss. This variation is the slowest among the three attenuation effects we are examining here.

(b) The *shadowing* loss is due to the absorption of the radiated signal by scattering structures. It is typically modeled by a random variable with log-normal distribution.

(c) The *fading loss* occurs as a combination of two phenomena, whose combination generates random fluctuations of the received power. These phenomena are *multipath propagation* and *Doppler frequency shift*. In the following we shall focus our attention on these two phenomena, and on mathematical models of the fading they generate.

Multipath propagation occurs when the electromagnetic field carrying the information signal propagates along more than one "path" connecting the transmitter to the receiver. This simple picture of assuming that the propagation medium includes several paths along which the electromagnetic energy propagates, although not very accurate from a theoretical point of view, is nonetheless useful to understand and to analyze propagation situations that include reflection, refraction, and scattering of radio waves. Such situations occur, for example, in indoor propagation, where the electromagnetic waves are perturbed by structures inside the building, and in terrestrial mobile radio, where multipath is caused by large fixed or moving objects (buildings, hills, cars, etc.).

Example 2.1 (Two-path propagation)

Assume that the transmitter and the receiver are fixed and that two propagation paths exist. This is a useful model for the propagation in terrestrial microwave radio links. The received signal can be written in the form

$$y(t) = x(t) + b\,x(t-\tau) \tag{2.6}$$

where b and τ denote the relative amplitude and the differential delay of the reflected signal, respectively (in other words, it is assumed that the direct path has attenuation 1 and delay 0). Equation (2.6) models a static multipath situation in which the propagation paths remain fixed in their characteristics and can be identified individually. The channel is linear and time invariant. Its transfer function

$$H(f) = 1 + b\,\mathrm{e}^{-j2\pi f\tau}$$

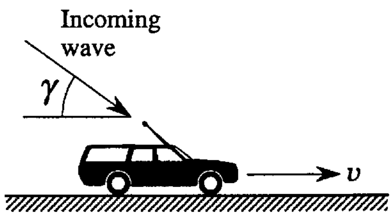

Figure 2.2: *Effect of movement: Doppler effect.*

in which the term $b \exp(-j2\pi f\tau)$ describes the multipath component, has magnitude

$$\begin{aligned} |H(f)| &= \sqrt{(1+b\cos 2\pi f\tau)^2 + b^2\sin^2 2\pi f\tau} \\ &= \sqrt{1+b^2+2b\cos 2\pi f\tau} \end{aligned}$$

For certain delays and frequencies, the two paths are essentially in phase alignment, so $\cos 2\pi f\tau \approx 1$, which produces a large value of $|H(f)|$. For some other values, the paths nearly cancel each other, so $\cos 2\pi f\tau \approx -1$, which produces a minimum of $|H(f)|$ usually referred to as a *notch*. □

When the receiver and the transmitter are in relative motion with constant radial speed, the received signal is subject to a constant frequency shift (the *Doppler shift*) proportional to this speed and to the carrier frequency. Consider the situation depicted in Figure 2.2. Here the receiver is in relative motion with respect to the transmitter. The latter transmits an unmodulated carrier with frequency f_0. Let v denote the speed of the vehicle (assumed constant), and γ the angle between the direction of propagation of the electromagnetic plane wave and the direction of motion. The Doppler effect causes the received signal to be a tone whose frequency is displaced (decreased) by an amount

$$f_D = f_0 \frac{v}{c} \cos\gamma \tag{2.7}$$

(the *Doppler frequency shift*), where c is the speed of propagation of the electromagnetic field in the medium. Notice that the Doppler frequency shift is either greater or lower than 0, depending on whether the transmitter is moving toward the receiver or away from it (this is reflected by the sign of $\cos\gamma$).

By disregarding for the moment the attenuation and the phase shift affecting the received signal, we can write it in the form

$$y(t) = A\exp[j2\pi(f_0 - f_D)t] \tag{2.8}$$

Notice that we have assumed a constant vehicle speed, and hence a constant f_D. Variations of v would cause a time-varying f_D in (2.8).

More generally, consider now the transmission of a bandpass signal $x(t)$, and take attenuation $\alpha(t)$ and delay $\tau(t)$ into account. The complex envelope of the received signal is

$$\tilde{y}(t) = \alpha(t)\mathrm{e}^{-j\theta(t)}\tilde{x}[t - \tau(t)]$$

where

$$\theta(t) = 2\pi\left[(f_0 + f_D)\tau(t) - f_D t\right]$$

This channel can be modeled as a time-varying linear system with low-pass equivalent impulse response

$$h(t;\, \tau) = 2\alpha(t)\,\mathrm{e}^{-j\theta(t)}\,\delta[t - \tau(t)]$$

2.3 Fading

In general, the term *fading* describes the variations with time of the received signal strength. Fading, due to the combined effects of multipath propagation and of relative motion between transmitter and receiver, generates time-varying attenuations and delays that may significantly degrade the performance of a communication system.

With multipath and motion, the signal components arriving from the various paths with different delays combine to produce a distorted version of the transmitted signal. A simple example will illustrate this fact.

Example 2.2 (A simple example of fading)

Consider now the more complex situation represented in Figure 2.3. A vehicle moves at constant speed v along a direction that we take as the reference for angles. The transmitted signal is again an unmodulated carrier at frequency f_0. It propagates along two paths, which for simplicity we assume to have the same delay (zero) and the same attenuation. Let the angles under which the two paths are received be 0 and γ. Due to the Doppler effect, the received signal is

$$y(t) = A\exp\left[j2\pi f_0\left(1 - \frac{v}{c}\right)t\right] + A\exp\left[j2\pi f_0\left(1 - \frac{v}{c}\cos\gamma\right)t\right] \tag{2.9}$$

We observe from the above equation that the transmitted sinusoid is received as a pair of tones: this effect can be viewed as a spreading of the transmitted signal frequency, and hence as a special case of frequency dispersion caused by the channel and due to the combined effects of Doppler shift and multipath propagation.

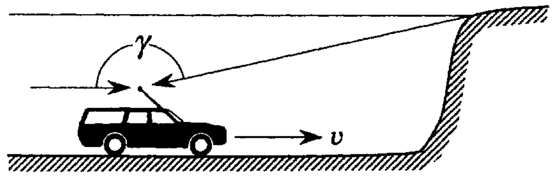

Figure 2.3: *Effect of a two-path propagation and movement.*

Equation (2.9) can be rewritten in the form

$$y(t) = A\left[\exp\left(-j2\pi f_0 \frac{v}{c} t\right) + \exp\left(-j2\pi f_0 \frac{v}{c}\cos\gamma t\right)\right] e^{j2\pi f_0 t} \tag{2.10}$$

The magnitude of the term in square brackets provides the instantaneous envelope of the received signal:

$$R(t) = 2A\left|\cos\left[2\pi \frac{v}{c} f_0 \frac{1-\cos\gamma}{2} t\right]\right|$$

The last equation shows an important effect: the envelope of the received signal exhibits a sinusoidal variation with time, occurring with frequency

$$\frac{v}{c} f_0 \frac{1-\cos\gamma}{2}$$

The resulting channel has a time-varying response. We have time-selective fading, and, as observed before, also frequency dispersion. □

A more complex situation, occurring when the transmission environment includes several reflecting obstacles, is described in the example that follows.

Example 2.3 (Multipath propagation and the effect of movement)

Assume that the transmitted signal (an unmodulated carrier as before) is received through N paths. The situation is depicted in Figure 2.4. Let the receiver be in motion with velocity v, and let A_i, θ_i, and γ_i denote the amplitude, the phase, and the angle of incidence of the ray from the ith path, respectively. The received signal contains contributions with a variety of Doppler shifts: in the ith path the carrier frequency f_0 is shifted by

$$f_i \triangleq f_0 \frac{v}{c} \cos\gamma_i, \qquad i = 1, 2, \ldots, N$$

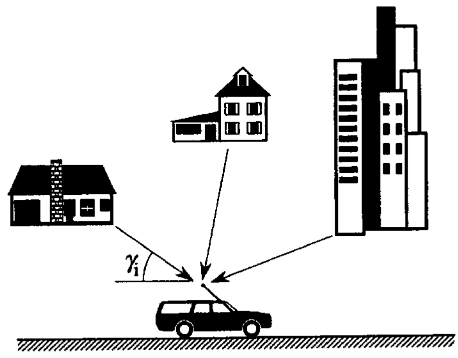

Figure 2.4: *Effect of N-path propagation and movement.*

Thus, the (analytic) received signal can be written in the form

$$y(t) = \sum_{i=1}^{N} A_i \exp j[2\pi(f_0 - f_i)t + \theta_i] \tag{2.11}$$

The complex envelope of the received signal turns out to be

$$R(t)e^{j\Theta(t)} = \sum_{i=1}^{N} A_i e^{-j(2\pi f_i t - \theta_i)}$$

□

2.3.1 Statistical models for fading channels

As we can observe from the previous examples, our ability to model the channel is connected to the possibility of deriving the relevant propagation parameters. Clearly, this is increasingly difficult and becomes quickly impractical as the number of parameters increases. A way out of this impasse, and one that leads to models that are at the same time accurate and easily applicable, is found in the use of the central limit theorem whenever the propagation parameters can be modeled as random variables (RV) and their number is large enough. To be specific, let us refer to the situation of Example 2.3. For a large number N of paths, we may assume that the attenuations A_i and the phases $2\pi f_i t - \theta_i$ in (2.11) are random variables that can be reasonably assumed to be independent of each other. Then, invoking the central limit theorem, we obtain that at any instant, as the number of contributing paths become large, the sum in (2.11) approaches a Gaussian RV. The complex envelope of the received signal becomes a lowpass Gaussian process whose real and imaginary parts are independent and have mean zero and the same variance σ^2. In these conditions, $R(t)$ and $\Theta(t)$ turn out to be independent processes, with $\Theta(t)$ being uniformly distributed in $(0, 2\pi)$ and $R(t)$ having a Rayleigh probability density function (pdf), viz.,

$$p_R(r) = \begin{cases} \frac{r}{\sigma^2} e^{-r^2/2\sigma^2}, & 0 \leq r < \infty \\ 0, & r < 0 \end{cases} \tag{2.12}$$

Here the average power of the envelope is given by

$$\mathbb{E}[R^2] = 2\sigma^2 \tag{2.13}$$

A channel whose envelope pdf is (2.12) is called a *Rayleigh fading channel.* The Rayleigh pdf is often used in its "normalized" form, obtained by choosing $\mathbb{E}[R^2] = 1$:

$$p_R(r) = 2r e^{-r^2} \tag{2.14}$$

An alternative channel model can be obtained by assuming that, as often occurs in practice, the propagation medium has, in addition to the N weaker "scatter" paths, one major strong fixed path (often called a *specular* path) whose magnitude is known. Thus, we may write the received-signal complex envelope in the form

$$R(t)e^{j\Theta(t)} = u(t)e^{j\alpha(t)} + v(t)e^{j\beta(t)}$$

where, as before, $u(t)$ is Rayleigh distributed, $\alpha(t)$ is uniform in $(0, 2\pi)$, and $v(t)$ and $\beta(t)$ are deterministic signals. With this model, $R(t)$ has the *Rice* pdf

$$p_R(r) = \frac{r}{\sigma^2} \exp\left\{-\frac{r^2 + v^2}{2\sigma^2}\right\} I_0\left(\frac{rv}{\sigma^2}\right) \tag{2.15}$$

for $r \geq 0$. ($I_0(\cdot)$ denotes the zeroth-order modified Bessel function of the first kind.) Its mean square is $\mathbb{E}[R^2] = v^2 + 2\sigma^2$. This pdf is plotted in Figure 2.5 for some values of v and $\sigma^2 = 1$.

Here $R(t)$ and $\Theta(t)$ are not independent, unless we further assume a certain amount of randomness in the fixed-path signal. Specifically, assume that the phase β of the fixed path changes randomly and that we can model it as a RV uniformly distributed in $(0, 2\pi)$. As a result of this assumption, $R(t)$ and $\Theta(t)$ become independent processes, with Θ uniformly distributed in $(0, 2\pi)$ and $R(t)$ still a Rice random variable.

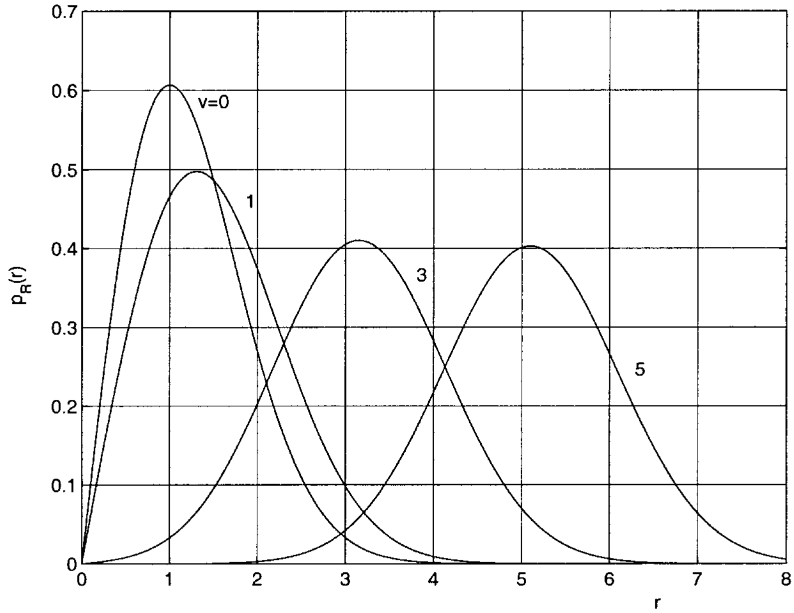

Figure 2.5: *Rice pdf with* $\sigma^2 = 1$.

Notice that, in (2.15), v denotes the envelope of the fixed-path component of the received signal, while $2\sigma^2$ is the power of the Rayleigh component (see (2.13) above). Thus, the "Rice factor"

$$K = \frac{v^2}{2\sigma^2}$$

denotes the ratio between the power of the fixed-path component and the power of the Rayleigh component. Sometimes the Rice pdf is written in a normalized form, obtained by assuming $\mathbb{E}[R^2] = v^2 + 2\sigma^2 = 1$ and exhibiting the Rice factor explicitly:

$$p_R(r) = 2r(1+K)\exp\left\{-(1+K)r^2 - K\right\} I_0\left(2r\sqrt{K(1+K)}\right) \tag{2.16}$$

for $r \geq 0$.

As $K \to 0$—i.e., as the fixed path reduces its power—since $I_0(0) = 1$, the Rice pdf becomes a Rayleigh pdf. On the other hand, if $K \to \infty$, i.e., the fixed-path power is considerably higher than the power in the random paths, then the Gaussian pdf is a good approximation for the Rice density.

Yet another statistical model for the envelope R of the fading is the Nakagami-m distribution. The probability density function of R is

$$p_R(r) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m r^{2m-1} e^{-mr^2/\Omega}, \qquad r \geq 0 \tag{2.17}$$

which has $\mathbb{E}[R^2] = \Omega$. The parameter m, called *fading figure*, is a ratio of moments:

$$m \triangleq \frac{\Omega^2}{\mathbb{V}[R^2]} \geq \frac{1}{2} \tag{2.18}$$

For integer values of m, (2.17) is the pdf of the RV

$$Y \triangleq \sqrt{\sum_{i=1}^{m} X_i^2} \tag{2.19}$$

where $X_1, \ldots, X_m$ are independent, Rayleigh-distributed RVs. As special cases, the choice $m = 1$ yields the Rayleigh distribution, while $m = 1/2$ yields a single-sided Gaussian distribution.

We observe that the Nakagami-m distribution is characterized by *two* parameters, and consequently it provides some extra flexibility if the mathematical model of the fading must be matched to experimental data.

2.4 Delay spread and Doppler-frequency spread

A simple yet useful classification of fading channels can be set up on the basis of the definition of two quantities called *coherence time* and *coherence bandwidth* of the physical channel.

Multipath fading occurs because different paths are received, each with a different Doppler shift: when the receiver and the transmitter are in relative motion with constant radial speed, the Doppler effect, in conjunction with multipath propagation, causes *time-* and *frequency-selective fading*. Consider these propagation paths, each characterized by a delay and attenuation, and examine how they change with time to generate a time-varying channel response. First, observe that significant changes in the attenuations of different paths occur at a rate much lower than significant changes in their phases. If $\tau_i(t)$ denotes the delay in the ith path, the corresponding phase is $2\pi f_0(t - \tau_i(t))$, which changes by 2π when $\tau_i(t)$ changes by $1/f_0$, or, equivalently, when the path length changes by c/f_0. Now, if the path length changes at velocity v_i, this change occurs in a time $c/(f_0 v_i)$, the inverse of the Doppler shift in the ith path. Consequently, significant changes in the channel occur in a time T_c whose order of magnitude is the inverse of the maximum Doppler shift B_D among the various paths, called the *Doppler spread* of the channel. The time T_c is called the *coherence time* of the channel, and we have

$$T_c \triangleq \frac{1}{B_D} \tag{2.20}$$

The significance of T_c is as follows. Let T_x denote the duration of a transmitted signal.[1] If it is so short that during transmission the channel does not change appreciably in its features, then the signal will be received undistorted. Its distortion becomes noticeable when T_x is above T_c, which can be interpreted as the delay between two time components of the signal beyond which their attenuations become independent. We say the channel is *time selective* if $T_x \gtrapprox T_c$.

The coherence time shows how rapidly a fading channel changes with time. Similarly, the quantity dual to it, called *coherence bandwidth*, shows how rapidly the channel changes in frequency. Consider paths i and j and the phase difference between them, i.e., $2\pi f(\tau_i(t) - \tau_j(t))$. This changes significantly when f changes by an amount proportional to the inverse of the difference $\tau_i(t) - \tau_j(t)$. If T_d, called the *delay spread* of the channel, denotes the maximum among these differences, a significant change occurs when the frequency change exceeds the inverse of T_d. We define the *coherence bandwidth* of the channel as

$$B_c \triangleq \frac{1}{T_D} \tag{2.21}$$

This measures the signal bandwidth beyond which the frequency distortion of the transmitted signal becomes relevant. In other words, the coherence bandwidth is the frequency separation at which two frequency components of the signal undergo independent attenuations. A signal with $B_x \gtrapprox B_c$ is subject to frequency-selective fading. More precisely, the envelope and phase of two unmodulated carriers at different frequencies will be markedly different if their frequency spacing exceeds B_c so that the cross-correlation of the fading fluctuations of the two tones decreases toward zero. The term *frequency-selective fading* expresses this lack of correlation among different frequency components of the transmitted signal.

In addition to coherence time and bandwidth, it is sometimes useful to define the *coherence distance* of a channel in which multiple antennas are used (see especially Chapter 10). This is the maximum spatial separation of two antennas over which

[1] Since we shall be considering *coded* signal for most of this work, from now on we may think of T_x as the duration of a code word.

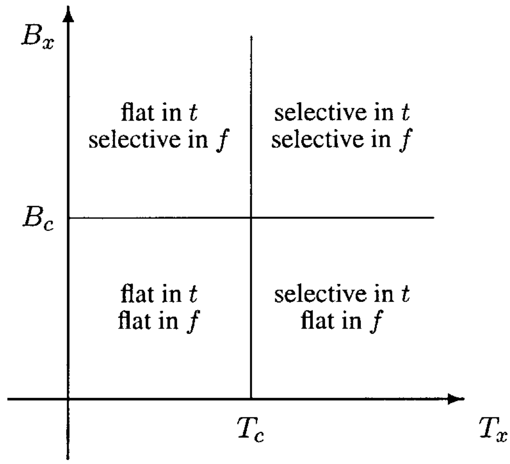

Figure 2.6: *Radio-channel classification.*

the channel response can be assumed constant: specifically, we say that the channel is *space selective* if the separation between antennas is larger than the coherence distance.

2.4.1 Fading-channel classification

From the previous discussion we have two quantities B_c and T_c describing how the channel behaves for the transmitted signal. Specifically,

(a) If $B_x \ll B_c$, there is no frequency-selective fading and hence no time dispersion. The channel transfer function looks constant, and the channel is called *flat* (or *nonselective*) in frequency. The fading affects the transmitted signal multiplicatively, by a factor which varies with time.

(b) If $T_x \ll T_c$, there is no time-selective fading, and the channel is called *flat* (or *nonselective*) in time.

Qualitatively, the situation appears as shown in Figure 2.6. The channel flat in t and f is not subject to fading either in time or in frequency. The channel flat in time and selective in frequency is called an *intersymbol-interference channel.* The channel flat in frequency is a good model for several terrestrial mobile radio channels. The channel selective both in time and in frequency is not a good model for terrestrial mobile radio channels, but it can be useful for avionic communications, in which high speeds (and hence short coherence times) combine with long delays due to earth reflections (and hence narrow coherence bandwidths).

The product $T_d B_D = 1/T_c B_c$ is called the *spread factor* of the channel. If $T_d B_D < 1$, the channel is said to be *underspread*; otherwise, it is *overspread.* Generally, if the spread factor $T_d B_D \ll 1$, the channel impulse response can be easily measured, and that measurement can be used by the receiver in the demodulation of the received signal and by the transmitter to optimize the transmitted signal. Measurement of the channel impulse response of an overspread channel is extremely difficult and unreliable, if not impossible. Since, in general, signal bandwidth and signal duration are such that $B_x T_x \gg 1$ (as otherwise there would be no hope for reliable communication, even in a nonfaded time-invariant channel, as, for example, the AWGN channel), it follows that a slowly fading, frequency nonselective channel is underspread.

Finally, we say that the channel is *ergodic* if the signal (i.e., the code word) is long enough to experience essentially all the states of the channel. This situation occurs when $T_x \gg T_c$. Thus, we discriminate between slow and fast fading and ergodic and nonergodic channels according to the variability of the fading process in terms of the whole code word transmission duration.

The preceding discussion is summarized in Table 2.1. (See [2.2] for further details.)

$B_x \ll B_c$	frequency-flat fading
$B_x \gtrapprox B_c$	frequency-selective channel
$T_x \ll T_c$	time-flat (slow) fading
$T_x \gtrapprox T_c$	time-selective (fast) channel
$T_c B_c > 1$	underspread channel
$T_c B_c \ll 1$	overspread channel
$T_x \ll T_c$	nonergodic channel
$T_x \gg T_c$	ergodic channel

Table 2.1: Classification of fading channels.

2.5 Estimating the channel

As we shall see in subsequent chapters, the performance of a transmission system over a fading channel may be greatly improved if the value taken on by the fading random variable affecting the propagation is known, at the receiver only or at both transmitter and receiver. Here we examine a technique for measuring a channel described as in Figure 2.1. We use "probing signals," to be transmitted in addition to information-bearing signals each time the channel changes significantly (and hence at least once every T_c).

A good set of probing signals is generated by a *pseudonoise (PN) sequence* $u(1), \ldots, u(N)$; it has the property that its autocorrelation $c(m)$ is approximately an ideal impulse. For simplicity we assume here that the channel is real, that the sequence is binary ($u(j) = \pm A$ for $1 \le j \le N$), and that we have exactly

$$c(m) \triangleq \sum_{j=1}^{N} u(j)u(j+m) = \begin{cases} A^2N, & m = 0 \\ 0 & m \neq 0 \end{cases} \tag{2.22}$$

where we take $u(j) = 0$ whenever $j < 1$ or $j > N$. Without noise, the channel response to the PN sequence is the convolution

$$r'(n) = \sum_{k=0}^{L} h(n;k)u(n-k) \tag{2.23}$$

This response can be nonzero only from time $n = 1$ to time $n = N + L$: in this period we assume that the channel, albeit random, remains constant, so that we can rewrite (2.23) as

$$r'(n) = \sum_{k=0}^{L} h(k)u(n-k) \tag{2.24}$$

with $h(k)$, $k = 0, \ldots, L$, a sequence of complex random variables. Now, correlate the noiseless channel output $r'(n)$ with the PN sequence. Using (2.22) we obtain

$$\begin{aligned} \rho'(-m) &\triangleq \sum_{n=m+1}^{m+N} r'(n)u(n-m) \\ &= \sum_{n=m+1}^{m+N} \sum_{k=0}^{L} h(k)u(n-k)u(n-m) \\ &= \sum_{k=0}^{L} h(k) \sum_{j=1}^{N} u(j+m-k)u(j) \\ &= \sum_{k=0}^{L} h(k)c(m-k) \\ &= A^2Nh(m) \end{aligned} \tag{2.25}$$

which is proportional to the mth sample of the channel impulse response.

Consider now the effect of an additive white Gaussian noise $w(n)$ with variance σ^2. The noisy-channel response to the PN sequence is

$$y(n) = y'(n) + w(n) \tag{2.26}$$

Correlating the channel output $y(n)$ with the PN sequence, we obtain

$$\rho(-m) = \rho'(-m) + \sum_{n=M+1}^{m+N} w(n)u(n-m) \tag{2.27}$$

where the additional term is again a Gaussian RV with mean zero and variance

$$\begin{aligned} \sum_{n=m+1}^{m+N} \sum_{n'=m+1}^{m+N} \mathbb{E}[w(n)w^*(n')]u(n-m)u(n'-m) &= \sigma^2 \sum_{n=m+1}^{m+N} u^2(n-m) \\ &= \sigma^2 N A^2 \end{aligned} \tag{2.28}$$

In conclusion, we observe a correlation $\rho(-m)$ which is the sum of two terms: one is proportional to N times the impulse-response sample that we wish to estimate, while the other is a noise term whose variance is proportional to the PN sequence length N. The resulting signal-to-noise ratio is proportional to N: thus, by increasing the sequence length (and hence the measurement length) we can make the channel measure arbitrarily good. Notice, however, that making N very long leads to an accurate estimate but decreases the data-transmission rate. Two techniques, used, for example, in the GSM standard of digital cellular telephony, allow one to increase the ratio between the information symbols and the probe symbols: the first one consists of placing the probe symbols in the middle of a data frame, the second one of interpolating between the previous and the next channel measurement.

2.6 Bibliographical notes

Ref. [2.2] contains an extensive review of the information-thoretical and communications aspects of fading channels. Engineering aspects of wireless channels and modeling problems are treated, for example, in [2.3–2.5].

References

[2.1] S. Benedetto and E. Biglieri, *Digital Transmission Principles with Wireless Applications*. New York: Kluwer/Plenum, 1999.

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