Preface

This book is intended to be an extension of Gurtin's book on continuum mechanics [5] by including the laws of thermodynamics and thus making it possible to study the mechanical behaviour of material bodies, the response of which involves variables such as entropy or temperature. In order to do that our departure point is Coleman and Noll's article [3] on the thermodynamics of elastic materials with heat conduction and viscosity which has been extended for the purpose at hand to the case of nonhomogeneous materials.

The present book has been used for many years as a textbook for graduate and undergraduate mathematics students at the University of Santiago de Compostela.

The first Chapter revisits the conservation principles of continuum thermomechanics, that is, the conservation of mass, linear and angular momentum balance and the first two principles of thermodynamics: namely, energy conservation and entropy inequality. All principles are introduced in integral form and in Eulerian coordinates. Local forms consisting of partial differential equations are then obtained. Writing these local equations in Lagrangian coordinates is the subject of Chapter 2.

Chapter 3 deals with the constitutive laws of continuum thermomechanics. First the notion of a material body characterised by its constitutive class is given. Then we introduce a general material body defined by Coleman and Noll in the above referenced article. By imposing the second principle of thermodynamics, we prove some relations to be satisfied by the response functions of such a material. Then, in Chapter 4, the principle of material frame-indifference is introduced and its consequences for the response functions of the Coleman-Noll materials are established. In Chapter 5, the partial differential equations governing a thermodynamic process are written replacing entropy with temperature.

Chapter 6 is devoted to isotropy. By using the representation theorems for isotropic tensor and vector-valued functions, we obtain simple forms for the response functions of Coleman-Noll materials. In Chapter 7, the equations satisfied by each thermodynamic process of these materials are written in Lagrangian coordinates. We also show that inviscid Coleman-Noll materials are hyperelastic.

The linear approximations of these equations about a static reference state are deduced in Chapter 8, assuming that the gradient of the displacement and the difference of temperature with respect to a reference state are both small. This is rigorously done through careful computation of the derivatives of the response functions. Isotropic materials are specifically considered. Thus, we obtain the partial differential system for linear thermoviscoelasticity; its numerical solution by incremental methods, in the inviscid quasi-static case, is addressed in Chapter 9.

Fluids are the subject of Chapter 10 where they are introduced as particular Coleman-Noll materials when the extended symmetry group is the unimodular group. We define the classical thermodynamic variables like specific heat, sound speed, volumetric thermal expansion, and write the conservation equations in terms of them. We also include a section dealing with fluid statics under a gravity field, and finally the approximate Boussinesq model for natural convection is introduced.

The linearized models obtained for general Coleman-Noll materials in Chapter 8 are specialized for fluids in Chapter 11. Thus, the standard models for dissipative and non-dissipative acoustics are properly deduced. Assuming that the body force is gravity force, we also deduce the equations for internal gravity waves.

Particular fluids called perfect gases are studied in Chapter 12 where the compressible Navier-Stokes and Euler systems of partial differential equations are deduced. Incompressible fluids are briefly examined in Chapter 13, but since these materials are not Coleman-Noll materials, they need to be studied separately. In particular, the incompressible Navier-Stokes and Euler equations are introduced.

A quick overview of turbulence models is the goal of Chapter 14.

Thermodynamics of mixtures of Coleman-Noll fluids and, more specifically, of mixtures of perfect gases is the subject of Chapter 15. In Chapters 16 and 17 some concepts of chemical kinetics are given, with special emphasis placed on modelling the chemical equilibrium by using Gibbs free energy. By using this methodology we study the flow of a mixture of reacting perfect gases in Chapter 18. In particular, the standard equations for modelling combustion are properly written both for finite-rate and equilibrium chemistry.

Chapter 19 deals with a computational method widely used in combustion: the mixture fraction method. The important case of equilibrium chemistry is specifically considered.

Finally, in Chapter 20, we give a short introduction to the interesting subject of turbulent reacting flows by recalling the fundamentals of the probability density function (PDF) method.

The appendices include some mathematical background in tensor algebra and analysis and then the formulation of conservation equations in so-called arbitrary Lagangian-Eulerian (ALE) coordinates is given. From a computational point of view, this formulation is very useful when dealing with free boundary flows or fluid-structure interaction problems.

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