

Decisions under Uncertainty

Probabilistic Analysis for Engineering Decisions

Decision-making and risk assessment are essential aspects of every engineer's life, whether this be concerned with the probability of failure of a new product within the warranty period or the potential cost, human and financial, of the failure of a major structure such as a bridge. This book helps the reader to understand the tradeoffs between attributes such as cost and safety, and includes a wide range of worked examples based on case studies. It introduces the basic theory from first principles using a Bayesian approach to probability and covers all of the most widely used mathematical techniques likely to be encountered in real engineering projects. These include utility, extremes and risk analysis, as well as special areas of importance and interest in engineering and for understanding, such as optimization, games and entropy. Equally valuable for senior undergraduate and graduate students, practising engineers, designers and project managers.

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For Christina

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Preface

Probabilistic reasoning is a vital part of engineering design and analysis. Inevitably it is related to decision-making – that important task of the engineer. There is a body of knowledge profound and beautiful in structure that relates probability to decision-making. This connection is emphasized throughout the book as it is the main reason for engineers to study probability. The decisions to be considered are varied in nature and are not amenable to standard formulae and recipes. We must take responsibility for our decisions and not take refuge in formulae. Engineers should eschew standard methods such as hypothesis testing and think more deeply on the nature of the problem at hand. The book is aimed at conveying this line of thinking. The search for a probabilistic ‘security blanket’ appears as futile. The only real standard is the subjective definition of probability as a ‘fair bet’ tied to the person doing the analysis and to the woman or man in the street. This is our ‘rule for life’, our beacon. The relative weights in the fair bet are our odds on and against the event under consideration.

It is natural to change one’s mind in the face of new information. In probabilistic inference this is done using Bayes’ theorem. The use of Bayesian methods is presented in a rigorous manner. There are approximations to this line of thinking including the ‘classical’ methods of inference. It has been considered important to view these and others through a Bayesian lens. This allows one to gauge the correctness and accuracy of the approximation. A consistent Bayesian approach is then achieved. The link to decisions follows from this reasoning, in which there are two fundamental concepts. In our decision-making we use probability, a concept in which the Bayesian approach plays a special rôle, and a second concept, utility. They are related dually, as are other concepts such as force and displacement. Utility is also subjective: probabilities and utilities attach to the person, not to the event under consideration.

The book was written over many years during a busy life as a practising engineer. It is perhaps a good moment to present the perspective of an engineer using probabilistic concepts and with direct experience of engineering decision-making. Whilst a consistent approach to probability has been taken, engineering practice is often ‘messy’ by comparison to the mathematical solutions presented in the book. This can best be dealt with by the engineer – numerical methods, approximations and judgement must come into play. The book focusses on the principles underlying these activities. The most important is the subjective nature of the two concepts involved in decision-making. The challenge very often is to obtain consensus between engineers on design parameters, requiring a judicious addition of conservatism – but not too much! – where there are uncertainties which have not been formally analysed.

An important aspect for engineers is the link to mechanics. One can think of probability distributions as masses distributed over possible outcomes with the total mass being unity; mean values then become centres of mass; variances become moments of inertia; and other analogies appear. The Stieltjes integral is a natural way to obtain the mean values and moments; it is potentially of great usefulness to the engineer. It unifies common situations involving, for example, concentrated or distributed masses in mechanics or probability. But this has been included as an option and the usual combination of summation for discrete random quantities and Riemann integration for continuous ones has been used in most instances. Inference is treated from the Bayesian standpoint, and classical inference viewed as a special case of the Bayesian treatment is played down in favour of decision theory. This leads engineers to think about their problem in depth with improved understanding. Half hearted or apologetic use of Bayesian methods has been avoided. The derivation of the Bayesian results can be more demanding than the corresponding classical ones, but so much more is achieved in the results.

Use of this book

The book is intended as an introduction to probability for engineers. It is my sincere hope that it will be of use to practising engineers and also for teaching in engineering curricula. It is also my hope that engineering schools will in the future allow more time in their programs for modelling of uncertainty together with associated decision-making. Often probability is introduced in one course and then followed by courses such as those in ‘Civil Engineering Systems’. This allows sufficient time to develop the full approach and not just pieces of ‘statistics’. The present work will be of use in such situations and where it is considered beneficial for students to understand the full significance of the theory. The book might be of interest also in graduate courses.

I have often used extended examples within the text to introduce ideas, for example the collision probability of the Titanic in Chapter 2. More specific examples and illustrations are identified as ‘examples’ in the text. The use of urns, with associated random walks through mazes, assists in fundamental questions of assigning probabilities, including exchangeability and extremes. Raiffa’s urn problem brings decision theory into the picture in a simple and effective manner. Simplified problems of decision have been composed to capture the essence of decision-making. Methods such as hypothesis testing have a dubious future in engineering as a result of their basic irrationality if applied blindly. Far better to analyse the decision at hand. But we do need to make judgements, for example, with regard to quality control or the goodness-of-fit of a particular distribution in using a data set. If wisely used, confidence intervals and hypothesis testing can assist considerably. The conjugate prior analysis in inference leads naturally to the classical result as a special case. Tables of values for standard distributions are so readily available using computer software that these have not been included.

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Optimization has been introduced as a tool in linear programming for game theory and in the maximization of entropy. But it is important in engineering generally, and this introduction might become a springboard into other areas as well. Chapter 11, dealing with data, linear regression and Monte Carlo simulation is placed at the end and collects together the techniques needed for the treatment of data. This may require a knowledge of extremes or the theory of inference, so that these subjects are treated first. But the material is presented in such a way that most of it can be used at any stage of the use of the book.

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Special mention should be made of two persons no longer alive. John Munro formerly Professor at Imperial College, London, guided me towards studying fundamental aspects of probability theory. Bruce Irons, of fame in finite element work, insisted that I put strong effort into writing. I have followed both pieces of advice but leave others to judge the degree of success. My former students, Maher Nessim, Marc Maes, Mark Fuglem and Bill Maddock have taught me possibly more than I have taught them. My colleagues at C-CORE and Memorial University have been most helpful in reviewing material. Leonard Lye, Glyn George and Richard McKenna have assisted considerably in reviewing sections of the book. Recent students, Paul Stuckey, John Pond, Chuanke Li and Denise Sudom have been of tremendous assistance in reviewing material and in the preparation of figures.

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Many of the exercises have been passed on to me by others and many have been composed specially. Other writers may feel free to use any of the examples and exercises without special acknowledgement.