
2. DEMAND MODELING IN PRODUCT LINE TRIMMING: SUBSTITUTABILITY AND VARIABILITY

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ABSTRACT

The number of products or stock keeping units (SKUs) in most product categories has been growing at a phenomenal rate. Even though the number of products increases, the average sale of products can decrease. Due to cannibalization, the sales of some products may even drop below a threshold that makes them unprofitable. This has spurred some firms to remove these under-performing products from their product lines. This ad-hoc “trim the lame duck” procedure can have an adverse effect on the firm’s profit for two reasons: first, the “lame duck” may not be the most substitutable product within the line, trimming it results in higher lost sales; second, the “lame duck” may be cheaper to keep with lower inventory cost due to less variability in sales. As an initial step to developing a better product elimination procedure, we use a model that explicitly captures product substitution phenomenon to examine various product portfolios. We compare the mean and the variance of the sales associated with two basic strategies: trimming and no trimming. Our results provide insight into when and which products could be trimmed.

1. INTRODUCTION

Companies are constantly seeking revenue growth. One marketing strategy that aims at revenue growth is *line extension*. The basic rationale for the strategy is that companies

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can increase their revenue if they cater to the diverse preferences of the consumers in the product categories. This strategy is perceived to be less risky since line extensions are launched in an existing product category that already has an established customer base. The wide-spread practice of this strategy has led to an unprecedented increase in the number of stock keeping units (SKUs) in most product categories (see for example Hays (1994), Khermouch (1995), Putsis and Bayus (2001)).

There are several criticisms of the line extension strategy. Bayus and Putsis (1999) show that line extension can significantly increase costs and the cost increase dwarfs the benefits derived from the increase in revenue. Quelch and Kenny (1994) suggested that many companies fail to account for the hidden costs of product proliferation associated with line extension. Perhaps, more critically, these authors challenged the wisdom “more products, more revenues” by arguing that “people do not eat more, drink more, brush their teeth more, or wash their hair more just because they have more products from which to choose.” In fact, they showed that the total sales of many product categories (such as cookies and shampoo) decreased despite a significant increase in the number of products. This argument seems plausible for certain mature product categories where growth is often minimal or stagnant.

Quelch and Kenny’s finding suggests that increasing the number of SKUs in a product category does not necessarily draw new customers to the category. Consequently, sales of new products must come from the cannibalization on the sales of existing products. This happens especially when the existing products are similar or inferior to the new extensions. This has prompted some companies to trim their product lines (e.g. Khermouch (1995)). There are several rationales for product line trimming. Firstly, the sales of some products can drop below a level which makes it unprofitable for firms to continue supplying them. Secondly, intense competition for limited grocery shelf space drives firms to remove the under-performing products, giving ‘space’ to more promising new products. In most cases, the companies tend to trim their product line in a *reactive* manner. Product obsolescence can be a cause of such reaction. Avlonitis and James (1982) reported, in a survey of 94 firms, that most firms trimmed their product lines because of product obsolescence. Factors contributing to product obsolescence include change in government regulations, change of buyers’ specification, and development of new products. In a mail survey of 166 companies, Hart (1988) concluded that firms also trimmed their product lines because of other factors such as resource shortage, poor product quality, rationalization due to merger and acquisition, and problems associated with raw materials.

On the *proactive* side, firms might want to trim their product lines for two reasons. First, a leaner product line enables a firm to regain its product focus. Some reports suggest that an over-crowded product category can lead to brand loyalty weakening (see for example Hays (1994) and Narisetti (1997)). This brand weakening phenomenon makes the product category very competitive because customers are easily swayed by price promotion (c.f. Quelch and Kenny (1994)). Second, a leaner product line allows a firm to streamline its manufacturing and logistics functions. This is particularly useful when product trimming has only a minimal or no impact on the sales of the firm’s entire product line. Indeed, a study conducted by the Food Marketing Institute

found that the retailers can reduce the number of SKUs by up to 25% without hurting sales or consumers' perception of variety (c.f. Teinowitz and Lawrence (1993)). However, trimming the wrong product can be costly. For instance, the removal of *Classic Coke* by Coca Cola Company in 1985 caused a strong negative reaction by its consumers. Consequently, the company had to re-introduce the product to reduce the loss of customer to its competitors.

Despite its strategic importance, empirical evidence has shown that most companies do not have a formal product elimination program. Hise and McGinnis (1975) reported that only 31.3% out of their sample of 96 firms have formal procedures. Hise *et al.* (1984) observed a similar pattern: 120 out of a sample of 299 firms have product elimination programs. But only 31% of the 120 firms have a documented program. A survey of U.S. and U.K. firms indicated that less than half of the sample ever utilize any formal methodology in making product elimination decision (c.f. Greenley and Bayus (1994)). The formality of a product elimination program appears to correlate with its frequency of use. In a sample size of 94 firms, Avlonitis (1985) observed that the more products a firm trims, the more formal the process gets; and that the longer it takes to trim a product, the less formal the process is. Because of the rapid rise of the number of SKU, we expect product trimming to receive more managerial attention in the near future. The need to formalize product trimming process can not be over-emphasized.

A formal procedure for product line trimming must address the following two fundamental questions: (1) Given the current line of products, should the firm trim its product line? (2) Suppose it is desirable to trim the product line, which product(s) should be eliminated? Several approaches were suggested in earlier works (see for example, Berenson (1963), Alexander (1964), Kotler (1965), Hamelman and Mazze (1972), Browne and Kemp (1976)). Most of these approaches use a rating scheme to rate the attractiveness of the products. Berenson (1963) based his model on five decision factors, namely, financial security, financial opportunity, marketing strategy, social responsibility and organized intervention. Management assigns weight to each factor to reflect its relative importance. The summation of five weighted scores becomes the overall rating of a product. Alexander (1964) used a similar procedure with a different set of factors. These are sales trend, price trend, profit trend, substitute products, product effectiveness and demand on executive time. Kotler (1965) did not provide a list of factors to consider. Instead, he suggested a six-step procedure where the first 3 steps were used by management to develop a comprehensive set of criteria and to generate ratings and weights for these criteria. A "product retention index", which is computed in a similar fashion as those of Berenson (1963) and Alexander (1964), is calculated for each product using the ratings and weights. A different index was suggested by Hamelman and Mazze (1972). Their "Selection Index Number (SIN_{*i*})" is a ratio of the square of product *i*'s percentage contribution margin to its percentage resource cost utilization. Browne and Kemp (1976) suggested a general multiple stage sequential review process. First, weak products are detected based on criteria specified by top management. Further analysis and evaluation of these products are done leading to an elimination decision and finally implementation.

These models are “macro” in the sense that they do not capture consumer choice process and product substitution explicitly. But both components are vital to an accurate assessment of the impact due to trimming. Knowing which products consumers will switch to in the absence of a product allows one to quantify precisely the impact of trimming. Our model incorporates the consumer choice process and product substitution phenomenon explicitly.

The intent of this paper is to highlight the importance of product *substitutability* and to examine its impact on product line trimming in terms of *expected demand* and *demand variability*. Urban (1969) studies the impact of product substitutability on the marketing strategy of a product line. Specifically, Urban (1969) models consumer response to different marketing mixes of a product line and use it to determine an optimal allocation of marketing mix for each product in a product line. In this work, we model consumer response to the removal of a product from a product line and use it to evaluate product trimming decisions.

There are several product portfolio models that incorporate, whether explicitly or implicitly, the expected demand (i.e. return) and demand variability (i.e. risk) in their analyses. These models either assimilate the financial portfolio analysis in their approaches (e.g. Mahajan and Wind (1985), Mahajan, Wind and Bradford (1982)) or use a mathematical programming formulation in their analyses (e.g. Larréché and Srinivasan (1982), Corstjens and Weinstein (1982)). The prime objective of these models is to determine the right product lines in the optimal product portfolio and to allocate marketing resources to these product lines. Our work is complementary to these models. We focus our analysis on the impact of product trimming on the expected demand and the variance of demand for the remaining products. The result of our analysis can potentially be used as an input to these optimization models.

One might argue that an alternative strategy to product line trimming is to lower the price of under-performing products.² However, this strategy may not be desirable (or even viable) for the following reasons. First, the products may differ only in product attributes such as taste (e.g. fruit drops), flavor (e.g. ice cream) or color (e.g. T-shirts, ball pen). The consumers do not usually expect any price difference in such cases. Second, the manufacturer may have limited freedom to charge different prices for similar products because this may not be acceptable to the retailers and consumers. For example, we can not price a product below another product having less feature or of a smaller package size. Third, in industries where the production lead time is relatively long, product line decision must precede pricing decision. Hence, we focus on product line decision.

To analyze the underlying issue whether to trim or not, we consider a situation in which a firm must select one of the following strategies: *Same Line* (i.e., no products should be eliminated from the line) and *Trim Line* (i.e., some products should be

2. Instead of price decrease, one might suggest price increase to realize a higher profit margin. This is a less viable option since sales volume tends to be low for the under-performing products.

eliminated from the line). Specifically, we analyze and compare the expected sales (or demand) and the variance of the sales associated with the two strategies. Besides expected demand, we analyze the variance of the demand, a variable commonly treated as noise by marketing scientists but studied extensively by operations researchers (see Silver and Peterson (1985)), because it is a good surrogate for the manufacturing and logistics costs. We present a simple framework that enables brand managers to better conceptualize the impact of product trimming. In this framework, we specify conditions under which the Same Line strategy is optimal. In the event that the Trim Line strategy may be optimal, our model suggests potential candidates to be trimmed from the line. Driven by its intuitive appeal, many firms tend to adopt the so-called *lame duck* heuristic: trim the product that has the lowest sales (see surveys by Hart (1988), and Avlonitis and James (1982)). We shall show later that this heuristic can be sub-optimal because it neglects demand interaction among products within the same category.

This paper is organized as follows. In the next section, we present a model that captures substitutability among products. Section 3 compares the ‘Same Line’ and ‘Trim Line’ strategies. In addition, we derive conditions under which the Same Line strategy is optimal. Section 4 deals with the situations when the Trim Line strategy may be optimal. We show why and when the “lame duck” heuristic can be seriously flawed. In section 5, we provide a numerical example to illustrate some of the insights generated from our analysis. Section 6 concludes the paper with some suggestions for future research.

2. THE MODEL

Consider a firm that manufactures and sells a line of N products in a selling season. The firm has accumulated some experience in selling the N products during previous seasons. At the beginning of the current season, the firm is contemplating if certain products should be removed from the product line. Essentially, the firm can either offer the same N products (i.e., adopt ‘Same Line’ strategy) or trim the line by removing some products (i.e., adopt ‘Trim Line’ strategy). In the event that the Trim Line strategy is preferred, the firm would like to determine the product(s) to be eliminated from the product line. In order to evaluate the pros and cons of the Same Line and Trim Line strategies, one needs to examine the impact of each strategy on the expected sales (or demand) and the variance of demand of the products must be determined. This observation motivates us to develop a simple model that captures the consumer purchasing processes and product substitution.

In our model, the product category has $N + 1$ products indexed by i where $i = 0, 1, \dots, N - 1, N$. For expository purpose, we aggregate all other products other than those of the firm and denote this ‘aggregated product bundle’ as product 0. Products 1 to N constitute the firm’s product line before line trimming. The product category consists of a population of consumers with heterogeneous product preferences. Each consumer is assumed to have a stationary and zero-order probabilistic product choice process. We consider the case in which the consumers are broadly

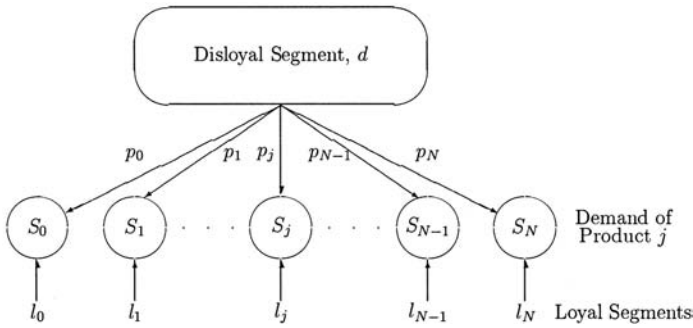


Figure 1. Demand of product j under the same line strategy.

classified into 2 segments: **loyal** and **disloyal**.³ The loyal segment consists of consumers whose choice of product is pre-specified, while the disloyal segment is comprised of consumers whose product selection is governed by their product choice probabilities. Each product has its own loyal segment but there is an aggregate segment of disloyalists.

We denote the size of loyal segment i by l_i ; the size of the disloyal segment by d . Both l_i and d are exogenously given parameters. Each customer in the disloyal segment is assumed to choose product i with probability p_i (see Figure 1). Obviously, $\sum_0^N p_i = 1$. Let D_i be the random variable denoting the demand for product i generated from the disloyal segment. Then $(D_0, D_1, \dots, D_{N-1}, D_N)$ are $N + 1$ multinomial random variables with parameter $(d; p_0, p_1, \dots, p_{N-1}, p_N)$. It is assumed that the firm has some knowledge about these parameters which can be estimated reliably from the sales data generated during previous selling seasons. The reader is referred to Grover and Srinivasan (1987) for a methodology that estimates these parameters using panel data.

2.1 The same line strategy

Under the Same Line strategy, the firm would sell the same N products as before. Let S_i be the total demand of product i within the selling season. Notice that the demand S_i generates from two pools of consumers: one from the loyal segment and the other from the disloyal segment. Thus, we have

$$S_i = l_i + D_i, \quad i = 0, \dots, N. \tag{2.1}$$

3. Our model is inspired by a model developed by Grover and Srinivasan (1987). Grover and Srinivasan propose a market segment methodology that allows for multiple disloyal segments. For notational convenience, we do not need such a refined partition. We conceptually ‘collapse’ these disloyal segments into a single aggregate segment and it should be clear later that this will not cause any loss in generalizability of our results. Within each market segment, customers are homogeneous in terms of product choice probabilities. Since each product has its own loyal customers, we have altogether $N + 1$ loyal segments.

It follows from (2.1) and the properties of multinomial random variables that the expected value and the variance of S_i can be expressed as:

$$\begin{aligned} E(S_i) &= l_i + dp_i, \quad i = 0, \dots, N, \\ \text{Var}(S_i) &= dp_i(1 - p_i), \quad i = 0, \dots, N. \end{aligned}$$

In this case, it is easy to check that the mean and variance of the total demand of all products in the firm's product line under the Same Line strategy are given by:

$$E\left(\sum_1^N S_i\right) = \sum_{j=1}^N l_j + d(1 - p_0) \quad (2.2)$$

and,

$$\text{Var}\left(\sum_1^N S_i\right) = d p_0 (1 - p_0). \quad (2.3)$$

2.2 The trim line strategy

Suppose that the firm has decided to trim product i from its product line. Then there are $N - 1$ remaining products in the line, namely, $1, \dots, i - 1, i + 1, \dots, N$. In this paper, we shall assume that the elimination of product i would have *direct* impact on the purchasing behavior of two specific groups (the loyal segment of product i and the disloyal segment) as follows:

2.2.1 The loyal segment of product i

When we eliminate product i from the product line, the customers in loyal segment of product i have to consider other alternatives. Each of these l_i customers may consider the following alternatives: (1) becomes loyal to other products; (2) joins the disloyal segment and selects the product according to some choice probabilities; or (3) becomes disenchanted and leaves the consumer base. Thus, for each customer who belongs to the loyal segment i , it is assumed that there is a probability q_{ij} that he/she becomes 'loyal' to product j , where $j = 0, 1, \dots, i - 1, i + 1, \dots, N$, that there is a probability x_i that he/she joins the disloyal group, and that there is a probability γ_i that he/she leaves the consumer base. Clearly, $\sum_{j \neq i} q_{ij} + x_i + \gamma_i = 1$, for $i = 1, \dots, N$.

Notice that the values of q_{ij} , x_i , γ_i depend on the nature of product i and the degree of its *substitutability* with other products in the category.⁴ When there exists a *highly* substitutable

4. Substitution between products can occur because products are often similar to some degree. Thus, when a product is not available for purchase, some customers may instead buy an alternative product. Intuitively, one would expect product substitution to occur more readily between a pair of similar products than between a pair of dissimilar products. This notion of similarity is implicit in most product positioning models; products that are positioned close to each other in the perceptual space are more similar to each other and are more easily substitutable (see Green and Krieger, 1993). In the classical attraction model (Bell, Keeney and Little, 1975), when a product i is eliminated from the consideration and choice set, the probability for product j ($j \neq i$) is increased from $A_j / \sum_k A_k$ to $A_j / \sum_{k \neq i} A_k$ where A_k is the level of attraction of product k . Thus, the attraction model restricts that the ratio of the purchase probabilities of any two remaining products stays constant. This restriction is not required in our model.

product j , it is reasonable to assume that $q_{ij} \approx 1$, and $\{q_{ik}, k \neq j\}$, $x_i, \gamma_i \approx 0$. Notice that the substitutable product j could be one of the remaining products (i.e., $j = 1, \dots, i - 1, i + 1, \dots, N$), or it could be one of the products that belongs to the competitors (i.e., when $j = 0$). However, when all other products are equally substitutable, customers may join the disloyal group (i.e., $x_i \approx 1$). Finally, when the salient features of product i are not captured in other products, it is possible that the i 's loyal consumers may leave the product category completely (i.e., $\gamma_i \approx 1$).⁵

Let L_{ij} be the size of the loyal segment j generated from the loyal segment l_i if the firm eliminates product i . Let X_i be the new *addition* to the disloyal segment generated from the loyal segment l_i after trimming product i . Let Y_i be the number of customers who leave the product category. In this case, it is easy to see that the random variables $(L_{i0}, \dots, L_{i,i-1}, L_{i,i+1}, \dots, L_{iN}, X_i, Y_i)$ are $N + 2$ multinomially distributed with parameters $(l_i; q_{i0}, \dots, q_{i,i-1}, q_{i,i+1}, \dots, q_{iN}, x_i, \gamma_i)$.

2.2.2 The disloyal segment

There are two disloyal segments that are affected by the elimination of product i . These two disloyal segments are: D_i , the original disloyal segment who intends to buy product i , and X_i , the new *addition* to the disloyal segment generated from the loyal segment l_i after trimming product i . Depending on its substitutability with each of the remaining products j , it is assumed that each consumer who belongs to these two disloyal segments will 'switch' to product j with probability α_{ij} .⁶ Notice that $\sum_{j=0, j \neq i}^N \alpha_{ij} = 1$. Let D_{ij} be the random variable denoting the demand for product j generated from the original disloyal segment after trimming product i from the product line. Then $\{D_{ij} : j = 0, \dots, i - 1, i + 1, \dots, N\}$ are N multinomial random variables with parameters $(D_i; \alpha_{i0}, \alpha_{i1}, \dots, \alpha_{iN})$. Similarly, let X_{ij} be the random variable denoting the demand for product j generated from the additional disloyal segment coming from the loyal segment of product i . Then $\{X_{ij} : j = 0, \dots, i - 1, i + 1, \dots, N\}$ are N multinomial random variables with parameters $(X_i; \alpha_{i0}, \alpha_{i1}, \dots, \alpha_{iN})$.

By considering the impact of trimming product i on the loyal segment of product i and the disloyal segment, the effective demand of product j under the Trim Line strategy can be expressed as:

$$T_{ij} = l_j + L_{ij} + D_j + D_{ij} + X_{ij}, j = 0, 1, \dots, i - 1, i + 1, \dots, N.$$

Notice that the total demand T_{ij} for product j is the sum of the various loyal and disloyal segments. The loyal segment consists of the old loyal segment of product j , l_j , and the new addition of loyal segment generated from a migration from the loyal segment of i , L_{ij} . The disloyal segment consists of the existing disloyal segment, D_j , the substitution from the existing disloyal segment of product i , D_{ij} , and the substitu-

5. The elimination of 'Classic Coke' is such a case when many consumers have threatened Coca-Cola company that they will abandon cola drinks and switch to other types of soda.

6. To simplify the exposition of our model, we assume that the 'switching' probabilities are the same for both disloyal segments; however, our analysis can be easily extended to the case when these probabilities are different.

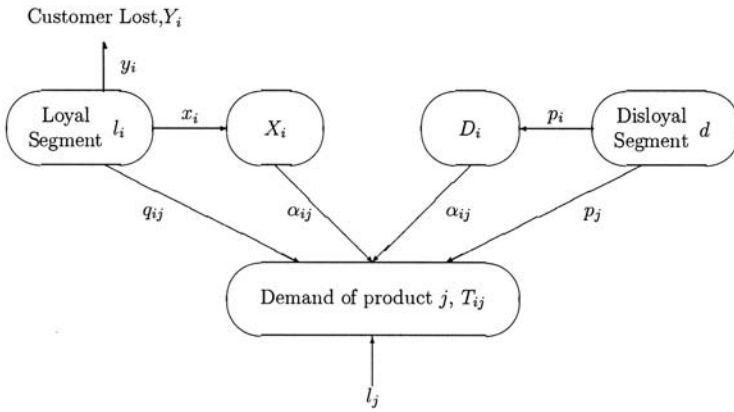


Figure 2. Demand of product j with product i trimmed.

tion from the new disloyal segment generated from a migration of the loyal segment i , X_{ij} (see Figure 2).

Applying the conditional expectation formula, the expected demand for product j after trimming product i can be expressed as:

$$\begin{aligned}
 E(T_{ij}) &= l_j + l_i q_{ij} + d p_j + \alpha_{ij} d p_i + \alpha_{ij} l_i x_i, \\
 &= l_j + d(p_j + \alpha_{ij} p_i) + l_i (q_{ij} + \alpha_{ij} x_i), \\
 j &= 0, 1, \dots, i-1, i+1, \dots, N.
 \end{aligned} \tag{2.4}$$

Similarly, we can apply the conditional variance formula to obtain the variance of the demand of product j after trimming product i . In this case, it is easy to show that:

$$\begin{aligned}
 Var(T_{ij}) &= Var(D_j) + Var(D_{ij}) + Var(L_{ij}) + Var(X_{ij}) \\
 &\quad + 2Cov(D_j, D_{ij}) + 2Cov(L_{ij}, X_{ij}), \\
 &= d p_j (1-p_j) + d \alpha_{ij} p_i (1 - \alpha_{ij} p_i) + l_i q_{ij} (1-q_{ij}) + l_i \alpha_{ij} x_i (1 - \alpha_{ij} x_i) \\
 &\quad - 2d \alpha_{ij} p_i p_j - 2l_i \alpha_{ij} x_i q_{ij}, \\
 &= d(p_j + \alpha_{ij} p_i)(1 - p_j - \alpha_{ij} p_i) + l_i (q_{ij} + \alpha_{ij} x_i)(1 - q_{ij} - \alpha_{ij} x_i).
 \end{aligned} \tag{2.5}$$

From Figure 2, we observe that the expressions for the expected demand and the variance of the demand of T_{ij} capture the fact that the demand of product j is generated from three sources: (a) the loyal segment of product j , l_j , (b) the original disloyal segment, d , where each customer will purchase product j with probability $(p_j + \alpha_{ij} p_i)$, and (c) the loyal segment of product i , l_i , where each customer will purchase product j with probability $(q_{ij} + \alpha_{ij} x_i)$.

Observe from equations (2.4) and (2.5) that the expected demand and the variance of T_{ij} depend on the substitutability of product j for product i . This substitutability effect is captured in q_{ij} (in the context of the loyal segment of product i) and α_{ij} (in

the context of the disloyal segment). To examine the expectation and the variance of the demand for the remaining product line (i.e., $\sum_{j=1, j \neq i}^N T_{ij}$), if we trim product i from the line, observe that:

$$\begin{aligned} \sum_{j=1, j \neq i}^N T_{ij} &= \sum_{j=1, j \neq i}^N \{l_j + D_j + D_{ij} + L_{ij} + X_{ij}\}, \\ &= \sum_{j=1}^N l_j + d - D_0 - L_{i0} - D_{i0} - X_{i0} - Y_i. \end{aligned}$$

By applying (2.4) and (2.5), it can be easily shown that:

$$E\left(\sum_{j=1, j \neq i}^N T_{ij}\right) = \sum_{j=1}^N l_j + d(1 - p_0) - \{\alpha_{i0}p_i + l_i(q_{i0} + \alpha_{i0}x_i + \gamma_i)\}. \quad (2.6)$$

and

$$\begin{aligned} \text{Var}\left(\sum_{j=1, j \neq i}^N T_{ij}\right) &= dp_0(1 - p_0) + d\alpha_{i0}p_i(1 - \alpha_{i0}p_i) \\ &\quad - 2d\alpha_{i0}p_i p_0 + l_i q_{i0}(1 - q_{i0}) \\ &\quad + l_i \alpha_{i0} x_i (1 - \alpha_{i0} x_i) + l_i \gamma_i (1 - \gamma_i) \\ &\quad - 2l_i q_{i0} \gamma_i - 2l_i q_{i0} \alpha_{i0} x_i - 2l_i \gamma_i \alpha_{i0} x_i, \end{aligned}$$

which can be simplified into:

$$\text{Var}\left(\sum_{j=1, j \neq i}^N T_{ij}\right) = d(p_0 - \alpha_{i0}p_i)(1 - p_0 - \alpha_{i0}p_i) + l_i(\gamma_i + q_{i0} + \alpha_{i0}x_i)(1 - \gamma_i - q_{i0} - \alpha_{i0}x_i). \quad (2.7)$$

3. SAME LINE VS. TRIM LINE STRATEGIES

In the last section, we derived the expressions for the expectation and the variance of the demand of the entire product line under the same line and trim line strategies (i.e. (2.2), (2.3), (2.6) and (2.7)). In this section, we develop a simple framework for comparing the Same Line and Trim Line strategies. Under the framework, the *pros* and *cons* associated with each of the strategies can be represented as a point on a two-dimensional map. The two dimensions on this map are the expected demand of the entire product line (vertical axis) and the variance of the demand of the entire product line (horizontal axis). The justification of this two-dimensional map is as follows. Observe that the expected revenue increases in expected demand, and that the expected costs (processing cost, production and capacity planning cost, and inventory cost) increases in the variance of the demand (The reader is referred to Silver and Peterson (1984) for an excellent discussion on how costs are affected by the variance

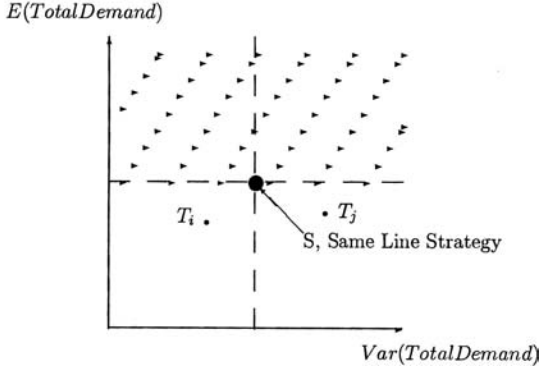


Figure 3. Same line strategy vs. trim line strategies.

of demand.). This observation implies that one can view the expected demand as a surrogate for the revenue and the variance of the demand as a surrogate for cost.

By using (2.2), (2.3), (2.6) and (2.7), we can map each of the strategies on this two dimensional map (see Figure 3). In Figure 3, the point s corresponds to the Same Line strategy that has coordinates $(E(\sum_{i=1}^N S_i), \text{Var}(\sum_{i=1}^N S_i))$. Similarly, the point T_i (or T_j) corresponds to the Trim Line strategy when product i (or product j) is trimmed from the line.⁷ In what follows, we shall show that (a) the relative position of each point T_i (or T_j) depends on the product to be trimmed; and (b) T_i and T_j can only be located in the unshaded region (in Figure 3).

In order to compare the Same Line and Trim Line strategy, let us examine the relative position of the point S associated with the Same Line strategy and the point T_i associated with the case when we trim product i under the Trim Line strategy. To do so, we shall first compare the expected demand of the entire product line under the Same Line and the Trim Line strategies, and then compare the variance of the total demand of the entire product line under the two strategies.

Based on equations (2.2) and (2.6), we define $Loss(i)$ as:

$$\begin{aligned}
 Loss(i) &= E\left(\sum_{j=1}^N S_j\right) - E\left(\sum_{j=1, j \neq i}^N T_{ij}\right), \\
 &= dp\alpha_{i0} + l_i(q_{i0} + y_i + x_i\alpha_{i0}) > 0.
 \end{aligned} \tag{3.1}$$

The term $Loss(i)$ represents the expected loss in sales as a result of trimming product i . Since the right hand side is always positive, we can conclude that:

7. Although we consider the case of trimming a single product, it is not difficult to see that repeated application of this two dimensional map will allow us to see the impact of trimming more than one product.

OBSERVATION 1: *The expected total demand associated with the Trim Line strategy is always lower than that of the Same Line strategy.*⁸

This observation implies that all points T_i will be located in the unshaded region in Figure 3; i.e., either at the south-west quadrant or the south-east quadrant from the point S . Note that the following remarks follow from (3.1):

- $Loss(i)$ increases in dp_i and l_i . Thus, products that have a smaller expected demand (i.e., $dp_i + l_i$) tend to result in a smaller loss in the demand for the entire product line. This result may appear to support the ‘lame duck’ heuristic. However, the expression for $Loss(i)$ also suggests that the loss is mediated by other parameters.⁹
- $Loss(i)$ increases in γ_i . Thus, highly unique products should not be trimmed because they are more likely to lead to customers leaving the product category, i.e., γ_i is high.
- $Loss(i)$ increases in x_i . In other words, products that exhibit significant ‘brand weakening’ phenomena, i.e. high probability of joining disloyal segment, should not be trimmed.
- $Loss(i)$ increases in α_{i0} and/or q_{i0} . It follows from the definitions of α_{i0} and q_{i0} that α_{i0} and q_{i0} increase when the competitive product 0 is a better substitute for product i than that of other products belonging to the firm. To reduce the expected loss of sales under the Trim Line strategy, it is desirable to trim a product i that has close substitutes offered by the firm but not offered by the competitors. This observation highlights the significance of product substitutability when deciding the product to be trimmed.

Next, based on equations (2.3) and (2.7), we define $Reduction(i)$ as:

$$Reduction(i) = Var\left(\sum_{j=1}^N S_j\right) - Var\left(\sum_{j=1, j \neq i}^N T_{ij}\right),$$

$$= dp_0(1 - p_0) - \{d(p_0 + \alpha_{i0}p_i)(1 - p_0 - \alpha_{i0}p_i) + l_i(\gamma_i + q_{i0} + \alpha_{i0}x_i)(1 - \gamma_i - q_{i0} - \alpha_{i0}x_i)\}, \quad (3.2)$$

$$= d\alpha_{i0}p_i(2p_0 + \alpha_{i0}p_i - 1) - l_i(\gamma_i + q_{i0} + \alpha_{i0}x_i)(1 - \gamma_i - q_{i0} - \alpha_{i0}x_i). \quad (3.3)$$

The term $Reduction(i)$ represents the reduction of variance in sales as a result of trimming product i . Notice from (3.3) that $Reduction(i)$ can be either positive or negative.

8. Note that our model does not incorporate the possibility that a smaller number of products may lead to increased sales because of less customer confusion.

9. Intuitively, $dp_i + l_i$ is the maximum potential loss and $Loss(i)$ is the expected loss after accounting for consumer switching behaviors.

This implies that the total variance can either decrease or increase as a result of trimming product i . In this case, we can conclude that:

OBSERVATION 2: *Depending on the values of the parameters, the variance of the total demand associated with the Trim Line strategy can be lower or higher than that of the Same Line strategy.*

Based on (3.2), we can make the following remarks:

- When $p_0 < 0.5$ and $(p_0 + \alpha_{i0} p_i) \leq 0.5$, we have $Reduction(i) < 0$.¹⁰ That is, if the firm holds a dominant position in the disloyal segment before line trimming (i.e., $\sum_{i=1}^N p_i = 1 - p_0 > 0.5$), trimming a product always lead to an increase in the variance of the demand of the entire product line. This finding implies that a leader in the disloyal segment is less likely to trim a product. Put differently, a dominant leader may have a wider product line than followers.
- When $p_0 > 0.5$, and $d \gg l_i$, we have $Reduction(i) > 0$. In words, if the firm has less than 50% market share in a large disloyal segment, then the firm can reduce its variance of the demand for the entire product line by trimming the product line.

Similarly, the following remarks follow from (3.3):

- $Reduction(i)$ increases in p_i when $p_0 \geq 0.5$. This implies that the larger the market share of a product in the disloyal segment where the competitor is a market leader, the more attractive (from the viewpoint of variance reduction) is trimming that product to the firm.
- $Reduction(i)$ increases in p_0 . If the firm has low market share in the disloyal segment, then the firm can reduce its variance by trimming the product line.
- $Reduction(i)$ decreases in q_{i0} if $y_i = q_{i0} + \alpha_{i0} x_i < 0.5$. So, if the trimmed product is similar to any of competitive products, the firm can experience an increase in the variance. This observation and the fact that $Loss(i)$ increases in q_{i0} imply that trimming a product similar to any competitive product reduces the expected value and simultaneously increases the variance of the demand for the entire product line.
- $Reduction(i)$ decreases in l_i . If each product has a sufficiently large loyal segment, then $Reduction(i) < 0$ for all i ; hence it does not pay to trim.

Based on observations 1 and 2, each point T_i would either located at the south-west quadrant or the south-east quadrant from the point S (see Figure 3). For any point T_i located at the south-east quadrant from the point S , there is a decrease in the expected demand and an increase in the variance of the demand if the firm trims product i from its product line. Clearly, it is undesirable to trim a product i when the corresponding point T_i is located at the south-east quadrant from the point S . This observation enables us to specify a *necessary condition* under which the same line strategy is optimal as follows:

10. Note that $x(1 - x)$ increases in x for $x < 0.5$ and decreases in x for $x > 0.5$.

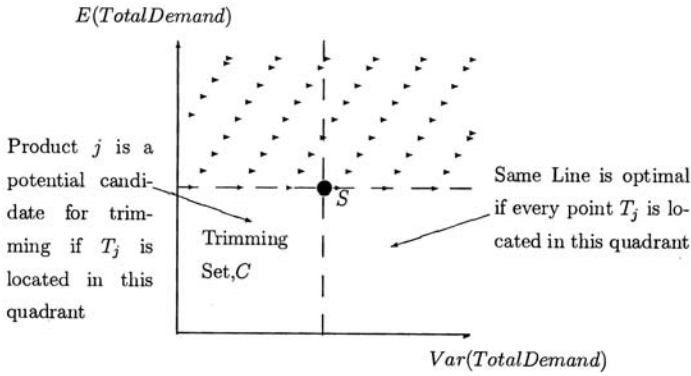


Figure 4. Optimality condition for same line strategy.

OBSERVATION 3: If $Reduction(i) < 0$ for each i , where $i = 1, \dots, N$, then the Same Line strategy is optimal.¹¹

If all T_i , ($i = 1, \dots, N$), are located at the south-east quadrant, the Same Line strategy is optimal (see Figure 4). When the above condition does not hold, then there exists a set of products, denoted by C , where

$$C = \{i : Reduction(i) > 0, i = 1, \dots, N\} \quad (3.4)$$

such that there will be a reduction in the variance of the demand as a result from trimming product $i \in C$ from the line. In this case, for each product i in the set C , the corresponding point T_i will be located at the south-west quadrant as depicted in Figure 4. Even though there is a reduction in variance when we trim a product $i \in C$, an expected loss, $Loss(i)$, would result from trimming product i . Hence, it is necessary to evaluate the trade-off between expected loss in demand and the reduction in the variance when selecting a product to trim. The actual trade-off would depend on the price/cost structure, which is beyond the scope of this paper.

In light of Observation 3, we now investigate the impact of substitutability, α_{ij} , on the Same Line and Trim line strategies. Since $\sum_{j=0, j \neq i}^N \alpha_{ij} = 1$, we can express α_{i0} , the substitutability of the competitive product for the firm's product, as $\alpha_{i0} = 1 - \sum_{j=1, j \neq i}^N \alpha_{ij}$. In this case, the substitutability with the competitive product, α_{i0} , decreases linearly as the substitutability with the firm's remaining product line, $\sum_{j=1, j \neq i}^N \alpha_{ij}$ increases. Hence, studying the impact of the substitutability with the competitive product, α_{i0} , is equivalent to studying the impact of substitutability with the firm's remaining product line, $\sum_{j=1, j \neq i}^N \alpha_{ij}$. For this reason, it suffices to focus

11. We assume that all benefits derived from trimming the product line are captured by the variance term. This is a reasonable assumption if the manufacturing technology is flexible enough that the unit cost is not affected by the number of products in the product line.

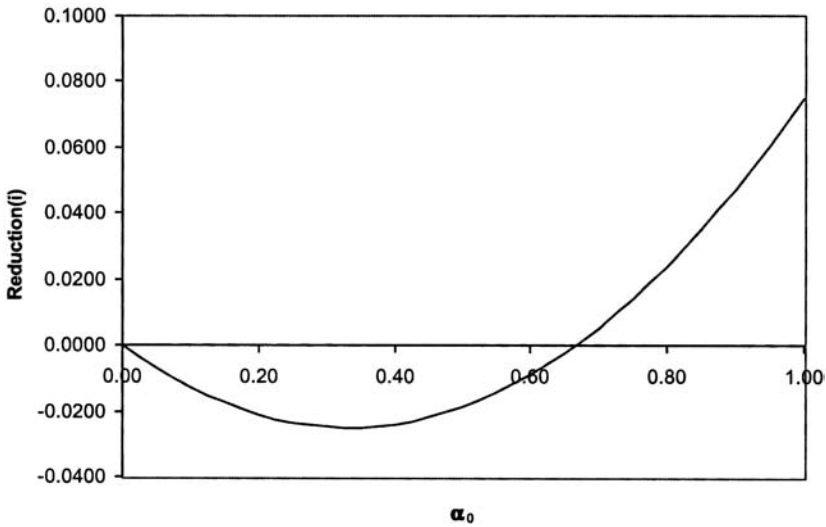


Figure 5. Reduction (i) as a quadratic function in α_0 .

on the impact α_0 on the Same Line and Trim Line strategies. In order to obtain some insights, we assume that there are no loyal segments (i.e., $l_i = 0, \forall i$).

Using Observation 3, we establish the condition for α_0 under which the Same Line strategy is optimal; i.e., the condition for α_0 such that $Reduction(i) < 0$ for all $i, i = 1, \dots, N$. It follows from equation (3.3) and the fact that $l_i = 0, \forall i$, we have:

$$Reduction(i) = d\alpha_0 p_i (2p_0 + \alpha_0 p_i - 1) \tag{3.5}$$

Recall from the second remark following observation 2 that $Reduction(i) > 0, \forall i$ when $p_0 \geq 0.5$, regardless of the value of α_0 . Therefore, it suffices to consider the case when $p_0 < 0.5$. Note from (3.5) that $Reduction(i)$ is a quadratic function in α_0 that has one root for $Reduction(i) = 0$ at $\alpha_0 = 0$. This implies that there exists another root $\theta_i > 0$ such that $Reduction(i) = 0$ at θ_i where $\theta_i > 0$ satisfies the equation $d\theta_i p_i (2p_0 + \theta_i p_i - 1) = 0$ or,

$$\theta_i = \frac{1 - 2p_0}{p_i} \tag{3.6}$$

This implies that $Reduction(i) < 0$ whenever α_0 is in the interval $0 < \alpha_0 < \theta_i$ (see Figure 5). It follows from Observation 3 that the Same Line strategy is optimal when $0 < \alpha_0 < \theta_i$. This leads to:

OBSERVATION 4: Consider a market consists entirely of disloyal segment (i.e., $l_i = 0, \forall i$). If the firm is a market leader in the disloyal segment (i.e., $p_0 < 0.5$) and the substitutabilities of competitive product for the firm's products are sufficiently low (i.e., when $\alpha_{i0} < \theta_i, \forall i$), then the Same Line strategy is optimal.

We now relate Observation 4 to the case in which product substitutability conforms to the classical attraction model of Bell, Keeney and Little (1975).¹² First, it can be shown (see Appendix) that the substitutability α_{ij} relates to the market share of product j in the disloyal segment p_j and the market share of the trimmed product i in the disloyal segment p_i as follows:

$$\alpha_{ij} = \frac{p_j}{1 - p_i} \quad (3.7)$$

and correspondingly for α_{i0} ,

$$\alpha_{i0} = \frac{p_0}{1 - p_i} \quad (3.8)$$

It follows from (3.6) and (3.8) that the condition $\alpha_{i0} < \theta_i$ can be rearranged into:

$$\frac{1 - 2p_0}{p_0} > \frac{p_i}{1 - p_i} \quad (3.9)$$

In this case, as stated in observation 4, Same Line strategy is optimal if (3.9) is true for all i . In other words, Same Line strategy is optimal if

$$\frac{1 - 2p_0}{p_0} > \max_i \left\{ \frac{p_i}{1 - p_i} \right\} \quad (3.10)$$

Notice that condition (3.10) is more likely to hold when the term $\max_i \{p_i/(1 - p_i)\}$ is minimized. Suppose that p_0 is fixed. Hence, $\sum_i p_i = 1 - p_0$ is also fixed. In this case, it is easy to see that $\max_i \{p_i/(1 - p_i)\}$ is minimized when the p_i 's are all equal. This implies that when the competitive market share p_0 is fixed, the Same Line strategy is more likely to be optimal if the market shares of the firm's products do not differ very much among themselves.

12. Assumption A4 of the classical attraction model by Bell, Keeney and Little (1975) assumes that the market share of a seller depends on the magnitude of the change in the attraction of other seller(s) but does not depend on which seller(s) is making the change. In other words, the change in the attraction of a seller does not impact *differentially* on other sellers (i.e. no asymmetry). By construction, our model allows for such an asymmetry because no specific form of substitutability, α_{ij} , is assumed

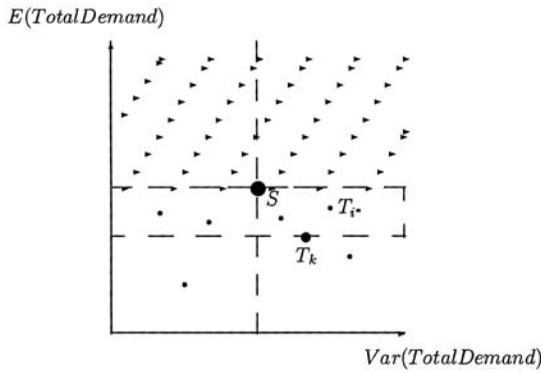


Figure 6. Lame duck strategy does not results in minimum loss.

4. THE LAME DUCK HEURISTIC

In the last section, we have presented a framework to analyze the conditions under which the Same Line strategy is optimal. In the event the same line strategy may not be optimal, the framework has also suggested a potential set of candidates to be trimmed. Now, we apply our framework to analyze the effectiveness of a common product trimming heuristic, i.e., the lame duck heuristic.¹³ Specifically, the lame duck heuristic prescribes the elimination of product k , where

$$k = \operatorname{argmin} \{j : E(S_j), j = 1, \dots, N\} = \operatorname{argmin} \{j : l_j + dp_j, j = 1, \dots, N\}. \quad (4.1)$$

By (4.1), it is obvious that the lame duck heuristic ignores the demand interactions and the substitutability among products.¹⁴ Without taking into account substitutability, the lame duck heuristic can be flawed for three reasons, as stated in the following three observations:

OBSERVATION 5: *In general, trimming the lame duck will not result in minimum loss in expected sales (i.e., $k \neq i^*$ where $i^* = \operatorname{argmin} \{j : \text{loss}(j), j = 1, \dots, N\}$). However, if $l_i = 0, \forall i$ and the classical attraction model for market share holds for the disloyal segment, then $k = i^*$.*

In Figure 6, the point T_k corresponds the lame duck whereas the point T_{i^*} corresponds to the product where trimming it results in minimum loss (or maximum expected demand after trimming). This can happen when the lame duck is a closer substitute to the competitive product than some products in the firm’s product line.

13. Various surveys in the literature cited in the introduction suggest that this is a common practice.

14. Assumption A4 of the attraction model by Bell, Keeney and Little (1975) basically implies that the impact of trimming is symmetric and linear. That is the impact of trimming product i on product j is proportionate to the market share of product j , which implicitly ignores the differences in substitutability. By trimming the product with smallest market share, the attraction model predicts a minimum loss of market share; hence, the attraction model essentially prescribes a lame duck selection.

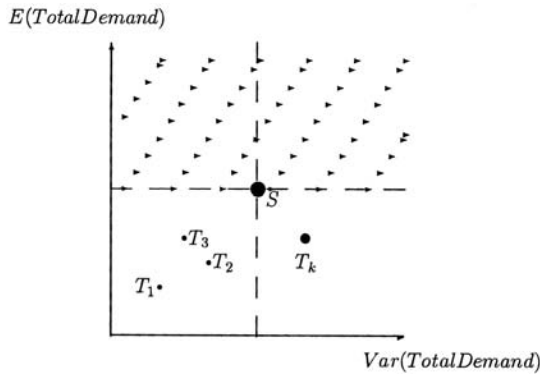


Figure 7. Lame duck strategy results in increased variance.

This can also happen when the lame duck is sufficiently unique that its removal results in a big portion of the loyal customers leaving the market.

Trimming lame duck will result in minimum loss when the attraction model is valid. To elaborate, let us consider the case in which the loyal segments are empty. From (3.1), we have $Loss(i) = d\alpha_i p_i$. If the substitutability of competitive product for product i , α_i , conforms to equation (3.8) (i.e., α_i agrees with the attraction model), then we have:

$$Loss(i) = d \frac{p_0}{1 - p_i} p_i = dp_0 \frac{p_i}{1 - p_i},$$

which can be minimized by choosing the product with the smallest p_i . Notice from (4.1) that the lame duck heuristic will select the product with the smallest p_i . In this case, we can conclude that the lame duck heuristic will produce minimum loss when the attraction model is valid.

OBSERVATION 6: *Under certain scenarios (discussed below), trimming the lame duck can result in a loss in sales and a simultaneous increase in demand variability, i.e., $Reduction(k) < 0$ and $loss(k) > 0$. Thus, it is sub optimal.*

Let us consider two cases. First, suppose that $Reduction(i) < 0$ for all i . Recall from the first remark following Observation 2 that this happens when the firm is a market leader (i.e. $p_0 < 0.5$) and each product does not have a significantly larger market share than others in the product line. In this case, Observation 3 implies that the Same Line strategy is optimal. Hence, the firm should not trim any product and the lame duck heuristic is definitely not appropriate. In the second case, suppose there exists some products j such that $Reduction(j) > 0$ for each j . Then the set C , as defined in (3.4), is not empty. Hence, it is quite reasonable to trim a product that belongs to the set C such that there is a reduction in the variance of the demand as a result of product trimming. Figure 7 depicts the case when $C = \{1,2,3\}$ and $k \notin C$. Since k

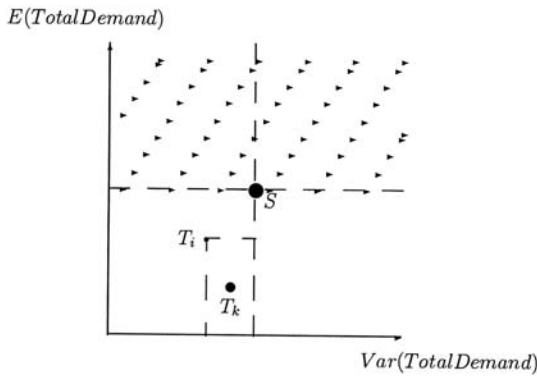


Figure 8. Lame duck strategy is dominated by trim i strategy.

$\notin C$, trimming product k would increase the variance of the demand (in addition to some loss in the expected sales, $loss(k)$). A possible scenario that could result in above situation is when the lame duck is sufficiently unique in the firm's product line, trimming the product results in the loyal customers either leaving the market completely (i.e. $y_k \gg 0$) or joining the disloyal segment (i.e. $x_k \gg 0$).

OBSERVATION 7: *Under certain scenario (discussed below), trimming the lame duck can be dominated even if it belongs to the trim set (i.e., $k \in C$).*

In other words, there exists at least one other product when trimmed is no worse off than trimming the lame duck and is strictly preferred on at least one of the two dimensions, i.e. expected demand and variance of demand. Figure 8 depicts such a case. In Figure 8, the point T_k corresponds the lame duck whereas the point T_i , located north-west from T_k , corresponds to a product when trimmed results in smaller loss and larger variance reduction than trimming the lame duck. One scenario where the lame duck is dominated is when the lame duck is highly substitutable by the competitive product in a market where the competitor has major market share in the disloyal segment.

5. AN ILLUSTRATIVE EXAMPLE

This section presents a numerical example that intends to serve two purposes: (1) to illustrate our findings in previous sections and (2) to show that lame duck heuristic can be flawed.

Consider a firm that offers two products, 1 and 2. Each product has a moderate size of loyal segment ($l_1 = 10, l_2 = 20$). The disloyal segment is large compared to the loyal segments ($d = 400$). Within the disloyal segment, the competitor is the market leader and product 1 has a larger market share than product 2. In our example, we have $p_0 = 0.7$, $p_1 = 0.2$ and $p_2 = 0.1$.

Based on managerial subjective estimates, the parameters associated with trimming product i are the same for both products 1 and 2. Specifically, upon trimming product

Table 1. Impact of same line and trim line strategies

Strategy	Same Line	Scenario 1: $\alpha_{10} = 0.05$ $\alpha_{20} = 0.10$		Scenario 2: $\alpha_{10} = 0.10$ $\alpha_{20} = 0.20$		Scenario 3: $\alpha_{10} = 0.20$ $\alpha_{20} = 0.40$	
		Trim 1	Trim 2	Trim 1	Trim 2	Trim 1	Trim 2
E(Total Demand)	150	143.8	141.2	139.6	136.4	131.2	126.8
Var(Total Demand)	84	84.08	86.01	82.46	84.67	78.98	81.57
$Loss(i)$	0	6.2	8.8	10.4	13.6	18.8	23.2
$Reduction(i)$	0	-0.08	-2.01	1.54	-0.67	5.02	2.43

$i, i = 1, 2$, each consumer in the loyal segment l_i would either join the disloyal segment with probability $x_i = 0.4$, or become loyal to the competitive product with probability $q_{i0} = 0.1$ (and $q_{12} = q_{21} = 0.4$), or leave the market completely with probability $y_i = 0.1$. Let us consider the following three scenarios with varying degree of substitutability with the competitive product:

Scenario 1: $\alpha_{10} = 0.05$ and $\alpha_{20} = 0.10$

Scenario 2: $\alpha_{10} = 0.10$ and $\alpha_{20} = 0.20$

Scenario 3: $\alpha_{10} = 0.20$ and $\alpha_{20} = 0.40$

Notice that products 1 and 2 become more likely to be substituted by the competitive product as we progress from Scenario 1 to Scenario 3. In addition, the ratio α_{20}/α_{10} is equal to 2 in each scenario, i.e. $\alpha_{20} > \alpha_{10}$. This implies product 2 is more likely than product 1 to be substituted by the competitive product. The impact of the Same Line and Trim Line strategies on the expected demand and the variance of demand for these three scenarios are summarized in Table 1.

Observe from table 1 that as product substitutability α_{i0} increases (as constructed for Scenario 1 through 3), both $Loss(i)$ and $Reduction(i)$ increase. This phenomenon is consistent with the fact that $Loss(i)$ is increasing in α_{i0} , as discussed in the fourth remark following Observation 1. Next, observe that $Reduction(i)$ increases as the product substitutability increases (as constructed for Scenario 1 through 3). This observation is consistent with the case depicted in (3.5) in which $Reduction(i)$ increases in α_{i0} when $p_0 > 0.5$. As the substitutability of competitive product for product i increases, trimming product i will result in increasing transfer of demand to the competitive product.

Let us plot the expected demand and the variance of demand for the three scenarios (as depicted in Table 1) in Figure 9. Each point $T_j^{(s)}$ in Figure 9 corresponds to the case where we trim product j in Scenario s . First, observe that all points $T_j^{(s)}, j = 1, 2, s = 1, 2, 3$ are located below the point S . It means that expected sales for Trim Line strategies are lower than that of Same Line strategy. This illustrates Observation 1. Second, each point $T_j^{(s)}$ either locates in the south-east quadrant or the south-west quadrant. This implies that the variance of demand could either increase or decrease as a result of trimming the product line. This illustrates Observation 2. Third, observe

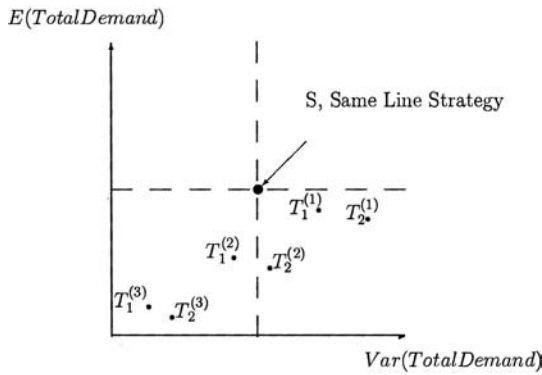


Figure 9. Comparing same line and trim line strategies.

that in Scenario 1, both $T_1^{(1)}$ and $T_2^{(1)}$ are located in the south-east quadrant. This implies that trimming either product 1 or product 2 would reduce expected demand and increase variance of demand; hence, Same Line strategy is optimal. This illustrates Observation 3.

We now illustrate when the lame duck heuristic could be flawed. In our example, product 2 is the lame duck, since $l_1 + d_{p1} = 90 > l_2 + d_{p2} = 60$; hence $k = 2$. Observe from Figure 9 that in each Scenario s , $T_1^{(s)}$ is located above $T_1^{(1)}$. This implies that the expected demand associated with trimming product 1 is consistently higher than that of product 2. This illustrates Observation 5 on why the lame duck heuristic could result in a lower sale. Next, consider Scenario 2. Observe that $T_1^{(2)}$ is located in the south-west quadrant from S while $T_2^{(2)}$ is located in the south-east quadrant from S . Hence, trimming product 1 would reduce the variance of total demand while trimming product 2 would increase the variance of total demand. Again, the lame duck heuristic leads to the wrong selection. This illustrates Observation 6. Finally, consider Scenario 3. Observe that $T_1^{(3)}$ is located north-west from $T_2^{(3)}$. This implies that trimming product 1 would result in a higher expected demand and lower expected variance than that of trimming product 2. In other words, product 2 is dominated by product 1. This illustrates Observation 7.

6. CONCLUDING REMARKS

Due to the increasing rate of new product introduction over the last two decades, many product lines have become over-crowded. Firms may improve their profits for its entire product line by trimming some of the products. In this paper, we developed a model to examine the impact of product trimming on the expected demand and demand variability for the entire product line. Our model is based on the consumer choice model developed by Grover and Srinivasan (1987). It captures the product substitution phenomenon. In addition, our model includes demand variability as an important determinant in making product trimming decision.

Our model exhibits the common observation that product line trimming leads to increased individual demands but to a reduced demand for the entire product line. Contrary to conventional wisdom, our model suggests that trimming the product with the minimal expected demand (i.e. the lame duck heuristic) can be flawed. We showed that this heuristic is deficient in three respects. First, it may not lead to minimal revenue loss. Second, even if it leads to minimal revenue loss, it can be sub-optimal if it leads to an increase in variance. Third, the lame duck heuristic may be dominated. Trimming another product may yield a lower loss and a higher variance reduction in expected demand. The heuristic appears to work well only when there is no demand substitution among products.

Carrying a smaller product line may not be strategically desirable for some firms. Smaller product line may translate to reduced presence at the retail front. Consider the grocery industry. Due to limited shelf space, supermarkets often allocate a fixed shelf space for carrying and displaying products from a particular manufacturer. Thus, the manufacturer may have to keep the breath of product line fixed to justify the shelf space. Consequently, the firm must determine which product to eliminate and which one to introduce. This product replacement decision is like a ‘football-team’ problem in which the ‘coach’ has to configure the best team to compete in the field.

Chong, Ho and Tang (2001) proposed a framework to capture the effect of product replacement on sales. Since their model conforms to the classical attraction model of Bell, Keeney and Little (1975), product substitutability implied by their model is similar to equation (3.7). Furthermore, they did not use sales variance in their modeling framework. By hybridizing our model with that of Ho and Tang (1995)¹⁵, we can have a model setup more general than Chong, Ho and Tang (2001). This general hybridized model seems like a logical next step for this research.

APPENDIX A: PROOF OF EQUATION (3.8)

By the definition of market share from Bell, Keeney and Little (1975), we have,

$$p_j = \frac{A_j}{\sum_{l=0}^N A_l}, \forall j,$$

where A_l is the level of attraction of product l , as defined in Bell, Keeney and Little (1975). If product i is trimmed, we have the corresponding revised market share:

$$p_j + \alpha_{ij} p_i = \frac{A_j}{\sum_{l=0, l \neq i}^N A_l c}, \forall j,$$

From the two equations above, we have,

15. The authors examined the issue of product line extension. The paper suggested conditions under which line extension is beneficial and related the benefits of line extension to market leadership and manufacturing capability.

$$\begin{aligned}
\alpha_{ij} &= \left\{ \frac{A_j}{\sum_{l=0, l \neq i}^N A_l} - \frac{A_j}{\sum_{l=0}^N A_l} \right\} / \frac{A_i}{\sum_{l=0}^N A_l}, \\
&= \frac{A_j}{A_i} \left\{ \frac{\sum_{l=0}^N A_l}{\sum_{l=0, l \neq i}^N A_l} - 1 \right\}, \\
&= \frac{A_j}{A_i} \left\{ 1 + \frac{A_j}{\sum_{l=0, l \neq i}^N A_l} - 1 \right\}, \\
&= \frac{A_j}{\sum_{l=0, l \neq i}^N A_l}, \\
&= p_j + \alpha_{ij} p_i.
\end{aligned}$$

This implies that,

$$\alpha_{ij} = \frac{p_j}{1 - p_i}$$

and also,

$$\begin{aligned}
\alpha_{i0} &= 1 - \sum_{j=1, j \neq i}^N \alpha_{ij}, \\
&= 1 - \sum_{j=1, j \neq i}^N \frac{p_j}{1 - p_i}, \\
&= \frac{p_0}{1 - p_i}.
\end{aligned}$$

REFERENCES

- Alexander, Ralph S., 'The Death and Burial of Sick Products,' *Journal of Marketing*, Vol 28, (April 1964), pp. 1-7.
- Avlonitis, G.J., 'Production Elimination Decision Making: Does Formality Matter?' *Journal of Marketing*, Vol. 49, (Winter 1985), pp. 41-52.
- Avlonitis, G.J. and James, B.G.S., 'Some Dangerous Axioms of Product Elimination Decision-Making,' *European Journal of Marketing*, Vol. 16, No. 1 (1982), pp. 36-48.
- Bell, D., Keeney, R. and Little, J., 'A market share theorem,' *Journal of Marketing Research*, Vol. 12 (August 1975), pp. 136-141.
- Berenson, Conrad, 'Pruning the Product-Line,' *Business Horizons*, Vol 6, (Summer 1963), pp. 63-70.
- Browne, W.G. and Kemp, P.S., 'A Three Stage Product Review Process,' *Industrial Marketing Management*, 5, (Dec 1976), pp. 333-342.
- Bayus, B. and Putsis, W., 'Product Proliferation: An Empirical Analysis of Product Line Determinants and Market Outcomes,' *Marketing Science*, Vol. 18, No. 2, (1999), pp. 137-153.
- Chong, J.K., Ho, T.H. and Tang, C.S., 'A Modeling Framework for Category Assortment Planning,' *Manufacturing & Service Operations Management*, Vol. 3, No. 3, (2001), pp. 191-210.
- Corstjens, M. and Weinstein, D., 'Optimal Strategic Business Units Portfolio Analysis,' *TIMS Studies in the Management Sciences*, 18, (1982), pp. 141-160.
- Green, Paul E. and Krieger, Abba M., 'Conjoint Analysis with Product Positioning Applications,' in

- J. Eliashberg and G.L. Lilien, Eds., *Handbooks in Operations Research and Management Science, Vol 5: Marketing*, North-Holland, Amsterdam, (1993), pp. 467–515.
11. Greenley, G., and Bayus, B., 'A Comparative Study of Product Launch and Elimination Decision in UK and US companies,' *European Journal of Marketing*, vol. 28, (1994), pp. 5–29.
 12. Grover, R. and Srinivasan, V., 'A Simultaneous Approach to Market Segmentation and Market Structuring,' *Journal Of Marketing Research*, Vol. 24 (may 1987), pp. 139–153.
 13. Hamelman P.W. and Mazze, E.M., 'Improving Product Abandonment Decisions,' *Journal Of Marketing*, Vol. 36, (April 1972), pp. 20–26.
 14. Hart, Susan J., 'The Causes of Product Deletion in British Manufacturing Companies,' *Journal of Marketing Management*, Vol. 3 No. 3, (1988), pp. 328–343.
 15. Hays, L., 'Too Many Computer Names Confuse Too Many Buyers,' *Wall Street Journal*, (June 29, 1994), B1, B6.
 16. Hise, R.T. and McGinnis, M.A., 'Product Elimination: Practices, Policies, and Ethics,' *Business Horizons*, 18, (June 1975), pp. 25–32.
 17. Hise, R.T., Parasuraman, A. and Viswanathan, R., 'Product Elimination: The Neglected Management Responsibility,' *The Journal of Business Strategy*, Vol. 4, No. 4, (Spring 1984), pp. 56–63.
 18. Ho, T and Tang, C.S., 'When is Product Line Extension Beneficial?' UCLA Working Paper, (1995)
 19. Khermouch, G., 'Amid Rampant Product Proliferation, Beverage Marketers Desperately Seeking Simplicity,' *Brandweek*, (May 29, 1995), 1, 9.
 20. Kotler, P., 'Phasing-out Weak Products,' *Harvard Business Review*, Vol. 43, (March-April, 1965), pp. 108–118.
 21. Larréché, J.C. and Srinivasan, V., 'STRATPORT: A Model for the Evaluation and Formulation of Business Portfolio Strategies,' *Management Science*, Vol. 28, (1982), pp. 979–1001.
 22. Mahajan, V. and Wind, Y., 'Integrating Financial Portfolio Analyses with Product Portfolio Models,' in H. Thomas and D. Garner, Eds., *Strategic Marketing and Management*, John Wiley & Sons, (1985), pp. 193–212.
 23. Mahajan, V., Wind, Y. and Bradford, J.W., 'Stochastic Dominance Rules for Product Portfolio Analysis,' *TIMS Studies in the Management Sciences*, 18, (1982), pp. 161–183.
 24. Narisetti, R., 'P&G, Seeing Shoppers Were Being Confused, Overhauls Marketing,' *Wall Street Journal*, (January 15, 1997), A1, A8.
 25. Putsis, W. and Bayus, B., 'An Empirical Analysis of Firms' Product Line Decisions,' *Journal of Marketing Research*, Vol. 38 (February 2001), pp. 110–118.
 26. Quelch, J. and Kenny, D., 'Extend Profits, Not the Product Lines,' *Harvard Business Review*, Vol. 72, (Sep–Oct, 1994), pp. 153–160.
 27. Silver, E.A. and Peterson, R., *Decision Systems for Inventory Management and Production Planning*. John Wiley & Sons, (1985).
 28. Teinowitz, Ira and Lawrence, Jennifer, 'Brand Proliferation Attacked,' *Advertising Age*, 1, (May 10 1993), pp. 49.
 29. Urban, G.L., 'A Mathematical Modeling Approach to Product Line Decisions,' *Journal of Marketing Research*, Vol. 6, (Feb. 1969), pp. 40–47.