2. Basic Principles

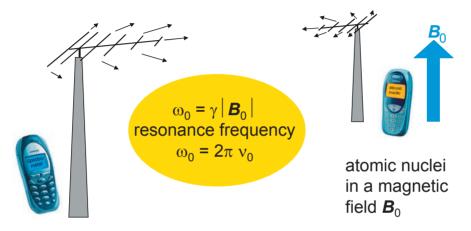
NMR spectrum Nuclear magnetism Rotating coordinate frame NMR spectrometer Pulse NMR Fourier transformation Phase correction Relaxation Spin echo Measurement methods Spatial resolution

Properties of Atomic Nuclei

- When exposed to magnetic fields, magnetic nuclei can receive and emit radio waves. Their *frequency* v₀ is proportional to the strength B₀ of the magnetic field: ω₀ = 2 π v₀ = γ B₀
- The constant of proportionality is the *gyro-magnetic ratio* γ. It is a characteristic constant of the nuclear isotope
- Examples of isotope abundance and radio frequencies are:

nuclear isotope	nat. abundance	v_0 at B_0 = 1.0 T
¹ H	99.98 %	42.57 MHz
¹⁴ N	99.63 %	3.08 MHz
¹⁹ F	100.00 %	40.05 MHz
¹³ C	1.108 %	10.71 MHz
¹²⁹ Xe	26.44 %	11.78 MHz

NMR is a Form of Telecommunication in a Magnetic Field

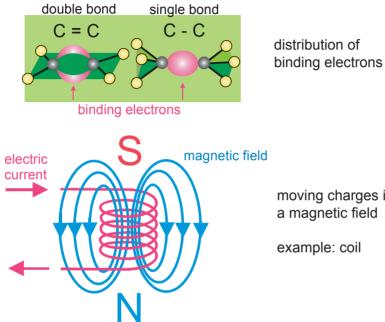


NMR spectrometer

Magnetic Shielding

- The NMR frequency is determined by the magnetic field at the site of the nucleus
- Atomic nuclei are surrounded by electrons
- In molecules, the electrons of the chemical bond are shared by different nuclei
- Electrons of atoms and molecules move in orbitals which are studied in quantum mechanics
- The orbitals of the binding electrons are characteristic of the *chemical structure* of the molecule
- Electrons carry an electric charge
- Electric charges in motion induce a magnetic field
- The internal magnetic field induced by the electrons moving in the external magnetic field **B**₀ is usually opposed to **B**₀. It shields the nucleus from **B**₀.

Electrons in Motion



moving charges induce a magnetic field

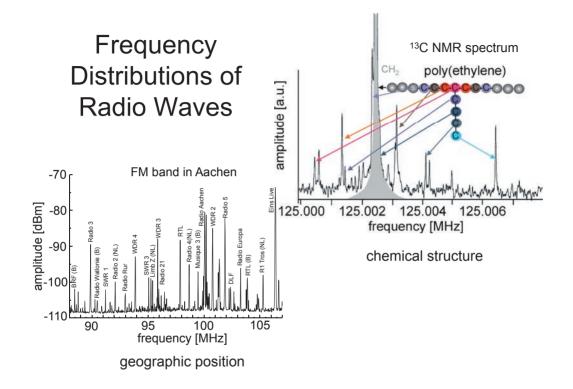
example: coil

Chemical Shift

• The induced magnetic field shifts the resonance frequency:

 $ω_{\rm L} = 2π v_{\rm L} = γ (1 - σ) B_0$

- $\bullet\,$ The quantity σ is the magnetic shielding for a given chemical group
- The quantity $\delta = (v_L v_{ref}) / v_{ref}$ is the *chemical shift* of a chemical group. It is independent of the magnetic field strength B_0 .
- The chemical shift can be calculated from tabulated chemical shift increments as well as *ab initio* from quantum mechanics
- The quantity v_{ref} is the reference frequency, for example, the resonance frequency of tetramethyl silane (TMS) for ¹H and ¹³C NMR
- Magnetically inequivalent chemical groups possess different chemical shifts
- In liquids narrow resonance signals are observed with typical widths of 0.1 Hz
- The distribution of resonance frequencies forms the NMR spectrum
- The NMR spectrum is a fingerprint of the molecular structure similar to a distribution of FM signals at a given location which is a fingerprint of the geographical position
- The acquisition of NMR spectra of molecules in solution is a standard method of analysis in following chemical synthesis



Nuclear Magnetism

- In a sample of material there are roughly 10²³ atomic nuclei per mole
- Some atomic nuclei have the properties of a magnetic dipole
- Examples: ¹H, ²H, ¹³C, ¹⁴N, ¹⁹F, ³¹P, ¹²⁹Xe
- Because atomic nuclei consist of a small number of elementary particles, the laws of classical physics do not apply. Instead the laws of *quantum mechanics* do
- According to quantum mechanics an elementary magnetic dipole with a dipole moment μ also possesses an *angular momentum* ħ I or *spin* I.
- In the laws of physics involving elementary particles Planck's constant h or ħ = h / (2π) appears
- A classical object with angular momentum is the *spinning top*
- A top spinning in a gravitational field formally follows the same laws as a spin in a magnetic field: it precesses around the direction of the field
- In NMR the precession frequency is called the Larmor frequency



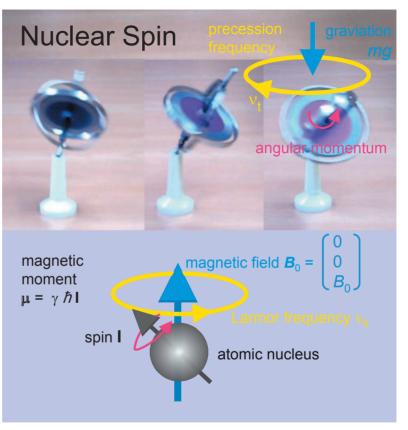
Arnold Sommerfeld, 1868 – 1951, Heisenberg's teacher, described the spinning top



Paul Adrien Maurice Dirac, 1902 – 1984, 1933 Nobel prize in Physics, postulated the existence of the spin



Otto Stern 1888 – 1963, 1943 Nobel prize in Physics, experimental discovery of the spin



Properties of Nuclear Spins

- Following Heisenberg's uncertainty principle, only the component of the spin in the direction of the magnetic field can be measured
- From quantum mechanics it is known that a spin with the *spin quantum number I* can assume 2*I* + 1 stable orientations in a magnetic field
- The projection of the spin angular momentum along the direction of the magnetic field is proportional to the *magnetic quantum number m*, where *m* = 1, 1-1, ..., -1
- $I = \frac{1}{2}$ is valid for the nuclei ¹H, ¹³C, ¹⁹F, ³¹P, ¹²⁹Xe and I = 1 for ²H, ¹⁴N
- For nuclei with spin *I* = ½ there are two possible orientations of it's projection along the axis of the magnetic field: ↑ und ↓
- Both orientations differ in the interaction energy $E_m = -\hbar \gamma m B_0$ of the nuclear magnetic dipoles with the magnetic field
- According to Bohr's formula $\Delta E = h v_0$ the energy difference $\Delta E = E_{-1/2} - E_{+1/2} = \hbar \gamma B_0$ associated with both orientations corresponds to the frequency $\omega_0 = 2\pi v_0 = \gamma B_0$
- $\bullet\,$ Here ν_0 is the precession frequency of the nuclear spins in the magnetic field



Niels Henrik David Bohr 1922 Nobel prize in Physics: $\Delta E = h v$



Felix Bloch. scholar of Heisenberg: 1905 - 1983. 1952 Nobel prize in Physics: NMR

Quantum Mechanics 1885 - 1962, energy $E = -\mu_z B_0$ magnetic field Bo $\mu_z = -\hbar\gamma/2$ **E**_{-1/2} $2\pi v_0 = \gamma B_0$ $\Delta E = h v_0$ magnetic, $\mu_z = +\hbar\gamma/2$ $E_{+1/2}$ frequencies at $B_0 = 7$ T: $v_0 = 300.5 \text{ MHz for }^{1}\text{H}$ $v_0 = 75.0 \text{ MHz for } {}^{13}\text{C}$

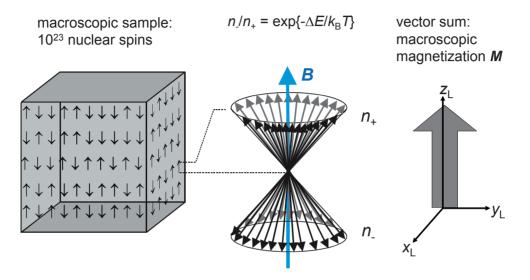


Edward Mills Purcell 1912 - 1997. 1952 Nobel prize in

Nuclear Magnetization in Thermodynamic Equilibrium

- All magnetic dipole moments are added as vectors; their components in each space direction are additive
- The sum of transverse components (if observable) vanishes
- The sum of longitudinal components constitutes the *longitudinal magnetization*
- This component is referred to as the *magnetic polarization* of the nuclei or the *nuclear magnetization*
- At room temperature only about 10¹⁸ spins of all 10²³ spins contribute to the macroscopic nuclear magnetization of the sample
- In the thermodynamic equilibrium state, the nuclear magnetization is oriented parallel to the direction of the magnetic field
- The direction of the magnetic field is referred to as the *z* direction of the *laboratory coordinate frame* LCF (index L)

Macroscopic Magnetization



Bloch's Equation

- When the magnetization *M* is not aligned with the *z*_L direction, it precesses around *z*_L with the frequency v₀ in complete analogy with the precession of a top spinning in a gravitational field *g*
- The precession is described by the equation for the magnetic *spinning top*:

$$\frac{\mathrm{d}}{\mathrm{d}t} \boldsymbol{M} = \gamma \boldsymbol{M} \times \boldsymbol{B}$$

- This equation states that any change d*M* of the magnetization *M* is perpendicular to *M* and *B*; therefore *M* precesses
- In general any macroscopic precessional motion is attenuated. This is why Felix Bloch introduced phenomenological attenuation terms:

$$\mathbf{R} = \begin{pmatrix} 1/T_2 & 0 & 0\\ 0 & 1/T_2 & 0\\ 0 & 0 & 1/T_1 \end{pmatrix}$$

• The resultant equation is the *Bloch equation*,

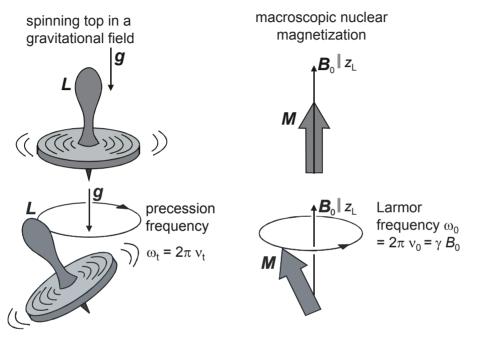
$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{M} = \boldsymbol{\gamma}\boldsymbol{M} \times \boldsymbol{B} - \boldsymbol{\mathsf{R}} (\boldsymbol{M} - \boldsymbol{M}_0)$$

where M_0 : initial magnetization, T_1 : longitudinal relaxation time,

 T_2 : transverse relaxation time

• Note: The Bloch equation formulates a left-handed rotation of the transverse magnetization. But for convenience sake a right handed one is followed throughout this text and many others in the literature

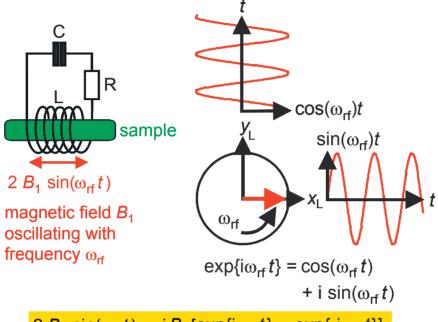
Precession of Nuclear Magnetization



Contacting Nuclear Magnetization

- Nuclear magnetization can be rotated away from the direction z_L of the magnetic polarization field B₀ by radio-frequency (rf) irradiation
- To this end one generates a magnetic field which rotates around ${\pmb B}_0$ with frequency ω_{rf}
- For maximum interaction of the rotating field with the nuclear magnetization the *resonance condition* $\omega_{rf} = \omega_0$ is chosen
- Because $v_0 = \omega_0/2\pi$ is a frequency in the radio-frequency regime, the rotating magnetic field is an electromagnetic *radio-frequency wave*
- High frequency electromagnetic waves are emitted from transmission antennas or oscillating electronic rf circuits
- An *electronic oscillator* consists of a *coil* with inductance *L*, a capacitor with capacitance *C*, and a resistor with resistance *R*
- The coil generates a linearly polarized, oscillating magnetic field $2B_1 \sin \omega_{rf} t$
- Two orthogonal, linearly polarized waves cosω_{rf}t und sinω_{rf}t generate a rotating wave
- A linearly polarized wave sinω_{rf}t can be decomposed into a right rotating wave ½ exp{iω_{rf}t} and a left rotating wave ½ exp{-iω_{rf}t}
- For optimum use of the oscillating magnetic field, the sample to be investigated is placed inside the coil

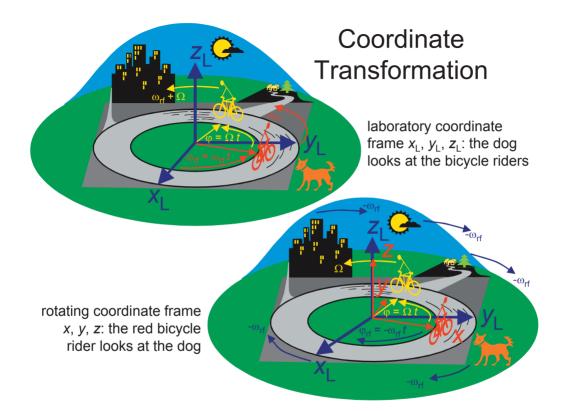
Magnetic Fields in an Oscillator Circuit



 $2 B_1 \sin(\omega_{rf} t) = -i B_1 [\exp\{i\omega_{rf} t\} - \exp\{-i\omega_{rf} t\}]$

Rotating Coordinate Frame

- Transformations from one coordinate frame into another change the point of view, i. e. they change the mathematics but not the physics
- As the precession of nuclear magnetization is a rotational motion and the rf excitation is a rotating wave, the magnetization is conveniently studied in a *rotating coordinate frame* (RCF)
- The dog at the traffic circuit is positioned in the *laboratory coordinate* frame (LCF): For him the bicycles are driving in the traffic circuit with angular velocities ω_{rf} and $\omega_{rf} + \Omega$
- The cyclists on the bicycles are viewing the world from the RCF. They are at rest in their respective RCF
- For the red cyclist the world is rotating against the direction of his bicycle with angular velocity - ω_{rf}
- For the red cyclist the yellow bicycle rides with angular velocity Ω in his RCF
- The connecting vectors from the center of the traffic circle to the bicycles correspond to the magnetization vectors in the transverse *xy* plane
- The angular velocity of the RCF as seen in the LCF corresponds to the frequency ω_{rf} of the rf wave

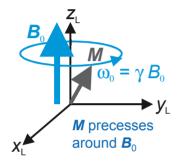


Radio-Frequency Pulses

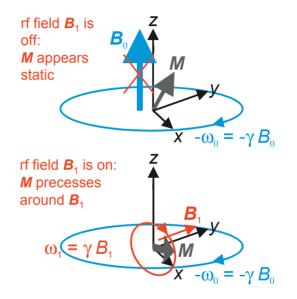
- In a coordinate system, which rotates with frequency ω₀ around the z axis the magnetization *M* appears at rest even if it is not parallel to the magnetic field *B*₀
- When the magnetization is not rotating, there is no magnetic field active in that frame which produces a torque on the magnetization
- On resonance $\omega_{rf} = \omega_0$, and the rf field **B**₁ is time independent and appears static in the RCF when turned on
- In the RCF, which rotates in the LCF with $\omega_{rf} = \omega_0$ around \boldsymbol{B}_0 , the magnetization rotates around the \boldsymbol{B}_1 field with frequency $\omega_1 = \gamma |\boldsymbol{B}_1|$ in analogy to the rotation with frequency $\omega_0 = \gamma |\boldsymbol{B}_0|$ around the \boldsymbol{B}_0 field in the LCF
- If B_1 is turned on in a pulsed fashion for a time t_P , a 90° pulse is defined for $\omega_1 t_P = 90^\circ$ and a 180° pulse for $\omega_1 t_P = 180^\circ$
- The phase φ₀ of the *rotating rf field* B₁exp{iω_{rf}t + i φ₀} defines the direction of the B₁ field in the xy plane of the RCF
- Using this phase the magnetization can be rotated in the RCF around different axes, e. g. 90°_y denotes a positive 90° rotation around the y axis of the RCF and 180°_x a positive 180° rotation around the x axis

Action of rf Pulses

laboratory coordinate frame



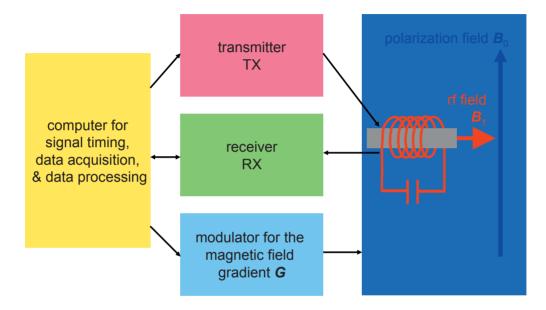
90° pulse: $\omega_1 t_p = \pi/2$ 180° pulse: $\omega_1 t_p = \pi$ rotating coordinate frame



NMR Spectrometer

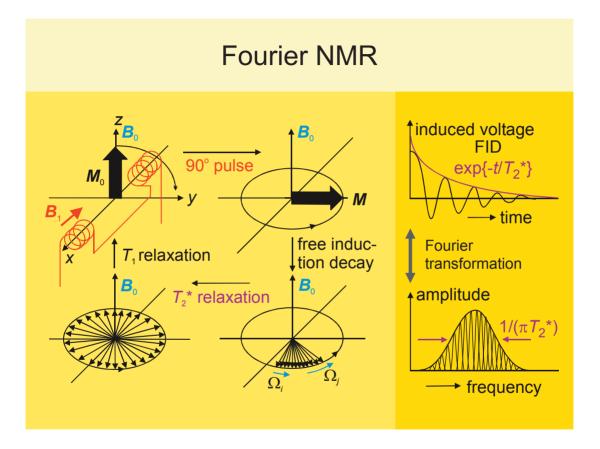
- The sample is positioned in a magnetic field B₀ inside a rf coil which is part of a rf oscillator tuned to the frequency ω_{rf}
- The oscillator is connected under computer control either to the rf *transmitter* (TX) or to the *receiver* (RX)
- A 90° rf pulse from the transmitter rotates the magnetization from the *z*_L direction of the *B*₀ field into the transverse plane
- Following the pulse, the transverse magnetization components precess around the z₁ axis of the LCF with frequency ω₀
- According to the dynamo principle, the precessing magnetization induces a voltage in the coil which oscillates at frequency ω_0
- In the receiver, this signal is mixed with a reference wave at frequency ω_{rf} , and the audio signal at the difference frequency is filtered for acquisition
- This step is the transition into the rotating coordinate frame
- Depending on the phase $\phi_0 = 0^\circ$ and 90° of the reference wave $\sin(\omega_{rf}t + \phi_0)$ the *quadrature components* $\sin(\omega_0 \omega_{rf})t$ and $\cos(\omega_0 \omega_{rf})t$ of the transverse magnetization are measured in the RCF, respectively
- Usually both quadrature components are measured simultaneously
- For imaging and flow measurements the spectrometer is equipped with switchable *gradient fields* in *x*_L, *y*_L, and *z*_L directions of the LCF

Spectrometer Hardware



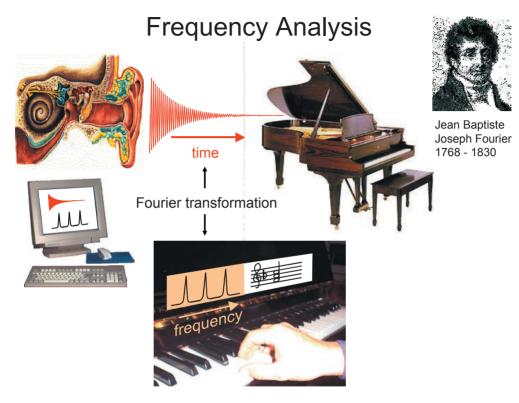
Pulse Excitation

- Outside a magnetic field the nuclear magnetic dipole moments are oriented in random directions in space
- When introducing the sample into the magnetic field B_0 , the *longitudinal* magnetization M_0 is formed parallel to B_0 with the time constant T_1 by aligning the nuclear magnetic moments according to the Boltzmann distribution: $M_z(t) = M_0 (1 \exp\{-t/T_1\})$
- A 90° rf pulse from the transmitter rotates the magnetization from the *z* direction of the magnetic field **B**₀ into the transverse plane of the RCF
- After the rf pulse the transverse components M_i of the magnetization precess around the z axis of the RCF with the difference frequencies $\Omega_i = \omega_{ri} - \omega_{rf}$
- Each component *M*_i corresponds to a different chemical shift or another position in the sample with a different magnetic polarization field
- The vector sum of the *transverse magnetization* components decays with the time constant T₂^{*} due to interference of the components with different precession frequencies Ω_i
- T_2^* is the *transverse relaxation time* due to time-invariant and timedependent local magnetic fields
- The signal decay is often exponential: $M_{xy}(t) = M_z(0) \exp\{-t/T_2^*\}$
- The signal induced in the coil after pulse excitation is the *free induction* decay (*FID*)
- The frequency analysis of the FID by *Fourier transformation* produces the *NMR spectrum* with a *linewidth* $\Delta\Omega = 1/(\pi T_2^*)$



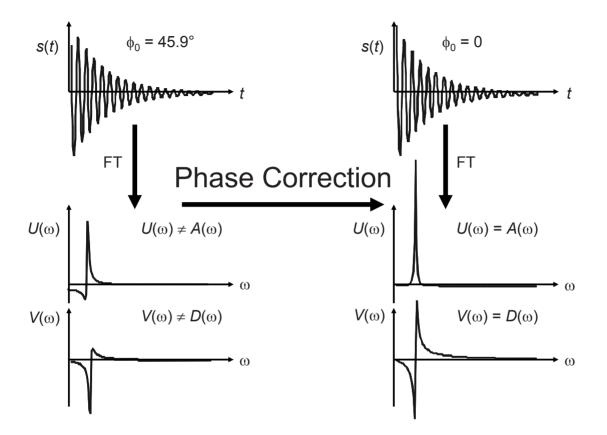
Fourier Transformation

- Fourier introduced the transformation named after him when studying thermal conductivity
- The Fourier transformation (FT) is a decomposition of a function s(t) into harmonic waves exp{i ωt} = cosωt + i sinωt with variable frequency ω
- In NMR the *FID* s(t) is transformed to the spectrum S(ω) of cosine and sine waves: S(ω) = ∫s(t) exp{-iωt}dt
- The spectrum S(ω) = U(ω) + i V(ω) consists of a real part U(ω) and an imaginary part V(ω)
- Often, only the magnitude spectrum $|S(\omega)| = [U(\omega)^2 + V(\omega)^2]^{1/2}$ is employed
- The Fourier transformation corresponds to the transformation of an acoustic signal into the colors of sound when listening to it
- For the discrete Fourier transformation there is a fast algorithm which was rediscovered in 1965 by J. W. Cooley and J. W. Tukey
- The algorithm requires the discrete representation of the time function s(t) and the spectrum $S(\omega)$ in steps Δt and $\Delta \omega$ of the variables t and ω
- The abscissa of the discrete spectrum corresponds to the keys of a piano
- The spectral amplitude corresponds to the volume of a given tone
- In NMR with pulsed excitation the Fourier transformation is part of processing the data
- Pulsed NMR is also called Fourier NMR
- The product of two Fourier conjugated variables, e. g. *t* and ω, is always an angle. It is referred to as *phase*



Signal Processing

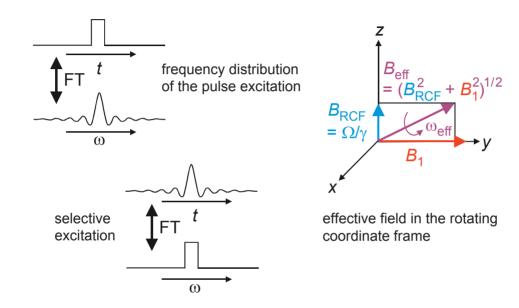
- Depending on the phase φ₀ of the *rotating coordinate frame*, the *FID* is measured as the sum of impulse responses s(t) = s(0) exp{-[1/T₂ iΩ] t + i φ₀} for each magnetization component with different Ω in the RCF
- For $\phi_0 = 0$ the real part $U(\omega)$ of the Fourier transform $S(\omega)$ is an absorption signal $A(\omega)$ and the imaginary part $V(\omega)$ a dispersion signal $D(\omega)$
- For φ₀ ≠ 0 the absorptive and dispersive components are mixed in U(ω) and V(ω), and the associated complex spectrum S(ω) = U(ω) + i V(ω) = [A(ω) + i D(ω)] exp{i φ₀} has to be corrected in phase by multiplication with exp{-i φ₀}
- The correction phase ϕ_0 consists of a frequency dependent and a frequency independent part
- The frequency-independent part can be adjusted by software before data acquisition via the rf reference phase of the spectrometer
- The frequency dependent part is determined by time the signals take to pass through the spectrometer and by the receiver *deadtime* following an excitation pulse
- For optimum resolution the spectrum is needed in pure absorptive mode
- A frequency dependent *phase correction* of the spectrum is a routine step in data processing of high-resolution NMR spectroscopy



Frequency Distributions

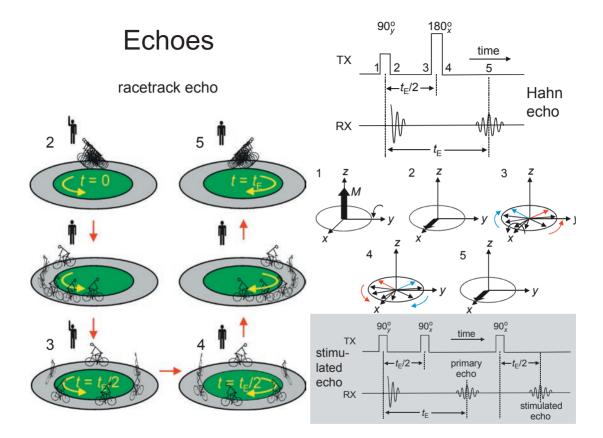
- $\bullet\,$ The rotating coordinate frame rotates with the rf frequency ω_{rf}
- In the laboratory frame the magnetization components M_i rotate with frequencies ω_{Li}
- The *rf pulse* with frequency ω_{rf} has to couple to several frequencies ω_{Li}
- The *bandwidth of the excitation* pulse is determined in approximation by the inverse of the pulse width t_p
- A better measure for the frequency dependence of the excitation is the *Fourier transform* of the excitation pulse
- For a rectangular pulse the Fourier transform is the sinc function
- Vice versa, the excitation can be made frequency selective by excitation with a rf pulse having a sinc shape in the time domain
- This simple Fourier relationship is a convenient approximation valid for small flip angles only
- In the RCF the magnetization components rotate with frequencies $\Omega_i = \omega_{Li} \omega_{rf}$
- For a given component the offset frequency Ω corresponds to a magnetic off-set field Ω/γ along the z axis of the RCF
- The magnetization always rotates around the *effective field* B_{eff}, which is the vector sum of the offset field Ω/γ and the rf field B₁
- The rotation angle of a pulse is then given by $\gamma B_{eff} t_p = \omega_{eff} t_p$
- The rotation axis is in the xy plane if $|B_1| >> |\Omega/\gamma|$
- If $|B_1| \ll |\Omega/\gamma|$, longitudinal magnetization cannot be rotated into the xy plane

RF Excitation and Effective Field



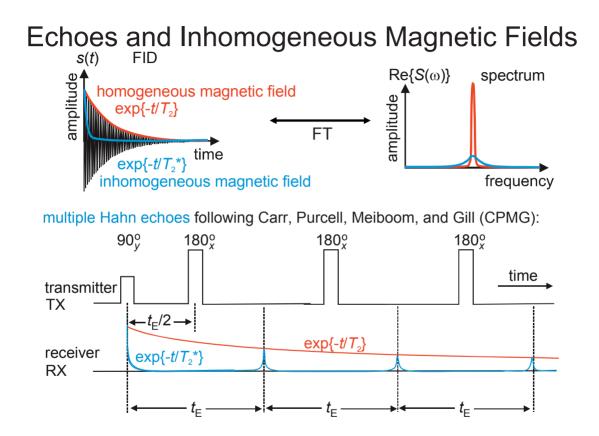
Relaxation

- Relaxation denotes the loss of transverse magnetization with the time constant T₂ and build-up of longitudinal magnetization with T₁
- The loss of transverse magnetization due to different time-invariant local magnetic fields can stroboscopically be reversed by formation of *echoes*
- For formation of a *racetrack echo* all bicyclists start at the same time but ride with different speeds. At a certain time all go back and meet at the starting line forming the *echo* after twice that time
- Their total riding time is the echo time t_E
- The NMR echo has accidentally been discovered in 1949 by Erwin Hahn
- For formation of a *Hahn echo* all transverse magnetization components are rotated by 180° around an axis in the *xy* plane
- The direction of precession is maintained with this change of positions on the circle, and all magnetization components refocus at time t_E
- If some components randomly change their precession frequencies, the *echo amplitude* is irreversibly reduced
- Random frequency changes arise from fluctuating local magnetic fields associated with molecules in motion
- T_2 relaxation denotes the irreversible loss of the echo amplitude
- Both *relaxation times* T_1 and T_2 are determined by the type and time scale of molecular motion
- By splitting the 180° pulse of the Hahn echo sequence into two 90° pulses separated by a time delay, one obtains the *stimulated echo* sequence



Multiple Echoes

- Transverse relaxation is often exponential with the time constant T_2
- In *inhomogeneous magnetic fields*, the FID decays faster with $T_2^* < T_2$
- The resonance signal in inhomogeneous magnetic fields is broad and small
- The envelope of the *FID* in homogeneous fields can be observed stroboscopically in inhomogeneous fields via the amplitude of many time shifted echoes
- Instead of many Hahn echoes with different echo times the echo envelope can be observed by a single train of multiple Hahn echoes
- The rf pulse scheme for excitation of multiple Hahn echoes is the *CPMG* sequence named after their discoverers Carr, Purcell, Meiboom, and Gill
- The repetition times of 5*T*₁ for regeneration of longitudinal magnetization between generation of different Hahn echoes are eliminated
- 5T₁ are needed to regain 99% of the thermodynamic equilibrium magnetization, because exp{-5} = 0.007
- Besides the Hahn echo and the CPMG echo train there are many more echoes and multiple-echo schemes to partially recover signal loss caused by the influence of different nuclear spin interactions on the resonance frequencies
- In the Hahn echo maximum, inhomogeneities in the B₀ field and the spread in chemical shifts do not affect the NMR signal



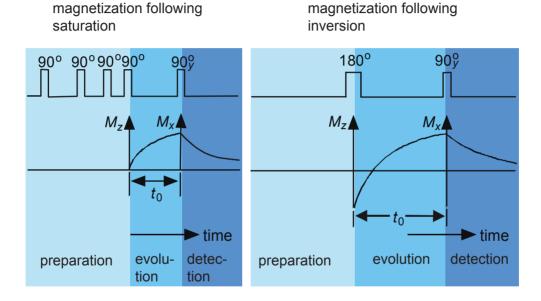
Determination of T_1

- Longitudinal magnetization cannot be directly observed
- Its momentary value can be interrogated via the amplitude of the FID following a 90° pulse
- There are two methods to measure the build-up of longitudinal magnetization: the recovery following saturation (*saturation recovery*) and the recovery following inversion of the magnetization (*inversion recovery*)
- For saturation, the spin system is irradiated with an aperiodic sequence of 90° pulses which destroys all magnetization
- Inversion of longitudinal magnetization is achieved by a 180° pulse following the establishment of equilibrium magnetization after a waiting time of $5T_1$
- After such preparation of the initial magnetization a variable evolution time t_0 follows for partial recovery of the *thermodynamic equilibrium* state
- Following the waiting time *t*₀, the momentary value of the longitudinal magnetization is converted into the amplitude of the *transverse magnetization* by a 90° pulse
- The transverse magnetization is measured and evaluated for different values of *t*₀
- In homogeneous spin systems, the longitudinal relaxation follows an exponential law

Pulse Sequences for Measurement of T_1

build-up of longitudinal

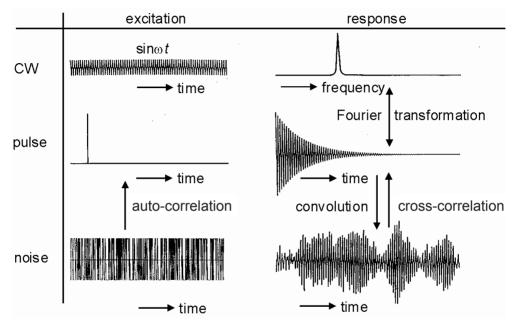
build-up of longitudinal



Measuring Methods

- *Pulsed excitation* and acquisition of an impulse response or an echo constitute the most successful class of methods to acquire NMR data
- In pulsed NMR, the signal measured can be conditioned by manipulating the initial magnetization in preceding preparation and evolution periods
- Pulsed NMR is uniquely suited for extension to multi-dimensional NMR
- With short pulses large spectral widths can be excited, and many frequency components can be simultaneously measured (*multiplex advantage*)
- When exciting the spins with *continuous waves* (CW), the frequency of the excitation wave is slowly scanned through the spectrum
- CW NMR is slow, because the frequency components of the spectrum are measured successively
- With *noise excitation* large bandwidths are excited and can be measured simultaneously (*stochastic NMR*)
- A division of the experiment into different periods such as preparation, evolution, and detection is not possible
- Such a partitioning of the time axis can be achieved during data processing by means of *cross-correlation* of excitation and response signals
- The *excitation power* in CW NMR and stochastic NMR is several orders of magnitude lower compared to that of pulsed NMR

CW-, Fourier, and Stochastic NMR



B. Blümich, Prog. Nucl. Magn. Reson. Spectr. 19 (1987) 331 - 417

Spatial Resolution

- By exploring the proportionality of the NMR frequency ω and the applied magnetic field *B*, signals from different positions in the sample can be discriminated if the magnetic field changes with position
- For a linear change of *B* with position, the NMR frequency ω is directly proportional to position
- Then, the magnetic field *B* is characterized by a space-invariant gradient *G*
- In such a gradient field, the linear frequency axis of an NMR spectrum can be directly replaced by a linear space axis
- The signal amplitude is determined by the number of nuclear spins at a particular position along the gradient direction
- This number is obtained by summation over all nuclei in the other two space directions
- Due to the large number of nuclei, the sum is written as an integral
- This integral over the spatially resolved magnetization $M_z(x,y,z)$ is called a *projection*
- $M_z(x,y,z)$ is also referred to as *spin density*
- From a set of projections acquired for different gradient directions an image of the object can be reconstructed in analogy to X-ray tomography

Space Encoding

