

Table of Contents

| | |
|---------------|---|
| Preface | V |
|---------------|---|

Part I. Representative Agent Models

| | |
|---|----|
| 1 Basic Models and Elementary Algorithms | 3 |
| 1.1 The Deterministic Finite Horizon Ramsey Model and Non-linear Programming | 4 |
| 1.1.1 The Ramsey Problem..... | 4 |
| 1.1.2 The Kuhn-Tucker Theorem..... | 6 |
| 1.1.3 Numerical Solutions | 8 |
| 1.2 The Deterministic Infinite Horizon Ramsey Model and Dynamic Programming | 11 |
| 1.2.1 Recursive Utility | 11 |
| 1.2.2 Euler Equations | 13 |
| 1.2.3 Dynamic Programming | 14 |
| 1.2.4 The Saddle Path | 17 |
| 1.2.5 Numerical Solutions | 20 |
| 1.3 The Stochastic Ramsey Model | 33 |
| 1.3.1 Stochastic Output..... | 33 |
| 1.3.2 Stochastic Euler Equations | 36 |
| 1.3.3 Stochastic Dynamic Programming..... | 38 |
| 1.3.4 Numerical Solutions | 39 |
| 1.4 Labor Supply, Growth, and the Decentralized Economy..... | 49 |
| 1.4.1 Substitution of Leisure..... | 49 |
| 1.4.2 Growth and Restrictions on Technology and Preferences..... | 50 |
| 1.4.3 The Decentralized Economy | 57 |

| | | |
|----------------------|--|-----|
| 1.4.4 | Numerical Solutions | 60 |
| 1.5 | Model Calibration and Evaluation | 64 |
| Appendices | | 73 |
| A.1 | Solution to Example 1.2.1 | 73 |
| A.2 | Restrictions on Technology and Preferences | 75 |
| Problems | | 80 |
| 2 | Linear Quadratic and Linear Approximation | |
| Methods | | 85 |
| 2.1 | The Linear Quadratic Model | 86 |
| 2.1.1 | Description | 86 |
| 2.1.2 | Derivation of the Policy Function | 87 |
| 2.1.3 | Certainty Equivalence | 88 |
| 2.1.4 | Derivation of the Euler Equations | 89 |
| 2.2 | LQ Approximation | 91 |
| 2.2.1 | An Illustrative Example | 91 |
| 2.2.2 | The General Method | 96 |
| 2.3 | Loglinear Approximation | 99 |
| 2.3.1 | An Illustrative Example | 99 |
| 2.3.2 | The General Method | 107 |
| 2.4 | Applications | 113 |
| 2.4.1 | The Benchmark Model | 113 |
| 2.4.2 | Time to Build | 120 |
| 2.4.3 | New Keynesian Phillips Curve | 125 |
| Appendices | | 139 |
| A.3 | Solution of the Stochastic LQ problem | 139 |
| A.4 | Derivation of the Loglinear Model of the New Keynesian Phillips Curve | 141 |
| Problems | | 149 |
| 3 | Parameterized Expectations | 155 |
| 3.1 | Characterization of Approximate Solutions | 156 |
| 3.1.1 | An Illustrative Example | 156 |
| 3.1.2 | A General Framework | 159 |
| 3.1.3 | Adaptive Learning | 161 |
| 3.2 | Computation of the Approximate Solution | 164 |
| 3.2.1 | Choice of T and ψ | 164 |

| | | |
|-------|--|-----|
| 3.2.2 | Iterative Computation of the Fixed Point .. | 166 |
| 3.2.3 | Direct Computation of the Fixed Point | 167 |
| 3.2.4 | Starting Points | 169 |
| 3.2.5 | Accuracy of Solutions | 171 |
| 3.3 | Applications | 172 |
| 3.3.1 | Stochastic Growth with Non-negative Investment | 172 |
| 3.3.2 | The Benchmark Model | 178 |
| 3.3.3 | Limited Participation in Financial Markets . | 181 |
| | Problems | 194 |
| 4 | Projection Methods | 197 |
| 4.1 | Characterization of Projection Methods | 198 |
| 4.1.1 | An Example | 198 |
| 4.1.2 | The General Framework | 201 |
| 4.1.3 | Parameterized Expectations and Projection Methods | 204 |
| 4.2 | The Building Blocks of Projection Methods | 205 |
| 4.2.1 | Approximating Function | 205 |
| 4.2.2 | Residual Function | 206 |
| 4.2.3 | Projection and Solution | 207 |
| 4.2.4 | Accuracy of Solution | 209 |
| 4.3 | Applications | 210 |
| 4.3.1 | The Deterministic Growth Model | 210 |
| 4.3.2 | The Stochastic Growth Model with Non-negative Investment | 215 |
| 4.3.3 | The Equity Premium Puzzle | 221 |
| | Problems | 235 |

Part II. Heterogeneous Agent Models

| | | |
|-----|--|-----|
| 5 | Computation of Stationary Distributions | 239 |
| 5.1 | A Simple Heterogeneous-Agent Model with Aggregate Certainty | 240 |
| 5.2 | Computation of the Stationary Equilibrium of a Heterogeneous Agent Economy | 247 |

| | |
|--|-----|
| 5.3 Applications | 268 |
| 5.3.1 The Risk-Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies | 268 |
| 5.3.2 Heterogeneous Productivity and Income Distribution | 275 |
| Problems | 292 |
| | |
| 6 Dynamics of the Distribution Function | 297 |
| 6.1 The Dynamics of Heterogeneous-Agent Economies | 298 |
| 6.2 Transition Dynamics | 301 |
| 6.2.1 Partial Information | 303 |
| 6.2.2 Guessing a Finite Time Path for the Factor Prices | 313 |
| 6.3 Aggregate Uncertainty | 317 |
| 6.4 Applications | 329 |
| 6.4.1 Costs of Business Cycles with Indivisibilities and Liquidity Constraints | 330 |
| 6.4.2 Business Cycle Dynamics of the Income Distribution | 339 |
| Problems | 354 |
| | |
| 7 Overlapping Generations Models | 357 |
| 7.1 Life-cycle Model with Perfect Foresight | 359 |
| 7.1.1 An Illustrative Example | 360 |
| 7.1.2 Computation of the Steady State | 364 |
| 7.1.3 Computation of the Transition Path | 373 |
| 7.2 Life-cycle Economies with Individual or Aggregate Uncertainty | 381 |
| 7.2.1 Overlapping Generations Models with Individual Uncertainty | 381 |
| 7.2.2 Aggregate Uncertainty | 395 |
| Problems | 408 |

Part III. Tools

| | | |
|-------|--|-----|
| 8 | Numerical Methods | 413 |
| 8.1 | A Quick Refresher in Linear Algebra | 413 |
| 8.1.1 | Complex Numbers | 413 |
| 8.1.2 | Vectors | 414 |
| 8.1.3 | Norms | 414 |
| 8.1.4 | Matrices | 415 |
| 8.1.5 | Linear and Quadratic Forms | 418 |
| 8.1.6 | Eigenvalues and Eigenvectors | 420 |
| 8.1.7 | Matrix Factorization | 421 |
| 8.1.8 | Givens Rotation | 424 |
| 8.2 | Function Approximation | 425 |
| 8.2.1 | Taylor's Theorem | 425 |
| 8.2.2 | Linear Interpolation | 428 |
| 8.2.3 | Families of Polynomials | 429 |
| 8.2.4 | Chebyshev Polynomials | 431 |
| 8.2.5 | Multidimensional Approximation | 439 |
| 8.2.6 | Neural Networks | 442 |
| 8.3 | Numerical Differentiation and Integration | 443 |
| 8.3.1 | Differentiation | 443 |
| 8.3.2 | Numerical Integration | 448 |
| 8.4 | Stopping Criteria for Iterative Algorithms | 453 |
| 8.5 | Non Linear Equations | 456 |
| 8.5.1 | The Newton-Raphson Method | 457 |
| 8.5.2 | A Globally Convergent Newton-Raphson Method | 462 |
| 8.6 | Numerical Optimization | 465 |
| 8.6.1 | Golden Section Search | 466 |
| 8.6.2 | Gauss-Newton Method | 469 |
| 8.6.3 | Quasi-Newton | 472 |
| 8.6.4 | Genetic Algorithms | 476 |
| 9 | Various Other Tools | 487 |
| 9.1 | Difference Equations | 487 |
| 9.1.1 | Linear Difference Equations | 487 |

| | |
|---|------------|
| 9.1.2 Non-Linear Difference Equations | 491 |
| 9.2 Markov Processes | 493 |
| 9.3 DM-Statistic | 499 |
| 9.4 The HP-Filter | 503 |
| Epilogue | 507 |
| References | 511 |
| Author Index | 527 |
| Subject Index | 531 |