1
View into the quantum world I: fundamental phenomena and concepts

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1.1 Introduction

With this chapter and the following one, which has the title "Entanglement and its consequences", an introduction into the world of quantum physics and its description by quantum theory will be given in a self-contained way. At the same time, the understanding of Chapters 3 to 10 should be made easier with this introduction. For didactic reasons, we will always return to two basic experiments — which used to be just gedankenexperiments for a long time — the transition of quantum objects through a single slit and through a double slit. This limitation should not be misunderstood. Apart from some exceptions, all experiments that are performed in experimental physics nowadays are based in one way or the other on quantum-physical phenomena. To be able to read the structures of quantum theory particularly clearly, it is, however, recommended to start from simple experiments.

The demands are growing from section to section in both chapters.[1] In Sections 1.2 to 1.5, fundamental experiences with quantum objects are described — based on the example of the slit and double-slit experiments. The approaches for the theoretical representation of quantum objects, which is the topic of Section 1.6, are also introduced. Thereby, however, only those elements of quantum theory that are actually required for the following sections are formulated. The quantum Zeno

1) Who wants to continue the ascent can find a more detailed and precise presentation of the whole subject for instance in the university test book Audretsch (2005).

Entangled World, Jürgen Audretsch

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effect described in Section 1.7 is a first application. This effect can be understood without any reference to mathematical relations. After introducing photons as the quantum objects of light in Section 1.8, the theoretical formalism can be illustrated for the first time with the example of interaction-free quantum measurements in Section 1.9. We will discuss the resulting picture about the quantum world in the concluding Section 1.10. The question about the structure of the reality in quantum physics will be at the center of this discussion.[2]

Anyone trying to understand quantum physics will quite soon be confronted with the question what the expression "to understand" really means in this context. Quantum physics is exactly not classical physics. Trying to understand quantum physics by transferring classical pictures is therefore useless. Classical physics can only be used by way of comparison in order to make particularly clear what is different in quantum physics. We will return to this problem several times again. The reader might become encouraged when learning that such difficulties of understanding are not uncommon at all. The famous physicist Lord Kelvin (1824–1907), who made important contributions to the theory of heat, wrote in 1884:

> "I am never content until I have constructed a mechanical model of the object that I am studying. If I succeed in making one, I understand; otherwise I do not. Hence I cannot grasp the electromagnetic theory of light."
> (Mason (1953))

Obviously Lord Kelvin already had to come to terms with the fact that electrodynamics could not be reduced to classical mechanics any longer.

Just one more word regarding the mathematical requirements: up to Section 1.5 we can do almost without mathematics. Strictly speaking, complex functions $\Psi(\underline{r}, t)$ and the formation of their absolute value $|\Psi(\underline{r}, t)|$ are necessary for the description to be correct. For a first understanding of the structure of quantum theory, it is totally sufficient to think of $\Psi(\underline{r}, t)$ as a real function. The same holds for Section 1.6. The visualization of the vectors in Fig. 1.8 is also carried out in the real vector space. Only in one place — which is in the

2) Regarding the literature quotations, no completeness is claimed. Reprints of the most important works can be found in Wheeler and Zurek (1983) and Macchiavello et al. (2000).

second half of Section 1.9 — is the imaginary number i actually introduced into a calculation, in order to represent the phase shift. Also in this case though, the physical effect of an interaction-free measurement is deduced first of all without physical equations. For further calculations however, complex vector spaces are required. From vector analysis, the vector addition and the scalar product (inner product) of two vectors are used starting with Section 1.6. A new terminology — compared to classical mechanics and electrodynamics for example — is going to be used for these calculations out of practical and historical reasons.

1.2 Diffraction at a single slit

The elementary building blocks of matter, like electrons, neutrons, protons etc. are called elementary particles. Therefore, they seem to be particles. At the same time one can frequently hear that these particles are supposed to have a wave nature. Can particles be waves at the same time? This is simply hard to imagine. Or are these elementary particles in certain situations behaving like particles and in others like waves? We will see that this conjecture is not totally wrong. Although it is expressed in a way that could be misunderstood and it will be our task to develop stepwise the precise ideas and formulations that let us describe experiments with elementary particles, including photons and also atoms and molecules. The theory that achieves this is the *quantum theory*. When the theory is limited to the description of objects with mass it is also called *quantum mechanics*. While studying the problem in a more systematic way, how can one actually get to the idea that the wave concept plays any role for the description of the physics of objects with small masses?

For a long time, the physics of massive objects has been described with great success by mechanics, which we more precisely call *classical mechanics*. Light is a phenomenon based on electromagnetic fields. We are going to start from these two *classical* theories and, in a first step, comment on two well-known experiments.

At first we give an account of an everyday experience, which can be described using classical mechanics. Tennis balls are flying in a perpendicular direction towards a wall in which we have a window opening. The tennis balls that come through hit the opposite wall. We mark the points where they hit this wall and determine the relative frequency $P_{cl}(x)$ with which these positions x are hit. In the case of a homogeneous current of tennis balls, one gets the curve in Fig. 1.1: we find hit points only directly opposite to the window opening. The same experiment can be repeated with beams of light. Again, this results in a regular distribution of brightness opposite to the window opening. The linear light rays at the edge define the shadow regions. This phenomenon can be explained by *geometrical optics.*

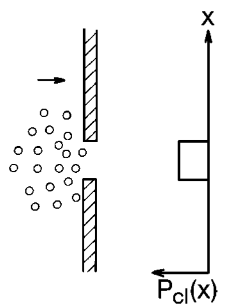

Fig. 1.1 Tennis balls flying through a slit are registered with the relative frequency $P_{cl}(x)$ on an opposite screen.

The propagation of light is in fact a wave process. The wave properties of light will show up when the dimension of the opening the light comes through is of the order of magnitude of the wavelength. Then the effects of *diffraction optics* replace geometrical optics. We will discuss this while looking at a single slit, which is lit by a plane light wave (Fig. 1.2). An intensity distribution $P_1(x)$ results now on a photoplate (screen) behind the slit The distribution shows a maximum directly opposite the center of the slit, and side maxima that are separated from each other by minima with vanishing intensities beyond the shadow limits. This is a diffraction image, which results from *interference.* Spherical waves caused by the plane wave that strikes the slit are sent off from different positions in the slit and they overlap behind the slit. When a wave maximum meets a wave maximum with the overlap at a given position, a higher wave maximum is created, and in the case of two wave minima a lower wave minimum correspondingly. When a wave maximum meets a wave minimum they cancel each other out. The wave picture behind the slit is therefore the result of an addition. This is also called the *superposition* of the elementary waves in this context. The intensity distribution

$P_1(x)$ can be recorded for example with the blackening of a photoplate. The plate, though, senses wave maxima and wave minima in the same way, therefore only the square of the wave amplitude gives a measure for the blackening. In short, one might say that the interference pattern is obtained following the rule "add first, then square". $P_1(x)$ in Fig. 1.2 shows the resulting relative frequency of blackening points.

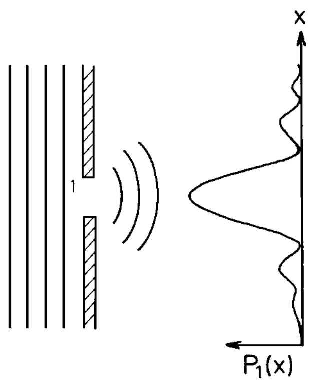

Fig. 1.2 A plane light wave (symbolic wave trains) hits a slit 1. The diffraction pattern $P_1(x)$ gives the intensity distribution, which is registered on an opposite screen (photoplate).

The appearance of a diffraction pattern is a direct indicator of the occurrence of interference. The result "light + light = darkness" cannot be produced with particles. Therefore, it must be mathematically a wave phenomenon that underlies this. Still, before discussing this we have to point out that geometrical optics and diffraction optics are two theories of light independent of each other, each one of them to be applied in special physical situations. The *electrodynamics*, that is the Maxwell Theory of electromagnetic fields from 1873, which also describes all light phenomena in a unified way, is a diffraction theory from its structure. The theory of geometrical optics is included as a limiting case. Electrodynamics, therefore, is the more general and comprehensive theory, but in special physical situations one can speak about light propagation along light beams to a good approximation.

Now we will return to our initial question, whether diffraction phenomena and thus interference exist also for molecules, atoms, elementary particles etc. In analogy to optics, we might expect the following relation between theories

geometrical optics	←	diffraction optics
classical particle physics	←	???

The arrows indicate the "transition in the limiting case". The following question arises now: is there any general theory for all material objects from elementary particles to stars, for which the superposition and the interference are central concepts and that, in certain physical situations, leads back to classical mechanics as a limit with its well defined particle paths? This theory would then be a general theory for the whole mechanics, which would also predict and describe a whole host of new phenomena. These are not necessarily diffraction effects in the strict sense because interference causes more than just diffraction. With *quantum mechanics*, such a theory has indeed been available for about 75 years. We name the objects described there *quantum objects* in order to avoid the misleading expression "particle".

Still, we should be cautious about using the analogy to electromagnetic phenomena. We will essentially just read from the similarities described above that in both cases interference and with that superposition play a central role in the mathematical description. We should expect that beyond this similarity quantum mechanics and electrodynamics are clearly different from each other both conceptually and formally. In particular, one theory will not be reducible to the other. It is rather the case that a quantum structure is also hidden behind electrodynamics. The corresponding quantum objects are the photons. Their existence and their quantum behavior are extraordinarily well confirmed with a whole host of experiments in high-energy physics and in quantum optics. We will return to this later. Since more than just objects with a mass are described, it is correct to speak about *quantum* theory rather than quantum mechanics.

However, we should first ask ourselves: do interference phenomena occur at all in connection with material objects?

1.3 Atom optics

During the first years of quantum mechanics it was just a gedanken experiment. In the meantime, it has been possible for many years to demonstrate diffraction phenomena for electrons, neutrons, atoms and molecules directly in experiments.[3] As for light, the diffracting arrangements like single slit, double slit or grating have to be suitably dimensioned. We are going to outline the scheme of such an experiment again for the example of a single slit. It is remarkable that almost all theoretical and conceptual elements leading towards the basic assumptions of quantum theory can be studied with this very clearly laid out experimental setup together with the measurement results that can be described in a very simple way. Therefore, it is justified that the diffraction of material objects at a slit or at a double slit have very often been chosen as the starting point for a description of quantum mechanics.[4]

We now take a look at atoms, all prepared with equal momentum, hitting a wall with a slit from a perpendicular direction. A screen on which the impact of the atoms can be registered, is located behind the slit and parallel to the wall. The details of this are irrelevant. Let us assume that the impact is documented by a blackening. When the incident current of atoms is not too dense, blackening spots of point shape are registered, which appear one after the other without regularity and thus randomly distributed over the screen (Fig. 1.3). It is possible to dilute the beam so much that there is always only one atom inside the setup and thus its registration on the screen takes place before the next atom is flying towards the slit.

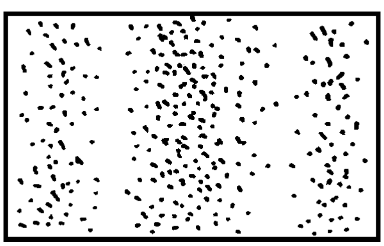

Fig. 1.3 Impact points of quantum objects (e. g. atoms) that passed through a slit show a blackening image in accordance with the diffraction at a slit.

3) A review about the experiments with atoms can be found in Berman (1997). For neutrons and electrons see Rauch and Werner (2000) or Tonomura (1998), respectively.

4) An example of this is Feynman et al. (1965).

Already at this point we can draw the first conclusions. Since single impacts are observed, we will be able to speak meaningfully about *single* quantum objects, in this case single atoms, which were inside the installation. A *position measurement* of a single quantum object is carried out on the screen due to the fact that we can exactly determine the position of the point-shaped impact. After collecting the impacts of many quantum objects, the resulting blackening picture looks not at all irregular. Amazingly one finds for the intensity of the blackening or the number of impacts per area element exactly the same distribution as in the case of light diffraction at a slit (Fig. 1.2). We have therefore obtained a diffraction image for quantum objects, too.

It is notable and actually even more surprising, that exactly the same intensity distribution can be realized experimentally in three very different ways:

1. For a specific arrangement of slit and screen, a dense current of quantum objects can be sent through a slit, so that many quantum objects hit the screen at the same time.

2. The current can be thinned out, so that there are always only single quantum objects one after the other inside the installation and hitting the screen.

3. Finally a huge number of the same experimental arrangements consisting of slit and screen can be built, and just exactly one quantum object is sent through each of these single arrangements and all the single hits from the many plates are marked in one single graph.

In all three cases we obtain the blackening distribution described above. Still, there has always been some chance at work. When, for example, the second procedure is repeated, the places hit by the first, second, etc., object are totally different each time. Nevertheless, the resulting overall picture after many impacts is again the same. We call this an *ensemble experience.*

We summarize once again: in all three cases it is guaranteed that the quantum objects cannot interact with each other. A single object, therefore, has absolutely no information about the places where other quantum objects have already hit the screen. Also, there is no prediction about the place where a single object is going to hit. The single impact takes place in an *undetermined,* which means random way. When the experiment is repeated with many quantum objects, still the same diffraction image emerges again and again. The formation of this overall picture is therefore a *deterministic* process. The overall picture can be precisely predicted. All these observations are of course *objective.* The experiments can in principle be repeated by anyone anywhere in the world, with the same result.

How can we describe these phenomena theoretically? The missing prediction in the single case and the definite prognosis for the combination of many results are well known from throwing dice. There, one can also throw the same dice several times or throw only once with several dice. With good approximation, $1/6$ of the cases will give, for example, the number 1. The *relative frequency* of the number 1 is $1/6$. When we want to make a prediction for the result of a single throw we say that the *probability* of getting for example the number 1 is $1/6$. We now transfer this form of a description cautiously to our quantum objects. While doing so, we must not conclude that there is anything behind the diffraction process of quantum objects that would be physically similar to the process of throwing dice in classical mechanics. We have no reasons for this and in fact we are going to show that this is really not the case.

For the mathematical description we will combine now the elements of determination and indetermination. We introduce a function $\Psi(\underline{r})$ of the position $\underline{r}$,[5] which describes the wave situation behind the slit. Similar to the electromagnetic field, the diffraction at the slit is expressed by the particular spatial behavior of this function. The square of the absolute value of the function, $|\Psi(\underline{r})|^2$, is interpreted physically as the impact probability $P(\underline{r})$ at a position in a small volume element dV around a position $\underline{r}$. This may be regarded as the prediction of the relative frequencies of the results of a position measurement. In other physical situations, Ψ and P will also depend on the time t. The function $\Psi(\underline{r})$ is called the *state function* or *Schrödinger function.* The screen can also be placed at different positions behind the slit. In any case, $|\Psi(\underline{r})|^2$ with the corresponding position vector $\underline{r}$ will give the correct impact probability on the screen. The state function describes the physical situation behind the slit but before the measurement. The function is determined by the

5) A complex time-dependent factor with an absolute value of one is suppressed.

width and the position of the slit. Different apertures will lead to different functions. One can also say that the function $\Psi(\underline{r})$ describes a specific *quantum state.*

In this description, we are strictly limiting ourselves to the prediction of position measurements. The state function serves only this purpose. Behind the slit there is no vibrating physical quantum substance or a "wave-pudding" of quantum objects imagined as being smeared out. The name "matter wave" for $\Psi(\underline{r}, t)$ is in this sense unfortunate. There are no classical particles with position and momentum, because in this case one should expect the result of Fig. 1.1. Instead, we speak about one or many quantum objects with an assigned quantum state, which is described by the function $\Psi(\underline{r}, t)$; $\Psi(\underline{r}, t)$ may have mathematically the form of a wave in certain situations.

After discussing the ensemble experience and formulating the elements of quantum theory, we shall propose, on this basis, a prognosis for a modified experimental arrangement. We let atoms fly, but instead of using one slit, we put two similar, aligned parallel and suitably dimensioned slits in their way. Our conjecture is that the analogy to the diffraction image of light waves passing through a double slit[6] will become evident in the resulting frequencies of impacts on different places of the screen. The superposition principle applies to light waves; this means that the single light waves, coming from slit 1 and slit 2, interfere behind the double slit. They add up as discussed above. When the square of the resulting wave field is determined, for example at the positions x on the screen, the normalized intensity distribution of the field $P(x)$ from Fig. 1.4 is obtained and thus the relative frequency of blackening points on the photoplate.

When atoms instead of light are incident on a double slit, a totally analogous image for the frequency of impacts of atoms on positions of the screen emerges. Again all three ensemble experiences are valid. This result is described in the same way by superposition, but in this case the state functions $\Psi_1(\underline{r})$ and $\Psi_2(\underline{r})$ are to be added. $\Psi_1(\underline{r})$ is the state function present when slit 2 is closed. This is the state function behind a single slit as discussed above. $\Psi_2(\underline{r})$ is the corresponding state function when slit 1 is closed. Therefore, we have superimposed the quantum state "through slit 1" and the quantum state "through

6) A modern double-slit experiment for atoms is described in this book by G. Rempe in Section 5.1.

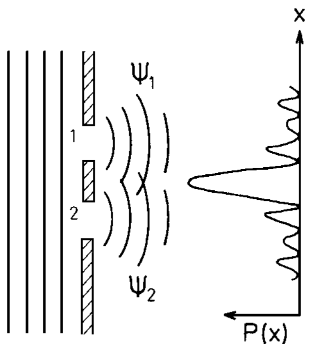

Fig. 1.4 The intensity distribution $P(x)$ of quantum objects diffracted at a double slit shows a maximum opposite to the bridge. The symbolic wave trains of the state function before and after the slit are drawn. Ψ_1 (Ψ_2) is the state function that is present when slit 2 (1) is closed. Ψ_1 and Ψ_2 are superimposing.

slit 2″:[7]

$$\Psi(\underline{r}) = \frac{1}{\sqrt{2}} \left(\Psi_1(\underline{r}) + \Psi_2(\underline{r})\right) \tag{1.1}$$

$$|\Psi(\underline{r})|^2 = \frac{1}{2}|\Psi_1(\underline{r})|^2 + \frac{1}{2}|\Psi_2(\underline{r})|^2 + \frac{1}{2}\left(\Psi_1^*(\underline{r})\Psi_2(\underline{r}) + \Psi_1(\underline{r})\Psi_2^*(\underline{r})\right) \tag{1.2}$$

The star indicates the complex conjugate function.[8]

Atom optics with a double slit shows in a particularly drastic way that quantum objects do not behave like small classical particles with a mass. The frequency of impacts is not highest behind the two slit apertures but behind the bar in between them. Again we can make the ensemble experiences. The frequency distribution of Fig. 1.4 can also be found in the case where there has been only one quantum object in the setup at a time. Equation (1.2) shows that this curve is not obtained by just adding the shifted curves for the frequency distribution of Fig. 1.2:

$$P(x) \neq \frac{1}{2}\left(P_1(x) + P_2(x)\right) \tag{1.3}$$

$$P(x) = |\Psi(x)|^2\,,\; P_1(x) = |\Psi_1(x)|^2\,,\; P_2(x) = |\Psi_2(x)|^2$$

7) The factor $1/\sqrt{2}$ is necessary because all state functions are normalized due to the probability interpretation. We will return to this in Section 1.6.

8) Note the comment about complex functions in the introduction.

Therefore, one cannot say that one half of the quantum objects went through slit 1 and the other half through slit 2. The last two terms in Eq. (1.2), which are not present on the right side of Eq. (1.3) anylonger, are responsible for the formation of the diffraction pattern of Fig. 1.4. Even the single quantum object behaves fundamentally differently at a double slit as compared to a single slit. Has the single quantum object passed through both slits? We will have to study later in detail if such a question that results from a particle picture makes any sense at all.

1.4 The quantum domain

We have spoken previously about the possibility to find a suitably dimensioned slit for the diffraction of atoms. How do we have to proceed? Since we are dealing with a wave phenomenon, this question is about the determination of the wavelength.[9] Let us take a look at the plane waves, which run towards the double slit. A momentum $\underline{p} = m\underline{v}$ can be attributed to the corresponding quantum object, which has a direction that agrees with the propagation direction of the plane wave. The absolute value of the momentum results from the preparation procedure. The quantum objects run, for example, through an acceleration voltage by which they gain a certain kinetic energy that can be used to calculate the magnitude of the momentum in the usual way. Since the details of the diffraction pattern depend on the wavelength, a relation between the absolute value of the momentum p and the wavelength λ can be read from an experiment. One finds the *De-Broglie relation*:

$$p = h/\lambda \tag{1.4}$$

This equation can be confirmed for other sorts of quantum objects and in other diffraction experiments. h is *Planck's constant* (or *Planck's "Wirkungsquantum"* in German)

$$h = 6.626 \times 10^{-34}\,\mathrm{kg\,m^2/s^2} \tag{1.5}$$

9) It is also possible to enter the quantum world by slowing down and thus cooling heavy particles like atoms. The current research field of the formation and manipulation of Bose–Einstein condensates is introduced in Section 4.3.

($\mathrm{kg\,m^2/s^2} = \mathrm{Watt\,s^2}$). Note that the value of h in the units of the quantities of everyday physics is extraordinarily small.

From the discussions so far we can also read in which way a quantum experiment typically proceeds. It starts at time t_0 with the *preparation* of a very particular quantum state $\Psi(\underline{r}, t_0)$. In our case, this state has mathematically the form of a plane wave with a definite wavelength and a definite propagation direction perpendicular to the slit. This quantum state is then subjected to an alteration by some influence from outside or an interaction. A charged particle for example can be exposed to a position- and time-dependent electrical potential. This is a process that can last for a certain time. The resulting continuous temporal alteration of the state can be calculated by means of quantum theory and it leads to the time-dependent function $\Psi(\underline{r}, t)$. This *dynamical evolution* of the state function as a function of the time t starting from an initial state is well defined and thus *deterministic*. In our case, this is the transition to the state $\Psi(\underline{r})$ of Eq. (1.1). Finally, the *measurement* is carried out. In our case, this is the position measurement on the screen. The state function at the moment of the measurement fixes the relative frequency of the different measurement results. When the experiment is always carried out just for one single quantum object in the setup at a time, it has to be repeated very often and the quantum objects must always be prepared in the same way.

We return now to the problems that we encountered in Section 1.2. The experiments described above have convincingly shown that massive objects exist, with a behavior that can fundamentally deviate from the behavior that we are used to in classical mechanics. Therefore, a *quantum domain* exists in nature. How wide is its range of application and, correspondingly, the range of validity of quantum mechanics extended? We have seen for the diffraction of light at a slit that the beams of the geometrical optics can be used for the projection of the slit, when the wavelength is very small compared to the width of the slit.[10] The pattern with the shadows of Fig. 1.1 emerges then from the diffraction image of Fig. 1.2. Using the De-Broglie relation, a wavelength can also be assigned to the quantum objects, and again one finds the same effect: when the wavelength is very small compared to the slit aperture, a fraction of the plane wave passes through

10) Also macroscopic quantum effects exist: superconductivity, superfluidity, Josephson effect, etc.

the slit aperture almost without any modification and the quantum objects cast a "shadow". With that, we are back again to the tennis ball experiment described above.

A quantum phenomenon is transformed to a classical phenomenon when the circumstances are changed. Does this mean that the domains of application of both theories touch without having any overlap? Or does the domain of application of quantum mechanics include classical mechanics (Fig. 1.5)? An extreme case of the first version was supported by the so-called Copenhagen interpretation of quantum mechanics. The research nowadays is pursuing the second approach, in which universal validity is attributed to the quantum theory. This approach follows the basic idea from our first section that electrodynamics also describes geometrical optics. It leads directly to an important, still unsolved problem. We have seen — and this will get clearer later — that quantum mechanics is very structured differently compared to classical mechanics. For macroscopic objects, no superposition of states exists. Cats, for example, are either dead or alive. Nobody has observed so far the hermaphrodite existence of a superposition of both states. How should one therefore proceed within quantum mechanics in order to describe objects of everyday physics? What must be the case so that an object behaves in the "classical" way?[11] For this approach not quantum physics, but classical physics is the unsolved problem. Where is the borderline between the quantum world and the classical world? Fullerenes with 60 and 70 carbon atoms still show diffraction at a grating (Arndt et al. (1999)).

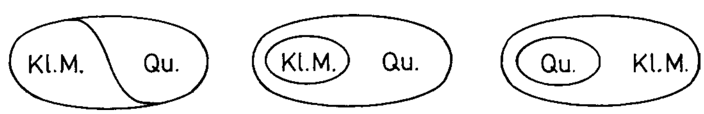

Fig. 1.5 Different possibilities of how the application domains of the classical mechanics (Kl. M.) and the quantum mechanics (Qu.) are related to each other. The middle version is favored in the article.

The third possibility, that quantum mechanics can be traced back completely to deterministic classical mechanics, still needs to be discussed (Fig. 1.5). This is an idea that is very attractive from some philosophical points of view. In the course of the preceding decades were made repeated attempts to base a viable theory on it. Today, it

11) Chapter 8 deals with this question.

can be proved by experiment that this is impossible. We will go into that in more detail in the following chapter. The quantum-mechanical probabilities are not reducible. This is the essential statement. Objective *chance* exists in nature. The situation is in fact *not* the same as for dice, where each throw goes really completely deterministic and only subjectively do we get the impression of chance, and a probabilistic description reproduces our experiences. Albert Einstein in 1949 argued against the probability interpretation of quantum mechanics in a polemic way with the famous saying: "God is not playing dice". He was right in a sense, when the phrase is understood in a different way from how it was meant by Einstein. Then it means that quantum objects are not classical objects like the dice and therefore no probability reducible to deterministic physics exists in this domain.

We would like to mention another consequence. From the fact that quantum mechanics cannot be reduced to classical mechanics follows that the classical concepts fail in the quantum domain. However, we have developed and practiced our intuition facing the everyday physics. When we are not educated in physics, only things that can be described and explained based on classical physics appear to be intuitive and obvious for us. If intuitive knowledge is understood in this way, quantum physics is necessarily *not intuitive or plausible*.[12] Also, colloquial language, which is used to express everyday phenomena, can handle phenomena only from classical mechanics without problems. We will see that quantum physics, in contrast to this, requires a "reduced" language in order to avoid suggesting via the wording propositions that are actually not valid in the quantum domain.

1.5 Quantum measurements

With the position measurement on the screen, we so far only know about a very simple type of quantum measurement. Measurement processes are actually another large field, in which quantum objects behave in a characteristic way that is totally different compared to objects of classical physics. We return to our double slit, which can be used to illustrate this. In Section 1.3, when discussing the double slit, we emphasized the fact that it is just not possible for the interference

12) Compare the problem of Lord Kelvin sketched in Section 1.1.

image to emerge from the sum of the probabilities, but instead from the superposition of both state functions Ψ_1 and Ψ_2, which belong to the two single slits 1 and 2. We have seen further that we are not able to decide whether the single quantum object came through slit 1 or through slit 2. Perhaps we have just missed collecting the necessary information by means of a measurement. Now we are going to make up for that.

For this purpose we use electrically charged quantum objects that are able to scatter light and irradiate the space directly behind the screen (Fig. 1.6).[13] We realize then, that for every object a flash occurs either behind slit 1 or behind slit 2 before the impact is recorded on the screen. Thus, we have made a measurement by light scattering to answer the question "Through which slit?" and obtained as a result either "through slit 1" or "through slit 2". A simultaneous flashing behind slit 1 and behind slit 2 never occurs. This would anyway require that the quantum object (for example an elementary particle) could somehow be split in two by the double slit. When taking a look at the impact points of many quantum objects on the screen, well-defined patterns are formed again. When the impacts corresponding to the flashes behind slit 1 are considered separately, exactly the intensity distribution belonging to the state Ψ_1 is obtained (cf. Fig. 1.2). In spite of having slit 2 open, a diffraction image was formed, which totally agrees with the image obtained when slit 2 was closed. In the same way, one gets for the impacts corresponding to the flashes behind slit 2 the diffraction image that belongs to Ψ_2. When all impacts are brought together in one graph, the intensity distributions simply add up. The result does therefore not agree with the interference result for the double slit (Fig. 1.4).

What should be expected from classical physics in contrast? It is characteristic of classical measurements that the measured object is not modified by the measurement. For example, a position measurement using radar has no influence on the state of motion of the object. This is obviously fundamentally different for measurements in the quantum domain. The position measurement with the result "through slit 1" transforms the state Ψ from Eq. (1.1) to the state Ψ_1 behind the screen. The same applies to slit 2. Here the state Ψ_2 is formed. So the state before the measurement is transformed into

13) Compare with Section 5.1.

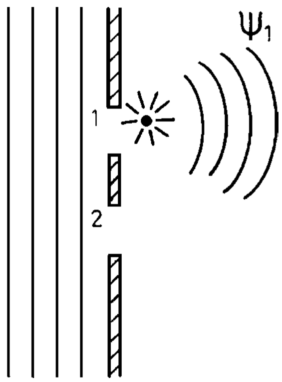

Fig. 1.6 Behind a double slit, the position where a quantum object comes through is measured. For the registration behind slit 1, a diffraction image is obtained on the screen, which corresponds to the state function Ψ_1.

a completely different, new state depending on the measurement result. A well-defined final state belongs to each measurement result that is also named the *eigenstate* of the measured quantity. When we know the measurement result, we also know precisely the state of the quantum object after the measurement. In this sense, a quantum measurement is *preparing* a new state.

When a single object after a first irradiation is irradiated again immediately, it flashes again behind the same slit at the same position like the first time and never behind the other slit. When the experiment is repeated for many objects and only the impacts on the screen belonging to double flashes behind the first slit are regarded, the diffraction image of the single slit of Fig. 1.2 is formed again. The second measurement to answer the question "through which slit?" — which is now made on the state Ψ_1 when the first flash occurred behind the first slit — has not modified this state Ψ_1. When the first measurement has given the result "through slit 1", the directly repeated measurement gives the same result. At least, the fact that an immediately repeated measurement reproduces the first measurement result is common for quantum measurements and classical measurements. One would rather not speak of a measurement, if this had not been the case.

Once again, we return to the single slit and the position measurement on the screen discussed in Section 1.2. We take a look at the quantum state Ψ_1 that belongs to a slit of width Δx (Fig. 1.7). We now carry out position measurements for many objects directly behind the slit. Then all positions behind the slit aperture occur as measuring points with the same frequency. The readings of the coordinate x *scatter* with a width Δx. When we look again at the situation in which there is only one quantum particle in this experimental setup at a time, we can say that the position of the quantum particle before the position measurement is undetermined with the *uncertainty* (square root of the mean square deviation) Δx. For a different experimental arrangement with a slit of the width $\Delta x'$, a state Ψ_1' is present with a positional uncertainty $\Delta x'$ and so on.

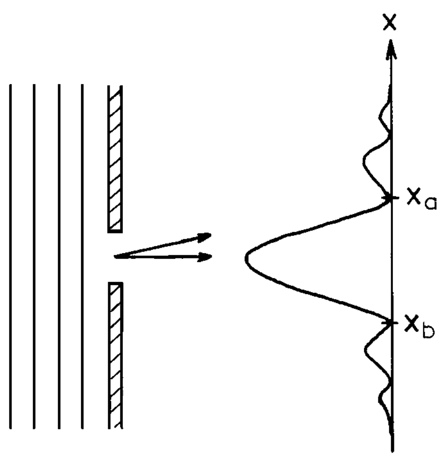

Fig. 1.7 The diffraction at a slit mirrors the uncertainty relation.

We return now to the diffraction image of the single slit (Fig. 1.7) on a distant screen and name the coordinates of the first two minima x_a and x_b. The position of a single impact on the screen can also be considered as an indirect measurement of the momentum direction. The momenta are scattering around the momentum direction of the incoming plane wave (which corresponds to the impact in the maximum) with an uncertainty Δp_x. This Δp_x can be roughly estimated by the distance $x_b - x_a$. It corresponds to an *uncertainty* Δp_x for the momentum.

The evaluation of the two types of experiments with many quantum objects shows

$$\Delta x \cdot \Delta p_x \geq h/4\pi \tag{1.6}$$

The larger symbol is due to the fact that impact points can be found outside the range between x_a and x_b. The inequality (1.6) is valid for any slit width, and thus also for the state Ψ_1' with $\Delta x'$ and the accompanying $\Delta p_x'$. The relation (1.6) is named *uncertainty relation.* Uncertainty relations also exist for measurement quantities other than position and momentum.

Sometimes, in this context, the expression indeterminacy is used. This is misleading, because all single position and momentum measurements have resulted in an exact measurement value in each case. We are not dealing with fuzzy measurements done with poor apparatus. The relation (1.6) is rather a statement about the scattering of many measurement results around a mean value using the same preparation of the state. Very often, either the position or the momentum is measured.

When the slit width is reduced, the distance $x_a - x_b$ increases and therefore the diffraction image is stretched. This leads to a limit that is also expressed in Eq. (1.6): $\Delta x \rightarrow 0$, $\Delta p_x \rightarrow \infty$. When a state is such that the result of a position measurement is exactly determined, the result of a momentum measurement is totally undetermined. When the width of the slit is increased, we get just the opposite case. Something else can be directly read from the inequality (1.6): it is not possible that Δx and Δp_x become zero at the same time. No quantum state exists, in which both momentum measurement and position measurement are obtained without scattering.

1.6 A theory for the quantum domain

We are going to summarize the previous experiences and generalize them to form the quantum theory. Such a theory should be founded on just a few basic assumptions and it should be completely formalized from the mathematical point of view. Only then can one clearly realize whether it is logically consistent. Furthermore, a good physical theory is expected to be simple in its mathematical structures and theoretical concepts. For our considerations, we will actually only need the knowledge of complex numbers and some vector algebra in two dimensions.

The quantum state introduced above is a fundamental concept of quantum theory. The use of vectors for the mathematical representation of states was found to be appropriate. The *state vectors* are written in brackets: $|u\rangle$, $|v\rangle$, ... in order to indicate their vector character. The *inner product* (scalar product) of two vectors is written as $\langle u|v\rangle$ and its value is allowed to be a complex number.

As a central physical operation, we have to express the *superposition* of states. This is represented by the vector addition:

$$|w\rangle = a|u\rangle + b|v\rangle \tag{1.7}$$

In general, a and b can be complex numbers. For the formation of the inner product, the dual vector for $|w\rangle$, $\langle w| = a^*\langle u| + b^*\langle v|$, is used. The star indicates the complex conjugate number.

When the passing of the quantum object through slit 1 is measured, the object is afterwards in the state $|\Psi_1\rangle$, and for slit 2 in state $|\Psi_2\rangle$, respectively. When no position measurement is carried out behind the slit, the state $|\Psi\rangle$ behind the slit is a superposition:

$$|\Psi\rangle = c_1|\Psi_1\rangle + c_2|\Psi_2\rangle \tag{1.8}$$

The slit numbers 1 and 2 are related to the possible *measurement results*.

The *probability* that the measurement result 1 is registered results in the theory from the square of the absolute value of an inner product:

$$P_1 = |\langle\Psi_1|\Psi\rangle|^2 \tag{1.9}$$

This applies to measurement value 2 accordingly. When the state $|\Psi\rangle$ from Eq. (1.8) is inserted, one finds that the rule "add first, then square" is expressed, which we learned about in Section 1.2 in the context of interference and diffraction (see also the Eqs. (1.1) and (1.2).

When $|\Psi\rangle$ is present, the probability that an object is found in the state $|\Psi\rangle$ equals one. For this reason it is required that all state vectors, including $|\Psi_1\rangle$, $|\Psi_2\rangle$, etc. are normalized: $\langle\Psi|\Psi\rangle = 1$. When the quantum object is in the state $|\Psi_1\rangle$, the probability to measure the result 2 equals zero. This means that the states $|\Psi_1\rangle$ and $|\Psi_2\rangle$ are orthogonal to each other: $\langle\Psi_2|\Psi_1\rangle = 0$. From this, using Eq. (1.8), the relation $|c_1|^2 + |c_2|^2 = 1$ follows, and from Eq. (1.9)

$$P_1 = |c_1|^2, P_2 = |c_2|^2 \tag{1.10}$$

This means in summary that the sum over all probabilities equals one: $P_1 + P_2 = 1$. One of the possible measurement values is found in each measurement.

When both probabilities P_1 and P_2 are the same, the state behind the double slit must have the form:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|\Psi_1\rangle + |\Psi_2\rangle \right) . \tag{1.11}$$

When the passage through one of the slits is made more difficult by means of a filter for example, the probabilities to register the quantum object with a measurement behind one of the slits is no longer the same. This is reflected by the general superposition (1.8).

Except when the scalar product is a complex number, we can use the vector arrows known from vector algebra of the real vector space for the graphical illustration of the states. Figure 1.8 then represents Eq. (1.8). When a filter is installed, we have $\alpha \neq 45°$.

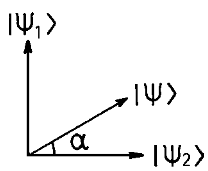

Fig. 1.8 The superposition of the two quantum states $c_1|\Psi_1\rangle$ and $c_2|\Psi_2\rangle$ gives the state $|\Psi\rangle$.

A *position measurement* gives as a measurement result the position vector $\underline{r}$ and transforms the state into the state $|\underline{r}\rangle$. We introduce the state function $\Psi(\underline{r})$, which was already used in Section 1.3 as an abbreviation: $\Psi(\underline{r}) = \langle\underline{r}|\Psi\rangle$. Then $|\Psi(\underline{r})|^2$ is a quantity analogous to P_1 from Eq. (1.9) (see Section 1.3). Precisely formulated, not directly $|\Psi(\underline{r})|^2$ but instead $|\Psi(\underline{r})|^2\,\mathrm{d}V$ is the probability to find the quantum object with a position measurement in a small volume $\mathrm{d}V$ around the position $\underline{r}$. Since the position is a continuous quantity, we have to refer to the volume element $\mathrm{d}V$. This is unnecessary for discrete measurement values.

Finally, the problem remains that we have to introduce the dynamics, or more precisely the different dynamics. In the force-free case or under the influence of external forces, which are taken into account with the corresponding potentials, the state of the system changes continuously between two times t_1 and t_2:

$$|\Psi(t_1)\rangle \rightarrow |\Psi(t_2)\rangle \tag{1.12}$$

Based on Fig. 1.8 this change would be illustrated by a continuous rotation of the vector $|\Psi\rangle$ with a time-dependent angle $\alpha(t)$. This evolution in time between the measurements is deterministic and causal. We are going to call it *dynamics I*. The differential equation that describes the evolution in time (1.12) in detail is the *Schrödinger equation*.

One important property of dynamics I should be mentioned here. Usually it is summarized in the following statement: "Quantum theory is linear". We ask ourselves how different states evolve under an identical influence from outside. *Linear* means that the coefficients a and b in Eq. (1.7) do not change during the dynamic evolution of the superposition. The evolution $|u(t_1)\rangle \rightarrow |u(t_2)\rangle$ and $|v(t_1)\rangle \rightarrow |v(t_2)\rangle$ implies the evolution

$$a\,|u(t_1)\rangle + b\,|v(t_1)\rangle \rightarrow a\,|u(t_2)\rangle + b\,|v(t_2)\rangle \tag{1.13}$$

of the superposition.

As a consequence of the *measurement*, the state is also transferred to a new state, but in a very different way. When we take the above example, we have $|\Psi\rangle \rightarrow |\Psi_1\rangle$ when result 1 is measured. In Fig. 1.8 this means that the state vector "jumps". We have seen before that this is a nondeterministic dynamics, since we are not able to predict before the measurement, in which one of the eigenstates $|\Psi_1\rangle$ or $|\Psi_2\rangle$ the initial state will jump. We are going to call this dynamics, which is present in the measurement, *dynamics II*. The probabilities P_1 and P_2 are the absolute squares of the projections of $|\Psi\rangle$ on the orthogonal vectors $|\Psi_1\rangle$ and $|\Psi_2\rangle$. The measurement is therefore sometimes also called a *projection measurement* or *state reduction*.[14]

14) In order to restrict the formalism to the essential, we have not introduced the concept of observables (operators that represent measurement quantities). Since they may be characterized by giving the vectors in which a state is transferred with the corresponding measurement, the knowledge of $|\Psi_1\rangle$ and $|\Psi_2\rangle$ is sufficient for our considerations. The uncertainty relation (1.6) could be deduced from the fact that position and momentum operators do not commute.

The fact that we need to postulate two dynamics for the quantum theory, one of them deterministic, the other nondeterministic, is certainly extremely unsatisfying. The search for unification is therefore another current research program. The attempts to formulate a satisfying *theory of the quantum measurement* start from dynamics I.

1.7 The quantum Zeno effect: How to stop the dynamical evolution

As a simple application, we study a dynamical process evolving freely or under the influence of outer forces (dynamics I), which is interrupted again and again by a measurement of the same type (dynamics II). It shows then as consequence of dynamics II an amazing quantum-mechanical effect for dynamics II: the dynamical evolution according to dynamics I can be completely suppressed by repeated measurements of a quantum system. The quantum system is "frozen" in its initial state. This effect is named the *quantum Zeno effect* in memory of the Greek philosopher Zeno (490–430 BC), who formulated a paradox, according to which any movement should be logically impossible, the so-called "paradox of Achilles and the tortoise". The infinitesimal calculus makes it possible to solve Zeno's paradox. A Zeno effect is impossible in classical mechanics. In contrast to this, the quantum Zeno effect is no paradox. It can be experimentally demonstrated (Itano et al. (1990)) and understood as a direct consequence of the peculiarities of the quantum measurement process.

Let us assume that we are measuring again a physical quantity, where the states $|\Psi_1\rangle$ and $|\Psi_2\rangle$ belong to the two measurement results. We further assume that the quantum system is in the initial state $|\Psi_2\rangle$ at an initial time t_0 as a result of a measurement. Under some influences from outside, which can for example be given by potentials, the system evolves according to dynamics I. This means for the state vector $|\Psi(t)\rangle$ that it is slowly rotating away from position $|\Psi_2\rangle$ with a time-dependent angle $\alpha(t)$ (see Fig. 1.8). After the next measurement, the probability to find the system either in the state $|\Psi_1\rangle$ or in $|\Psi_2\rangle$ is given by the square of the projection of $|\Psi(t)\rangle$ on these states. When the measurement is repeated after a very short time, the angle α is still very small and the probability for the state to be projected back to the state $|\Psi_2\rangle$ is much larger than the probability to "jump" to the state $|\Psi_1\rangle$. When the first case happens, the evolution therefore starts again with the state $|\Psi_2\rangle$. When the measurement is made once more after a very short time, the above applies again. In total, the probability to find the system still in the state $|\Psi_2\rangle$ after a very fast sequence of measurements of the same kind is high. The experiment confirms this.

At least theoretically we can consider now the limiting case that the time intervals approach zero. In this case one can show that the dynamical evolution is completely prevented and the system remains in its initial state $|\Psi_2\rangle$. This, remarkably, also holds when the initial state is not an eigenstate of the corresponding measurement quantity. The theoretical reason for this is quite simple. However, this is beyond the scope of this discussion. Strictly speaking, the limiting case cannot be realized due to the final duration of each measurement process. Investigations about the existence of exceptions, where special dynamics of type II prevent the quantum Zeno effect, are a topic of current research.

1.8 Photons: quantum objects of light

Once again we return to electrodynamics and ask ourselves if there might be any structure behind the electromagnetic fields, which is similar to the structure of quantum mechanics. This is indeed the case. First, we take a look at the particle aspect. For the objects of quantum mechanics this aspect became apparent in the always well-defined quantities of mass and charge. Electromagnetic fields do not have these two properties. A quantum-type structure manifests itself in a different way.

The photoelectric effect shows that energy can only be taken from a light field in portions of size $E = h\nu$. ν is the frequency of light and h is Planck's constant. This holds for all electromagnetic fields correspondingly. Atoms can be accelerated or decelerated with laser light. When the experiments are analyzed in detail, one finds that the momentum transfer is also a quantum-type exchange in "packages" of size $p = h\nu/c$. These mass- and charge-free exchanged energy-momentum packages are named *photons.* They move with the speed of light c, and they are a new type of quantum object. All effects in quantum optics are based on this quantum nature of light.

The electromagnetic waves and diffraction phenomena are now explained in the same way as for the material quantum objects: the description of Section 1.6 can be adopted correspondingly. Therefore our quantum domain is extended. Again we are dealing with many photons prepared in the same way. In special cases the current of photons can be thinned out, so that only one photon is inside the experimental setup.

Electromagnetic waves oscillate perpendicularly to their propagation direction. When we look at this linear polarization of the photons, the analogy to the formalism described above for material quantum objects becomes particularly clear. Light that is propagating in the z-direction can be polarized for example in x-direction by means of a polarization filter (polarizer). When this light falls on a second polarization filter (analyzer) with a perpendicular orientation in the y-direction, no light passes through. When the polarizer is turned by 45° into a diagonal orientation, only half the intensity will come through an analyzer aligned in the x-direction and also only half of the intensity will come through the analyzer in the y-direction. In the general case, the polarization is turned by an angle α against the x-direction. The intensities for both analyzer directions result from exactly the same rule that we established for the probabilities in Section 1.6. Again we get to the ensemble experience described in Section 1.3. In summary, we can conclude that for the description of photons a quantum state "linear polarization" has to be introduced, to which the same rules apply as the ones that we have already studied for the special case of our quantum state behind the double slit.

1.9 Is it possible to see in the dark? Interaction-free quantum measurements

Of course it is impossible to see in the dark. A photon must be scattered by an object so that one can see it — but this photon, of course, is not present in absolute darkness. However, we can reformulate the question: is it possible to prove the presence of an object at a defined position without the object being hit even by a single photon? We are going to demonstrate that it is indeed possible. This can already be shown with very simple considerations, which we are going to discuss first (Elitzur and Vaidman (1993)). In the second part of this section, this effect will serve as a simple application of the quantum-theoretical formalism that we discussed in Section 1.6.

A Mach–Zehnder interferometer is shown in Fig. 1.9. It consists of a light source (star), two semipermeable beam splitters A and B, two ideal mirrors K and L as well as the two detectors D_d and D_h. The paths 1 and 2 have exactly the same length. Light should be transmitted and reflected by the beam splitter to the same extent. With each reflection, either by the beam splitter or by the mirror, the phase of the electromagnetic wave is changed by $\pi/2$. In contrast, no phase shift occurs when the wave passes through the beam splitter. The "rules of the game" for the interferometer are defined by this characterization of the optical elements. Interference occurs on each way from B to one of the two detectors.

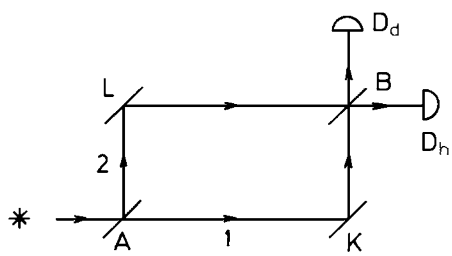

Fig. 1.9 Schematic representation of a Mach–Zehnder interferometer consisting of a light source (star), two semipermeable beam splitters A and B, two ideal mirrors K and L and the two detectors D_d and D_h.

At first we shall consider classical electromagnetic waves. When following the sequence of phase shifts, one realizes that two waves with different phase shifts run from B towards the detector D_d: one wave with phase shift $3\pi/2$ coming along path 2 and another with phase shift $\pi/2$ coming along path 1. Both waves are superimposed behind B and due to a resulting phase shift of π they cancel out each other. Therefore, the detector D_d is not responding, while the detector D_h is, as can be shown by analogous considerations.

We pointed out above that light consists of single photons. Therefore, it is quantum-theoretically possible and feasible that there is only one single photon within the interferometer. This nonclassical situation leads to new effects for the optical interferometer. Similar to the case of the double slit, no quantum object can be found in areas where the classical waves cancel each other by interference. According to this, the single photon in the case discussed here is registered always only in detector D_h but never in detector D_d.

Now we are going to insert an object into path 2, which is either absorbing or scattering a striking photon by an interaction, so that the photon can no longer reach the beam splitter B. In the same way as before, only one single photon enters the interferometer. We give a very simplified description of what happens in terms of paths and turn to a more correct one afterwards. At beam splitter A it is reflected to path 2 with a probability of $1/2$ and hits the object. Also with a probability of $1/2$ it enters path 1, where it reaches the beam splitter B. From there, it goes through to the detector D_d or the detector D_h with a total probability of $1/2 \times 1/2 = 1/4$. So having an object in beam path 2, a photon is registered in detector D_d in one quarter of all cases. Without the object, no photon is observed at all in detector D_d. Therefore it is clear: we only register a photon in detector D_d when there is an object in beam path 2. Since only one single photon has been used and this one was registered in detector D_d, no interaction of the photon with the object could have occurred, because in such a case the photon would not have reached one of the detectors.

To illustrate this, we are going to discuss the following practical example. Let us assume that the fuse of a bomb, which ought to be deactivated, is so sensitive that the explosion would be triggered by the impact of one single photon. It is impossible to search for such a bomb with classical light. With the arrangement described above there is, however, not too bad a chance — namely a probability of $1/4$ — to find the bomb without igniting it. The fact that once again it is only possible to make a statement about a probability, is characteristic of quantum-theoretical effects. In situations where such a prediction is "better than nothing", the use of quantum effects opens up unexpected technical possibilities. Quantum computers are another application of this.[15]

The interaction-free quantum measurement is a simple example, which can be used to test for photons the formalism of quantum theory from section 1.6. The state that describes a photon running in

15) Quantum computer and quantum information theory are discussed in more detail in the chapters of R. F. Werner (Chapter 7) and H. Weinfurter (Chapter 6) of this book.

the x-direction is marked by $|x\rangle$; for the y-direction we use $|y\rangle$ correspondingly. With a reflection at the mirror or the beam splitter, a phase jump of $\pi/2$ occurs. This means for the corresponding state a multiplication with i because of the phase factor $\exp(\mathrm{i}\ \pi/2) = \mathrm{i}$. Therefore, the mirrors in L and K cause the following state changes:

$$|x\rangle \to \mathrm{i}\,|y\rangle\,, \quad |y\rangle \to \mathrm{i}\,|x\rangle\,. \tag{1.14}$$

The beam splitters A and B transfer the state to a superposition, where the phase jump for the reflected outgoing state has to be taken into consideration:

$$|x\rangle \to \frac{1}{\sqrt{2}}\left(|x\rangle + \mathrm{i}\,|y\rangle\right) \tag{1.15}$$

$$|y\rangle \to \frac{1}{\sqrt{2}}\left(|y\rangle + \mathrm{i}\,|x\rangle\right) \tag{1.16}$$

These are our quantum-theoretical "rules of the game" for the state modifications by the optical elements.

Now we follow again a photon that enters with the state $|x\rangle$. The complete evolution of the state, when there is no scattering or absorbing object in one beam path, is given by the following sequence of state transitions:

$$\begin{aligned} |x\rangle &\to \frac{1}{\sqrt{2}}\left(|x\rangle + \mathrm{i}\,|y\rangle\right) \to \frac{1}{\sqrt{2}}\left(\mathrm{i}\,|y\rangle - |x\rangle\right) \to \\ &\to \frac{1}{2}\left(\mathrm{i}\,|y\rangle - |x\rangle\right) - \frac{1}{2}\left(|x\rangle + \mathrm{i}\,|y\rangle\right) = -\,|x\rangle \end{aligned} \tag{1.17}$$

The first arrow describes the transition at the beam splitter A. After the reflection at the mirrors L and K, the state after the second arrow is present. The effect of the beam splitter B is illustrated with the third arrow. This beam splitter, according to Eqs. (1.15) and (1.16), causes the transition to further superpositions, which are added up on the right side of the equation. After the beam splitter, our photon is thus in the state $-|x\rangle$ and it arrives at the detector D_h. The detector D_d therefore never responds in this configuration.

Now we imagine having a scattering or absorbing object inserted in the path between L and B. The state $|x\rangle$ that is running towards this object is transferred to the photon state $|s\rangle$ by scattering or absorption. When we take a look at the state evolution, Eq. (1.17) has to be modified accordingly:

$$\begin{aligned} |x\rangle &\rightarrow \frac{1}{\sqrt{2}}\left(|x\rangle + \mathrm{i}\,|y\rangle\right) \rightarrow \frac{1}{\sqrt{2}}\left(\mathrm{i}\,|y\rangle - |x\rangle\right) \rightarrow \\ &\rightarrow \frac{1}{\sqrt{2}}\left(\mathrm{i}\,|y\rangle - |s\rangle\right) \rightarrow \frac{1}{2}\left(\mathrm{i}\,|y\rangle - |x\rangle\right) - \frac{1}{\sqrt{2}}\,|s\rangle \end{aligned} \tag{1.18}$$

The last state is present after the influence of the beam splitter B. The probabilities are obtained as usual from the squares of the absolute of the prefactors. The probability of the response of detector D_h or detector D_d is $1/4$ in each case. The probability for a photon to be scattered or absorbed by an object is $1/2$. It is crucial that after inserting the object, the state $|y\rangle$ also appears in the resulting superposition of Eq. (1.18), in contrast to Eq. (1.17). This means that the detector D_d can now respond, which was impossible without the object. When the detector actually responds, we obtain the information that an object has been in path 2. In the first case, the single photon that was used to carry out our experiment has arrived unharmed in detector D_d, neither being scattered nor absorbed. In this sense, we were able to prove the existence of an object without having any interaction.[16] An experimental realization can be found in Kwiat et al. (1995).

1.10 What is real? Interpretations of quantum theory

When in classical mechanics or quantum mechanics the state of a system can be specified, one has the greatest possible knowledge about this system in the following sense: A prediction can be made for all possible measurements on this system. In quantum mechanics this prediction is only statistical but the expectation value (average value) and the variance are well defined. We have already seen that the quantum-mechanical process of measuring, as opposed to the classical measurement, alters the state. Quantum mechanics is for equally prepared states *non deterministic* in respect of the results, which are obtained in a specific measurement on these states. It is *acausal* (already because of dynamics II). The quantum probabilities are primary

16) Although generally used, the denotation "interaction-free" is not entirely appropriate. The third arrow in Eq. (1.18) represents an interaction. For a more detailed analysis see Audretsch (2005).

and fundamental, and real chance accordingly exists. Even if we ensure that there is always only one object within the experimental setup, the ensemble result for the double slit is different from the superposition of the results from two single slits. Oversimplified, one might say: the single object "notices" the presence of both slits. In this sense, quantum mechanics and quantum physics are generally *nonlocal*.

From classical mechanics, we know for flying tennis balls how to imagine the states behind the slits. But what is there in quantum physics? Obviously this leads to the question about the interpretation of quantum theory. First, we should clarify what we understand by an interpretation, because this term is not uniformly used.[17]

A physical theory is on the one hand a system of mathematical symbols with rules applied to them, which are typically used to derive the results from basic equations. This is the *syntax*. The advantage of a mathematical syntax for a physical theory is that it makes it easier to track down errors and inconsistencies. The ways to draw conclusions become intersubjectively compelling and in this sense, an objectivity of the physical knowledge is created.

Of course, a physical theory has to be more than just a mathematical theory: it should make statements about a part of reality. Therefore, there are *mappings* between some of the mathematical symbols on one side and objects in reality on the other side. For example, $\underline{r}$ denotes the space of a classical body, m its mass, and t the time on a clock. We need, as an essential core part of a physical theory, mapping rules or *correspondence rules* that link certain mathematical quantities with the pointer positions of measuring instruments, and thus with the results of the measurements. Whether we can say about the numerous other mathematical quantities that appear in the physical theory that they are related to something in reality and thus have a physical meaning (*semantics*) remains an open question.

In classical physics it is no different. In electrodynamics there is a mathematical quantity $\underline{E}(\underline{r}, t)$ that is named the electric field. This is

17) See also the chapters of C. Held (Chapter 3) and M. Esfeld (Chapter 10) in this book. There are many discussions of conceptional problems of quantum theory. Such, with a close reference to the theoretical formalism, can be found for example in Primas (1981), Readhead (1987), Mittelstaedt (1989), Omnès (1994), Peres (1995), Home (1997), Mittelstaedt (1998), Espagnat (1999) and Auletta (2000). For a review, see also Audretsch and Mainzer (1990/1996) and Audretsch (2005).

a *theoretical term*, because primarily we are only observing the mechanical behavior of charges. There is the option to regard $\underline{E}(\underline{r}, t)$ and the differential equations for $\underline{E}(\underline{r}, t)$ as purely mathematical auxiliary quantities and relations that allow us to deduce statements that can be related to measurement results. These, for example, can be statements about the dynamical behavior of charged spheres. The physical "existence" of electric fields then would not be assumed. With this kind of an approach to physical theories, the only goal of the theory would be the prediction of experimental results. The one who chooses this position negates the existence of an objective reality that is independent of what observers are recording. But one might also aim for an understanding of the physical world by means of a theory. In that case, one would say $\underline{E}(\underline{r}, t)$ is actually describing something real. There is an element of reality described by $\underline{E}(\underline{r}, t)$, which is called the electric field. This approach is common among physicists, but it is important to realize that it is not necessary to share it when the purpose of a physical theory is solely seen in the prediction of measurement results.[18]

What is considered to be *real* is obviously dependent on how a theory is interpreted. Already classical physics is usually given an *interpretation* beyond the correspondence rules of the essential core. These interpretations go beyond the nonreducible core statements, which relate directly some terms of the theory with measurement results. However, different interpretations can be attributed to the same experimental consequences: they cannot be distinguished experimentally. They cannot be falsified. This situation becomes even more complex in quantum mechanics.

The admission of different interpretations, though, must not be mistaken with the proposal of a theory for the quantum domain which differs from quantum theory but nevertheless leads for all experiments to the correct data. This would then not be a new interpretation of an old theory, but a truly new competitive theory. It is conceivable that there are experiments for which this theory predicts results other than quantum theory. In such a case, a decision

18) The many-worlds interpretation (Everett (1957)) is usually regarded as being more than an interpretation in the above sense. Theoretical problems in connection with the quantum process are solved in a very speculative way in which finally a connection with the state of mind of the observer is established. With this, a theory is asserted, which goes far beyond the present range of application of quantum theory.

between both theories can be forced by an experiment. We will come to an example in Sections 2.4 and 2.6.[19]

The interpretations, however, are on the other hand not a part of philosophy by themselves. They still belong together with the physical theory. It is one of the aims of a physical theory to lead beyond the prediction of measurement results to a conception about the physical world. Interpretations are in this sense physics but not metaphysics. Nevertheless, they have philosophic or metaphysical implications. On these, natural philosophy typically sets in.

In quantum mechanics, interpretations also try to give an answer to the question: What is *real*? What, based on the theory, can be said about reality beyond the prediction of measurement results? As a starting point one could ask: What is the state vector? It describes a state — but of what?

The interpretation that one might name the *Copenhagen Interpretation*, because ideas from the early days of quantum mechanics are assimilated within, is today only of historical relevance.[20] A phrase from Nils Bohr is placed at its center: "There is no quantum world."[21] There is no quantum world and no quantum objects. Only the phenomena are real. "Behind it" there is nothing. The quantum world is a mental construct. The state vector is a purely mathematical auxiliary quantity without correspondence in reality. It serves for the calculation of probabilities of macroscopic events, for example of measurement results. The "quantum object" is nothing but a manner of speaking that facilitates the communication about a computational procedure. A term like "electron" is hence just a practical abbreviation that is referring to a whole complex of calculations. The measurement instruments are classical devices, which are not to be described quantum mechanically. The calculations are finally just providing statements about the classical states of the measurement instruments. The complementarity of position and momentum in the way it is shown by the uncertainty relation has its origin in the fact that no measurement instrument exists for a combined measurement of position and momentum. For the supporters of this extremely pragmatic and

19) 2 refers to the article "View into the quantum world II" hereinafter. 2.4 indicates the Section 2.4 in there.

20) The addition "Copenhagen" is so dazzling and ambiguous that strictly speaking it should be replaced. A description of the historical situation can be found in the following article by C. Held.

21) Quoted according to Primas (1981), page 101.

minimalistic interpretation, the two challenges "establish dynamics II from dynamics I" and "trace the behavior of classical objects back to quantum mechanics" are completely irrelevant. This shows very clearly that interpretations can definitely be of great consequence for the design of research programs and for the motivation of scientists.

In contrast to this interpretation, the two problems set out above are absolutely meaningful from the point of view of an *ensemble interpretation.* In this case, the quantum world exists. The statements of quantum mechanics and with it the state vector $|\Psi\rangle$, refer to a statistical ensemble of infinitely many systems, all prepared in the same way. Therefore, statements are never made about a single quantum object. The relative frequency of measurement results can be predicted using the state vector $|\Psi\rangle$. Since in practice only finite numbers of systems are available, this represents an approximation. With this interpretation one remains entirely on the deterministic level. The reality represented by $|\Psi\rangle$ is precisely a matter of the entirety of measurements already performed. Again, there is a strong limitation on statements about experimental data. Therefore, this is often called a *minimal interpretation.* It contains all that the present-day physicists can agree on without problems. For the example discussed above, it is the interference pattern on the screen. A single object within the experimental apparatus, for example at the double slit, has no counterpart in the theory. Nothing is stated about its reality. It is questionable whether it appears to be meaningful within this framework to carry out the transition from quantum objects to large molecules, biological systems and classical objects.

Nowadays, single atoms and ions can be stored and manipulated in traps. The reality of these objects is generally assumed beyond question. Therefore, they should be represented. This is the common opinion among physicists today. In this *single-system interpretation,* the state vector $|\Psi\rangle$ now directly refers to real single objects and their attributes. The single objects, besides, are mostly of a microphysical nature. In this interpretation, the measurement results do not appear as the primary *references* of theory anylonger, but instead they are the single objects on which the measurements are performed. The quantum object really exists also before and after the measurement. A justification for this interpretation should be searched for by the fact that properties like mass, charge and size of the spin of a quantum object always have the same value, independent of the preparation of the quantum system and the performed experiments. The quantum object by itself "has" these *classical properties*. They do not rely on "relations" between the system and the apparatus for its preparation or its measurement. These properties therefore can be specified as being *objective* and *real*.[22] The quantum system obtains an "existence" in between preparation and measurement. The state refers now to the preparation procedure. At the same time, the question about the nonclassical attributes becomes important.

A measurement transfers the object to a state that corresponds to the measured value. When the same measurement is immediately repeated, the same measured value results. For example, the repetition of a position measurement finds the object in the same position. The position uncertainty of the object after the first position measurement is zero. When a reliable prediction for the result of the measurement of a quantity (for example the position) can be made for a quantum state, one would say that this property (of having a position) must be assigned to the object in that state.

When a physical quantity is undetermined in a given state (for example the position if $\Delta x \neq 0$), the object does not have the attribute (of having a position); this attribute does not exist at all in this case. In the extreme case of a momentum eigenstate ($\Delta p_x = 0$), each measurement of the component p_x of the momentum gives a well-defined value but no prediction about the result of a position measurement is possible ($\Delta x = \infty$). Therefore, one would have to say that objects in the momentum eigenstate have the property of a momentum but the property of a position cannot be assigned to them. In the position measurement and thus under participation of the measuring instrument and in the course of a state modification, the object receives the property of having a position. A subsequent momentum measurement not only destroys the value of the property, but the property itself. As an expression of the uncertainty relation, the object will in general be in a state in which *neither* the attribute position *nor* the attribute momentum can be assigned in this sense. Position and momentum are then *potential* properties. Not until the measurement are they *actualized.*

22) To deduce the reality of atoms from the reality of the apparatuses that are used for the preparation and detection takes a great formal effort. Whoever would like to gain some insight may consult the book by Ludwig (1985).

Such *neither-nor-objects* are unknown from our everyday surroundings. However, this kind of object can be drawn, as in Fig. 1.10: it consists neither of three tubes nor of two boxes. As a matter of fact, when watching exclusively the upper part one can see three tubes, when watching the lower part there are two boxes. Only as the result of a specific measurement "look at the top" does the object get the attribute to consist of three tubes. This cannot be assigned before. It is only a potential attribute inherent to the object. This is equivalent for the two boxes. Both attributes mutually exclude each other. When the measurement "look at the middle" is made, we can learn neither about the attribute "tubes" nor about the attribute "boxes".

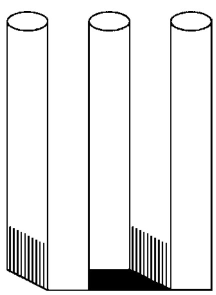

Fig. 1.10 A neither-nor-object.

When the comparison with classical physics is made, one realizes that not only the concept of "causality" but also the concepts "attribute" or "property" have become a great deal weaker, more general and more flexible. This reflects once again the fact that quantum theory is the more general and more comprehensive theory. It requires therefore — compared to the everyday physics-oriented common speech which is everyday physics-oriented — a *reduced level of speach.* When talking about quantum objects and their behavior we must not suggest, for example, that objects are particles or waves or that they always have all properties at the same time, by inconsiderately using common speech formulations.

Finally, it should be pointed out that the concept of "information" plays a completely new role compared to classical physics. There, information obtained by measurement tells us what has already existed before and is still there after the measurement, whereas in quantum physics the single measurement yields only information about the state after the measurement. In this sense, the quantum theory is of course more general but the statements are also weaker. This has consequences that will be discussed in Section 2.7. In reverse, a state $|\Psi\rangle$, from Fig. 1.10, which can assume all orientations α, can obviously be used to store more information than two bits. When these two peculiarities of quantum physics are combined with the entanglement of states described in the following chapter, an information theory of a completely unusual kind unfolds, with a wealth of new possibilities for information processing and transmission. This quantum information theory applies also to quantum computers that are constructed using entangled quantum systems. A fast-developing section of quantum physics has emerged in this direction over the past few years.[23]

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23) Details can be found in the articles from R. Löw, T. Pfau, H. Weinfurter and R. F. Werner in this book

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