1 Introduction

Drawing is the first method to shape our understanding of the world, for a child, for an artist, for an engineer, and for a mathematician. At school we learn how geometry can be abstracted from the images that are meant to describe some real object, and which are studied then without respect to their content. Things in space are projected onto a plane and we learn to figure out what happens to their form. We remember the curious properties of a triangle, for instance, that we can drop perpendiculars from the vertices, and that they meet at one point, that the hypotenuse of a right-angled triangle is the diameter of a circle around the triangle, and that the square on the hypotenuse of a right-angled triangle equals the sum of the squares on the other two sides. Some of us remember the logical compactness found in the axiomatic approach. Thales, Pythagoras, and Euclid are watching us.

Time seems to be different from space. Usually, it is not mentioned in geometry, and physics produces the impression that without Leibniz's and Newton's calculus one cannot say much about it. Forms in space have an aspect of stability, time is change instead. It was Einstein's theory of relativity that demonstrated the deep connection between space and time, and between geometry and physics. It became evident that elementary geometry is to be applied to the union of space *and* time. It became equally evident that physical observation decides which geometry of space and time is to be applied to real-world phenomena, and that a careful and elementary analysis of measurements is necessary to avoid misconceptions.

Usually, one does not imagine the motions of objects as geometrical figures in the union of space and time. For the insider, it is much faster to calculate analytically. Newton already solved the geometrical problems of the *Académie Française* analytically before embedding the result in a geometrical proof. Figures are drawn as auxiliary sketches at most. The outsider understands the theory of relativity as a system of more or less complicated formulas that avoid intuition. The following will show that the foundations of the relativity theory are fully subject to geometric intuition, and that relativistic kinematics is nothing else than the elementary geometry of the union of space and time. We shall learn how to use the drawing plane and space as space–time diagrams with one or two spatial dimensions and one dimension time.

A theoretical construction represented by elementary geometry and understood as an object of immediate geometrical experience leads to a strong expectation of internal consistency, more than an analytical derivation does for the outsider. For this reason, we wish to show in this book how elementary geometry, mechanics, and fundamental properties of the universe are interconnected. We intend to do this without the rigor that may be found quite readily in the literature. Instead, we wish to expose the real constructions and the relationships that produce the often aesthetically striking character of geometry. That is, we intend to fall in

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between all the stools available. However, we will discover many unexpected and astonishing relationships and associations. We shall consider the geometry of space and time and demonstrate by elementary means

- how physically elementary experiments receive a geometrical interpretation,
- how physical experiments restrict the properties of applicable geometries, and
- how geometrical properties determine correct physical formulations.

The figures of this book are produced with IDL. The programs can be requested by e-mail *deliebscher@aip.de*.

In Chapter 2 we introduce the notion of timetables as elementary representations of space–times. We shall learn the first means to draw in a space–time plane. The question of the definition of distances in timetables is left open here. Chapter 3 introduces the fundamental role of reflections. This role is a bit surprising because *real* motions are split into two reflections that produce only *virtual* images. However, in our timetables reflections are real and much simpler than other motions. We use this to get a first notion of the strangeness of the geometry in a timetable. Chapter 4 presents the central problem of Einstein's (special) theory of relativity. This was the first occasion to consider geometries different from the Euclidean geometry of space in the framework of physics. We correct the reflection procedure of Chapter 3 to solve the central problem and obtain the geometry of the space–time called the Minkowski geometry. The relativity theory and its paradoxes are considered in Chapter 5 with the help of this geometry. The elementary metric properties of the Minkowski geometry are compared with their Euclidean analogs in Chapter 6. Chapter 7 extends the relation between the Euclidean and Minkowski geometries of the plane to homogeneously curved surfaces, always trying to keep contact with physical examples. We obtain new, but characteristically similar, geometries. Chapter 8 presents the initial notions of projective geometry, which in Chapter 9 unites the geometries in one family, i.e., the Cayley–Klein geometries. This family can be characterized axiomatically as one expects for geometry. Chapter 10 deals with some general questions connected with the physical interpretation of these geometries.

All the notions explained in this volume are the subject of well-founded and strictly defined and formalized theories. It is not our aim to repeat these here, because we are interested in the interface, where these notions sometimes have to be unsharp enough to see that they fit. The necessary formal background for geometry is given in the appendices. Appendix A explains groups of motions and their generation by sets of generating elements interpreted as reflections. Appendix B considers questions connected with the physical introduction of coordinate systems, which, since the time of Descartes, have permitted the application of arithmetic methods to calculate and prove geometrical results. It explains in detail the transformations connected with changes in reference and introduced in the Riemannian geometry as far as these notions are concerned. Appendices C and D formalize the notions of projective and projective–metric geometry used in Chapters 8 and 9. Appendix E formalizes the classification of the Cayley–Klein geometries and, finally, gives the formal representation of the metric in projective metric spaces. In order to provide for a rapid access to definitions of the various notions used or touched in the book, a glossary is given instead of an index.

You will find many books about geometry or theory of relativity. Here only that part is cited that has some connection with our topic. Geometric and graphic presentation of the theory of relativity can be found in [1–7]. There are elementary [8–11] and less elementary [12, 13] introductions to the theory of relativity, in the general theory [14] and cosmology [15–18]. The descriptive and projective geometry can be learned in older [19–24] and more recent books [25]. Detailed information about the non-Euclidean geometry can be found in [26, 27]. General introduction to geometry is provided in [28–31]. The spatial imagination is trained in [32, 33]. And [34] is dedicated to computer graphics in our context.