Part II

Applications

1. Volume II in Overview

The work *Celestial Mechanics: Theory and Applications* consists of two volumes and three parts. This is Volume II, containing Parts II *Applications* and III *Program System*. Three key applications are discussed in the applications part, eight programs are described in Part III, *Program System*, which is used throughout the two volumes of this work.

1.1 Review of Volume I

Chapter I- 2 of Volume I briefly reviews the development of classical Celestial Mechanics, but also the developments related to the motion of artificial satellites.

In Chapter I- 3 the equations of motion were derived for three types of problems, namely

- the classical N-body problem with point masses in general and our planetary system in particular,
- the N-body problem with extended mass distributions in general and the three body problem Earth-Moon-Sun in particular, and
- the motion of an artificial Earth satellite,

which are considered in detail in this Volume.

In Chapter I- 4 the classical two- and three-body problems were developed and the extensions required for the relativistic treatment of these problems were specified. The definition of the classical orbital elements and the concepts of osculating and mean orbital elements will be assumed known in this Volume.

The variational equations, i.e., the differential equations for the partial derivatives of a particular solution of the equations of motion w.r.t. one of the parameters defining the initial (or boundary) values or the force field acting on the celestial body considered, were derived in Chapter I- 5. The solutions of the variational equations have to be known when determining the orbit of a celestial body or when studying the stability of a trajectory. Variational equations will be needed in particular in Chapter 4 of this Volume.

In Chapter I- 6 we derived differential equations for the osculating orbital elements. The set of the six first order differential equations for the six osculating elements (per point mass considered) is mathematically equivalent to the set of three second order differential equations for the Cartesian coordinates of the same body. The advantage of using the equations for the elements resides in the fact that they may be solved approximately. In this Volume the technique is used to interpret the osculating (and mean) elements emerging from the numerical solution of satellite orbits.

Numerical analysis, in particular the numerical solution of ordinary differential equations, was reviewed in Chapter I- 7. Starting from the most general problem, that of numerically solving a non-linear system of ordinary differential equations of order $n \geq 1$, algorithms for solving linear equations and for evaluating definite integrals (numerical quadrature) were developed. The so-called collocation methods were found to be very fruitful from the theoretical and from the practitioner's point of view. A collocation method provides an approximating function of the true solution, allowing it to compute (approximations of) the solution vector (and its derivatives) for any epoch contained within the integration interval. Multistep methods, but also the famous Gaussian methods for numerical quadrature, were recognized as special cases of collocation methods. Numerical integration techniques are the basis of the computer programs ERDROT, SATORB, and PLASYS accompanying the three main applications to be dealt with below. Not reviewing Chapter I- 7 before studying one of the chapters of Part II just implies that numerical integration is used as a "black box".

Orbit determination and parameter estimation is the concluding chapter of Volume I. As a matter of fact, this topic contains aspects of both, theory and application. Chapter I- 8 makes the distinction between first orbit determination (a non-linear parameter estimation problem) and orbit (parameter) improvement, which may be dealt with by linearizing and iteratively improving the orbits. In satellite geodesy orbit determination (improvement) often cannot be separated from the determination of other parameters. Many of the results discussed in Chapter 2 originally stem from such general parameter estimation processes.

1.2 Part II: Applications

Rotation of Earth and Moon. Chapter 2 deals with all aspects of the N-body problem Earth-Moon-Sun-planets. All developments and analyses are based on the corresponding equations of motion developed in Chapter I- 3; the illustrations, on the other hand, are based almost exclusively on the computer program ERDROT (see section 1.3).

In order to fully appreciate the general characteristics of Earth (and lunar) rotation, it is necessary to understand the orbital motion of the Moon in the first place. This is why the orbital motion of the Moon is analyzed before discussing the rotation of Earth and Moon.

The main properties of the rotation of Earth and Moon are reviewed afterwards under the assumption that both celestial bodies are rigid. Whereas the characteristics of Earth rotation are well known, the rotational properties of the Moon are usually only vaguely known outside a very limited group of specialists. Despite the fact that the structure of the equations is the same in both cases, there are noteworthy differences, some of which are discussed in this chapter. The analysis pattern is the same for the two bodies: The motions of the rotation axis in the body-fixed system and in the inertial system are established by computer simulations (where it is possible to selectively "turn off" the torques exerted by the respective perturbing bodies); the simulation results are then explained by approximate analytical solutions of the equations of motion. The simulations and the approximate analytic solutions are compared to the real motion of the Earth's and Moon's rotation poles. Many, but not all aspects are explained by the rigid-body approximation.

This insight logically leads to the discussion of the rotation of a non-rigid Earth. This discussion immediately leads in turn to very recent, current and possible future research topics. Initially, the "proofs" for the non-rigidity of the Earth are provided. This summary is based mainly on the Earth rotation series available from the IERS (International Earth Rotation and Reference Systems Service) and from space geodetic analysis centers. Many aspects of Earth rotation may be explained by assuming the Earth to consist of one solid elastic body, which is slightly deformed by "external" forces. Only three of these forces need to be considered: (1) the centrifugal force due to the rotation of the Earth about its figure axis, (2) the differential centrifugal force due to the rotation of the Earth about an axis slightly differing from this figure axis, and (3) the tidal forces exerted by Sun and Moon (and planets). The resulting, time-dependent deformations of the Earth are small, which is why in a good approximation they may be derived from Hooke's law of elasticity.

The elastic Earth model brings us one step closer to the actual rotation of the Earth: The difference between the Chandler and the Euler period as well as the observed bi-monthly and monthly LOD (Length of Day) variations can be explained now.

The elastic Earth model does not yet explain all features of the observed Earth rotation series. There are, e.g., strong annual and semi-annual variations in the real LOD series, which may *not* be attributed to the deformations of the solid Earth. Peculiar features also exist in the polar motion series. They are observed with space geodetic techniques because the observatories are attached to the solid Earth and therefore describe the rotation of this body (and not of the body formed by the solid Earth, the atmosphere and the oceans). Fortunately, meteorologists and oceanographers are capable of deriving the angular momentum of the atmosphere from their measurements: by comparing the series of AAM (Atmospheric Angular Momentum) emerging from the meteorological global pressure, temperature, and wind fields with the corresponding angular momentum time series of the solid Earth emerging from space geodesy, the "unexplained" features in the space geodetic observation series of Earth rotation are nowadays interpreted by the exchange of angular momentum between solid Earth, atmosphere and oceans – implying that the sum of the angular momenta of the solid Earth and of atmosphere and oceans is nearly constant.

Even after having modelled the Earth as a solid elastic body, partly covered by oceans and surrounded by the atmosphere, it is not yet possible to explain all features of the monitored Earth rotation. Decadal and secular motions in the observed Earth rotation series still await explanation. The explanation of these effects requires even more complex, multi-layer Earth-models, as, e.g., illustrated by Figure 2.55. The development of these complex Earth models is out of the scope of an introductory text. Fortunately, most of their features can already be seen in the simplest generalization, usually referred to as the Poincaré Earth model, consisting of a rigid mantle and a fluid core (see Figure 2.56). It is in particular possible to explain the terms FCN (Free Core Nutation) and NDFW (Nearly-Diurnal Free Wobble). The mathematical deliberations associated with the Poincaré model indicate the degree of complexity associated with the more advanced Earth models. It is expected that such models will be capable of interpreting the as yet unexplained features in the Earth rotation series – provided that Earth rotation is continuously monitored over very long time spans (centuries).

Artificial Earth Satellites. Chapter 3 deals with the orbital motion of artificial Earth satellites. Most illustrations of this chapter stem from program SATORB, which allows it (among other) to generate series of osculating and/or mean elements associated with particular satellite trajectories.

The perturbations of the orbits due to the oblate Earth, more precisely the perturbations due to the term C_{20} of the harmonic expansion of Earth's potential, are discussed first. The pattern of perturbations at first sight seems rather similar to the perturbations due to a third body: No long-period or secular perturbations in the semi-major axis and in the eccentricity, secular perturbations in the right ascension of the ascending node Ω and in the argument ω of perigee. There are, however, remarkable peculiarities of a certain practical relevance. The secular rates of the elements Ω (right ascension of ascending node) and ω (argument of perigee) are functions of the satellite's inclination i w.r.t. the Earth's equatorial plane. The perturbation patterns allow it to establish either sun-synchronous orbital planes or orbits with perigees residing in pre-defined latitudes $\phi \leq \pm i$.

The orbital characteristics are established by simulation techniques (using program SATORB), then explained with first-order general perturbation methods (based on simplified perturbative forces). Higher-order perturbations due to the C_{20}-term and the influence of the higher-order terms of the Earth's potential (which are about three orders of magnitude smaller than C_{20}) are studied subsequently. The attenuating influence of the Earth's oblateness term C_{20} on the perturbations due to the higher-order terms C_{ik} is discussed as well.

If a satellite's revolution period is commensurable with the sidereal revolution period of the Earth, some of the higher-order terms of Earth's potential may produce resonant perturbations, the amplitudes of which may become orders of magnitude larger than ordinary higher-order perturbations. Resonant perturbations are typically of very long periods (years to decades), and the amplitudes may dominate even those caused by the oblateness. Two types of resonances are discussed in more detail, the (1:1)-resonance of geostationary satellites and the (2:1)-resonance of GPS-satellites. In both cases the practical implications are considerable. In the case of GPS-satellites the problem is introduced by a heuristic study, due to my colleague Dr. Urs Hugentobler, which allows it to understand the key aspects of the problem without mathematical developments.

The rest of the chapter is devoted to the discussion of non-gravitational forces, in particular of drag and of solar radiation pressure. As usual in our treatment, the perturbation characteristics are first illustrated by computer simulations, then understood by first-order perturbation methods. Atmospheric drag causes a secular reduction of the semi-major axis (leading eventually to the decay of the satellite orbit) and a secular decrease of the eccentricity (rendering the decaying orbit more and more circular). Solar radiation pressure is (almost) a conservative force (the aspect is addressed explicitly), which (almost) excludes secular perturbations in the semi-major axis. Strong and long-period perturbations occur in the eccentricity, where the period is defined by the periodically changing position of the Sun w.r.t. the satellite's orbital plane.

The essential forces (and the corresponding perturbations) acting on (suffered by) high- and low-orbiting satellites are reviewed at the end of the chapter.

Evolution of the Planetary System. The application part concludes with Chapter 4 pretentiously entitled *Evolution of the Planetary System*. Three major issues are considered: (a) the orbital development of the outer system from Jupiter to Pluto over a time period of two million years (the past million years and the next million years – what makes sure that the illustrations in this chapter will not be outdated in the near future, (b) the orbital development of the complete system (with the exception of the "dwarfs" Mercury and Pluto), where only the development of the inner system from Venus to Mars is considered in detail, and (c) the orbital development of minor planets (mainly of those in the classical asteroid belt between Mars and Jupiter).

The illustrations have three sources, namely (a) computer simulations with program PLASYS, allowing it to numerically integrate any selection of planets of our planetary system with the inclusion of one body of negligible mass (e.g., a minor planet or a comet) with a user-defined set of initial orbital elements (definition in Chapter I- 2), (b) orbital elements obtained through the MPC (Minor Planet Center) in Cambridge, Mass., and (c) spectral analyses of the series of orbital elements (and functions thereof) performed by our program FOURIER.

By far the greatest part of the (mechanical) energy and the angular momentum of our planetary system is contained in the outer system. Jupiter and Saturn are the most massive planets in this subsystem. Computer simulations over relatively short time-spans (of 2000 years) and over the full span of two million years clearly show that even when including the entire outer system the development of the orbital elements of the two giant planets is dominated by the exchange of energy and angular momentum between them. The simulations and the associated spectra reveal much more information.

Venus and Earth are the two dominating masses of the inner system. They exchange energy and angular momentum (documented by the coupling between certain orbital elements) very much like Jupiter and Saturn in the outer system. They are strongly perturbed by the planets of the outer system (by Jupiter in particular). An analysis of the long-term development of the Earth's orbital elements (over half a million years) shows virtually "no long-period structure" for the semi-major axis, whereas the eccentricity varies between $e \approx 0$ and $e \approx 0.5$ (exactly like the orbital eccentricity of Venus).

Such variations might have an impact on the Earth's climate (annual variation of the "solar constant", potential asymmetry between summer- and winter-half-year). The eccentricity is, by the way, approaching a minimum around the year 35′000 A.D., which does not "promise" too much climate-relevant "action" in the near future – at least not from the astronomical point of view. The idea that the Earth's dramatic climatic changes in the past (ice-ages and warm periods) might at least in part be explained by the Earth's orbital motion is due to Milankovitch. Whether or not this correlation is significant cannot be firmly decided (at least not in this book). The long-term changes of the orbital characteristics (of the eccentricity, but also of the inclination of the Earth's orbital plane w.r.t. the so-called invariable plane) are, however, real, noteworthy and of respectable sizes.

Osculating orbital elements of more than 100′000 minor planets are available through the MPC. This data set is inspected to gain some insight into the motion of these celestial objects at present. The classical belt of minor planets is located between Mars and Jupiter. Many objects belonging to the so-called

Edgeworth-Kuiper belt are already known, today. Nevertheless, the emphasis in Chapter 4 is put on the classical belt of asteroids and on the explanation of (some aspects of) its structure. The histogram 4.43 of semi-major axes (or of the associated revolution periods) indicates that the Kirkwood gaps must (somehow) be explained by the commensurabilities of the revolution periods of the minor planets and of Jupiter. After the discussion of the observational basis, the analysis of the orbital motion of minor planets is performed in two steps:

- In a first step the development of the orbital elements of a "normal" planet is studied. This study includes the interpretation of the (amazingly clean) spectra of the minor planet's mean orbital elements. These results lead to the definition of the (well known) so-called proper elements. It is argued that today the definition of these proper elements should in principle be based on numerical analyses, rather than on approximate analytical theories as, e.g., developed by Brouwer and Clemence [27]. A few numerical experiments indicate, however, that the results from the two approaches agree quite well.
- Minor planets in resonant motion are studied thereafter. The Hilda group ((3:2)-resonance) and the (3:1)-resonance are considered in particular. The Ljapunov characteristic exponent is defined as an excellent tool to identify chaotic motion. A very simple and practical method for its establishment (based on the solution of one variational equation associated with the minor planet's orbit) is provided in program PLASYS. The tools of numerical integration of the minor planet's orbit together with one or more variational equations associated with it, allow it to study and to illustrate the development of the orbital elements of minor planets in resonance zones. It is fascinating to see that the revolutionary numerical experiments performed by Jack Wisdom, in the 1980s, using the most advanced computer hardware available at that time, nowadays may be performed with standard PC (Personal Computer) equipment.

1.3 Part III: Program System

The program system, all the procedures, and all the data files necessary to install and to use it on PC-platforms or workstations equipped with a WINDOWS operating system are contained on the CDs accompanying both volumes of this work. The system consists of eight programs, which will be briefly characterized below. Detailed program and output descriptions are available in Part III, consisting of Chapters 5 to 11.

The program system is operated with the help of a menu-system. Figure 1.1 shows a typical panel – actually the panel after having activated the program system *Celestial Mechanics* and then the program PLASYS. The top line of each panel contains the buttons with the program names and the help-key offer real-time information when composing a problem.

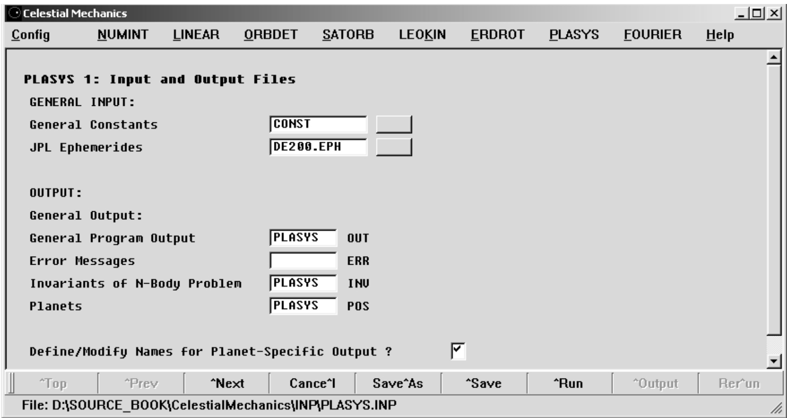

Fig. 1.1. Primary menu for program system *Celestial Mechanics, PLASYS*

The names of input- and output-files may be defined or altered in these panels and input options may be set or changed. By selecting ≪ Next Panel ≫ (bottom line), the next option/input panel of the same program are activated. If all options and file definitions are meeting the user's requirements, the program is activated by selecting ≪ Save and Run ≫ . For CPU (Central Processing Unit) intensive programs, the program informs the user about the remaining estimated CPU-requirements (in %).

The most recent general program output (containing statistical information concerning the corresponding program run and other characteristics) may be inspected by pressing the button ≪ Last Output ≫ . With the exception of LEOKIN all programs allow it to visualize some of the more specific output files using a specially developed graphical tool compatible with the menu-system. The output files may of course also be plotted by the program user with any graphical tool he is acquainted with. All the figures of this book illustrating computer output were, e.g., produced with the so-called "gnu"-graphics package. The gnu-version used here is also contained on the CD. The programs included in the package "Celestial Mechanics" are (in the sequence of the top line of panel 1.1):

1. **NUMINT** is used in the first place to demonstrate or test the mutual benefits and/or deficiencies of different methods for numerical integration. Only two kinds of problems may be addressed, however: either the motion of a minor planet in the gravitational field of Sun and Jupiter

(where the orbits of the latter two bodies are assumed to be circular) or the motion of a satellite in the field of an oblate Earth (only the terms C_{00} and C_{20} of the Earth's potential are assumed to be different from zero).

The mass of Jupiter or the term C_{20} may be set to zero (in the respective program options), in which case a pure two-body problem is solved.

When the orbit of a "minor planet" is integrated, this actually corresponds to a particular solution of the problème restreint. In this program mode it is also possible to generate the well known surfaces of zero velocity (Hill surfaces), as they are shown in Chapter I- 4.

2. **LINEAR** is a test program to demonstrate the power of collocation methods to solve linear initial- or boundary-value problems. The program user may select only a limited number of problems. He may test the impact of defining the collocation epochs in three different ways (equidistant, in the roots of the Legendre and the Chebyshev polynomials, respectively).

3. **SATORB** may either be used as a tool to generate satellite ephemerides (in which case the program user has to specify the initial osculating elements), or as an orbit determination tool using *either* astrometric positions of satellites or space debris as observations *or* positions (and possibly position differences) as pseudo-observations. In the latter case SATORB is an ideal instrument to determine a purely dynamical or a reduced-dynamics orbit of a LEO. It may also be used to analyze the GPS and GLONASS ephemerides routinely produced by the IGS (International GPS Service).

   The orbit model can be defined by the user, who may, e.g.,

   - select the degree and the order for the development of the Earth's gravity potential,
   - decide whether or not to include relativistic corrections,
   - decide whether or not to include the direct gravitational perturbations due to the Moon and the Sun,
   - define the models for drag and radiation pressure, and
   - decide whether or not to include the perturbations due to the solid Earth and ocean tides.

   Unnecessary to point out that this program was extensively used to illustrate Chapter 3.

   When using the program for orbit determination the parameter space (naturally) contains the initial osculating elements, a user-defined selection of dynamical parameters, and possibly so-called pseudo-stochastic pulses (see Chapter I- 8).

Programs ORBDET and SATORB were used to illustrate the algorithms presented in Chapter I- 8.

4. **LEOKIN** may be used to generate a file with positions and position differences of a LEO equipped with a spaceborne GPS-receiver. This output file is subsequently used by program SATORB for LEO orbit determination. Apart from the observations in the standard RINEX (Receiver Independent Exchange Format), the program needs to know the orbit and clock information stemming from the IGS.

5. **ORBDET** allows it to determine the (first) orbits of minor planets, comets, artificial Earth satellites, and space debris from a series of astrometric positions. No initial knowledge of the orbit is required, but at least two observations must lie rather close together in time (time interval between the two observations should be significantly shorter than the revolution period of the object considered).

   The most important perturbations (planetary perturbations in the case of minor planets and comets, gravitational perturbations due to Moon, Sun, and oblateness of the Earth (term C_{20}) in the case of satellite motion) are included in the final step of the orbit determination. ORBDET is the only interactive program of the entire package.

   The program writes the final estimate of the initial orbital element into a file, which may in turn be used subsequently to define the approximate initial orbit, when the same observations are used for orbit determination in program SATORB.

6. **ERDROT** offers four principal options:

   - It may be used to study Earth rotation, assuming that the geocentric orbits of Moon and Sun are known. Optionally, the torques exerted by Moon and Sun may be set to zero.
   - It may be used to study the rotation of the Moon, assuming that the geocentric orbits of Moon and Sun are known. Optionally, the torques exerted by Earth and Sun may be set to zero.
   - The N-body problem Sun, Earth, Moon, plus a selectable list of (other) planets may be studied and solved.
   - The program may be used to study the correlation between the angular momenta of the solid Earth (as produced by the IGS or its institutions) and the atmospheric angular momenta as distributed by the IERS (International Earth Rotation and Reference Systems Service).

   This program is extensively used in Chapter 2.

7. **PLASYS** numerically integrates (a subset of) our planetary system starting either from initial state vectors taken over from the JPL (Jet Propulsion Laboratory) DE200 (Development Ephemeris 200), or using the approximation found in [72]. A minor planet with user-defined initial

osculating elements may be included in the integration, as well. In this case it is also possible to integrate up to six variational equations simultaneously with the primary equations pertaining to the minor planet. Program PLASYS is extensively used in Chapter 4.

8. **FOURIER** is used to spectrally analyze data provided in tabular form in an input file. The program is named in honour of Jean Baptiste Joseph Fourier (1768–1830), the pioneer of harmonic analysis. In our treatment Fourier analysis is considered as a mathematical tool, which should be generally known. Should this assumption not be (entirely) true, the readers are invited to read the theory provided in Chapter 11, where Fourier analysis is developed starting from the method of least squares. As a matter of fact it is possible to analyze a data set using

   - *either* the *least squares* technique – in which case the spacing between subsequent data points may be arbitrary,
   - *or* the *classical Fourier analysis*, which is orders of magnitude more efficient than least squares (but requires equal spacing between observations), and where *all* data points are used,
   - *or* FFT (Fast Fourier transformation), which is in turn orders of magnitude more efficient than the classical Fourier technique, but where usually the number of data points should be a power of 2 (otherwise a loss of data may occur).

   In the FFT-mode the program user is invited to define the decomposition level (maximum power of 2 for the decomposition), which affects the efficiency, but minimizes (controls) loss of data. The general program output contains the information concerning the data loss.

   The program may very well be used to demonstrate the efficiency ratio of the three techniques, which should produce identical results. FOURIER is a pure service program.

The computer programs of Part III are used throughout the two volumes of our work. It is considered a minimum set ("starter's kit") of programs that should be available to students entering into the field of Astrodynamics, in particular into one of the applications treated in Part II of this work. Some of the programs, NUMINT, LINEAR, and PLASYS are also excellent tools to study the methods of numerical integration.