Preface

The cover picture shows a smooth quartic surface in space, the simplest example of a projective model of a $K3$ surface. In the following pages we will encounter many more examples of models of such surfaces.

The purpose of this volume is to study and classify projective models of complex $K3$ surfaces polarized by a line bundle L such that all smooth curves in $|L|$ have non-general Clifford index. Such models are in a natural way contained in rational normal scrolls.

These models are *special* in moduli in the sense that they do not represent the general member in the countable union of 19-dimensional families of polarized $K3$ surfaces. However, they are of interest because they fill up the set of models in $\mathbf{P}^g$ for $g \leq 10$ not described as complete intersections in projective space or in a homogeneous space as described by Mukai, with a few classificable exceptions.

Thus our study enables us to classify and describe all projective models of $K3$ surfaces of genus $g \leq 10$, which is the main aim of the volume.

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