Preface

This book presents a survey of modern methods for dynamic modelling of gas turbine engines. The manuscript was prepared using the results of research and unique practical experience of digital controller design for aero gas turbine engines. Experience of different research teams from Russia and the United Kingdom in the field of modelling of aero engines is summarised.

Dynamic models are traditionally used for describing behaviour of various systems. Dynamic models allow the design, analysis and identification of systems. Moreover, dynamic models can also be used for condition monitoring of complex systems providing information redundancy. The book describes various approaches to building and applying dynamic models. The approaches are compared and results of experiments on real engines are supplied.

The research team of Prof. G. Kulikov from Ufa State Aviation Technical University (Department of Automated Control and Management Systems) has a long history of participation in aero engine control design projects with Russian and Ukrainian industry. Chapters 1-4, 7, 10-13, 15 were contributed by Prof. G. Kulikov, Prof. V. Arkov and Dr. T. Breikin. Chapter 14 was contributed by Dr. O. Lyantsev.

The research team of Prof. P. Fleming works in close collaboration with British industry in the framework of the Rolls-Royce University Technology Centre for Control and Systems Engineering (Automatic Control and Systems Engineering Department, the University of Sheffield, UK). The introduction was contributed by Prof. P. Fleming and Prof. H. Thompson. Chapter 9 was contributed by Prof. P. Fleming, Prof. H. Thompson and Dr. K. Rodriguez-Vazquez.

The team of Dr. D. Rees (University of Glamorgan, UK) participates in research projects investigating modelling of Rolls-Royce aero engines. Chapters 5, 6 and 8 were contributed by Dr. D. Rees and Dr. N. Chiras.

Recently, a joint research programme was performed by all the mentioned teams funded by the British government enabling extended experimentation to be carried out at the Rolls-Royce test facility at DERA. Various identification techniques were demonstrated and compared during this project, and results are also featured in the book.

The monograph should be useful for control engineers, scientists and students.

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Chapter 2
Models and the Control System Design Cycle

2.1 Introduction

In this chapter techniques for mathematical modelling of gas turbine engines are presented considering the different stages in the life cycle from design, to demonstration, to commercial control system development, and finally through to in-service use. Through these stages the fidelity of the engine model is refined, as more information becomes available. The chapter thus considers evolution of models from the initial stages of design, where the general characteristics of the engine are known, to the latter stages, where a controller is tailored to the specific characteristics of the engine. The models required over the life cycle are discussed in greater detail in subsequent chapters. The key modelling stages are as follows:

- nonlinear performance–based modelling;
- dynamic characteristic modelling;
- linear dynamic modelling;
- piecewise linear dynamic modelling for real-time development.

In this chapter, a brief introduction is given to each of these techniques to highlight the use of the models at the different life cycle stages. It should be noted that the controller is an essential part of a GTE (as well as of the whole aircraft power unit). In practice, the engine and controller are designed simultaneously and are cooperatively tested. Therefore, the engine and controller are an inseparable integrated unit, consisting of transducers, converters, digital control electronics and actuators. The system has to be modelled as a whole considering the transducers and actuators to be part of the plant. Within this framework the control algorithms can be adjusted to provide the desired levels of performance.

2.2 Mathematical Models and Controller Life Cycle

The life cycle of the engine and its controller can be split into the following stages: scientific research, engineering design, demonstration, mass production, and the usage stage.

The *controller design stage* requires construction of a system with standard modules (transducers, actuators, processors, and system software), control algorithms, software development, controller synthesis and operational development. At this stage, mathematical models of the plant are widely used. After controller performance has been tested with test beds and engines, flight tests are performed (Figure 2.1). Joint GTE and controller tests enable experimental data to be gathered for system identification and controller operational development. Hence, design models are refined to reflect the individual characteristics of a particular engine, and the control algorithms, which are originally developed using a general engine model, are improved using more precise individual models.

Different accuracies and degrees of detail for the mathematical models are required at various stages of the life cycle of a controller (see Table 2.1). The degree of approximation to individual engine characteristics also differs.

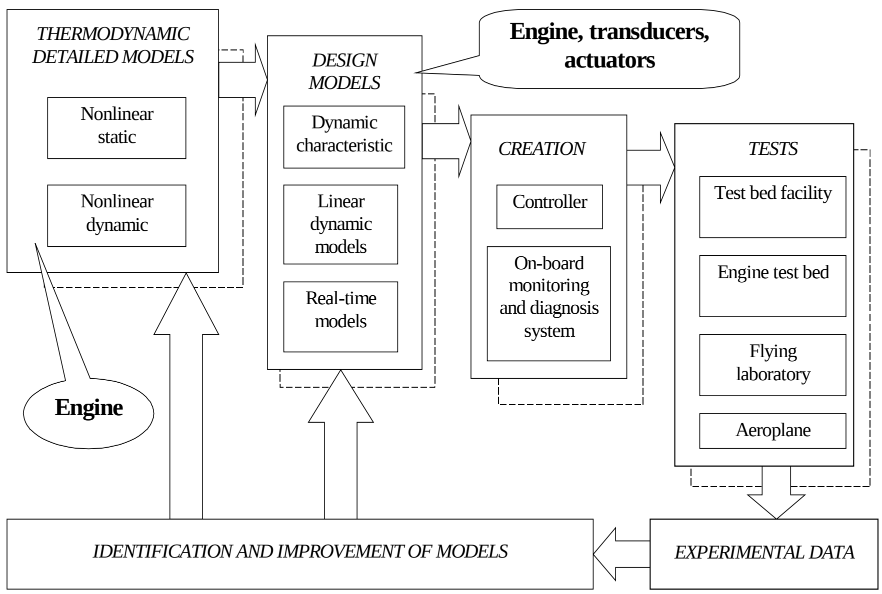

Figure.2.1. Design of controllers for gas turbine engines

Table 2.1. Engine modelling through controller life cycle

Stage	Models	Purpose
Design	General deterministic (simplified thermodynamic)	Control algorithms
Demonstration, tuning, production	Improved general + stochastic	Quality control
In-service use	Individual models	Monitoring and diagnosis, adaptation and optimisation

2.2.1 Design Stage Models

At the design stage, only general characteristics and parameters of the plant are necessary and the mathematical models are greatly simplified compared with the detailed nonlinear models used in engine design [1]. The design model of an engine is obtained by means of simplification of thermodynamic models. During the controller design stage, it is necessary to take into account that engine characteristics vary within broad limits due to flight and atmospheric conditions as well as due to gradual aging and deterioration of engine elements. Some change in characteristics also occurs after repair and replacement of elements of the engine and controller. Hence, the design model should reflect general characteristics of a whole class of engines. As a result, a controller is designed for an average plant. This is followed by individual tuning after installation of the controller on a particular power plant.

2.2.2 Experimental Demonstration and Production Stage Models

During the experimental demonstration and production stage, mathematical models are used for monitoring the fulfilment of requirements with respect to quality of control. Test bed facilities are designed for testing manufactured digital controllers. The term *test bed* refers to testing the controller, with the engine imitated by mathematical models. Such substitution provides a decrease in the amount of experimental research required on real engines and a considerable decrease in the research cost. Existing test beds for electronic controllers contain an electronic model of the controlled plant, imitators of transducers and actuators, and a logging device for recording and analysing experimental data. The most important components of such test beds are real-time dynamic models. Usually, these are deterministic linear dynamic models combined with nonlinear static models.

2.2.3 In-Service Use Models

In-service models can be used for health monitoring and on-line optimisation. These need to take into account the characteristics of the specific engine and the current operating conditions. A promising approach in this area is the use of identification techniques to produce accurate estimated models both off-line and on-line. A schematic of how this could be used in practice for condition monitoring is shown in Figure 2.2. Here, identification techniques are shown being used to aid in preflight check and scheduled maintenance to monitor the condition of the engine. The main types of engine modelling for control engineering purposes are discussed more thoroughly in the sections below; new trends in the development and application of such models are also described.

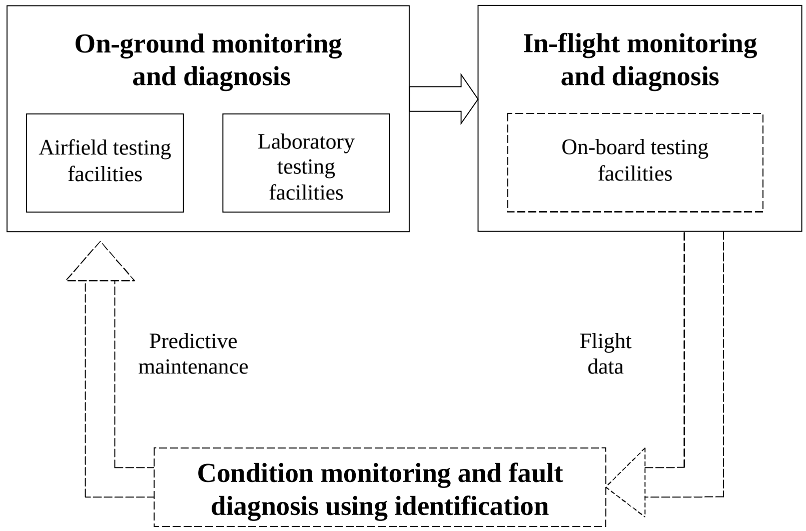

Figure.2.2. Use of identified models in scheduled maintenance

2.3 Dynamic Modelling Techniques

During the controller design stage, a number of dynamic models are derived from a detailed nonlinear performance-based model comprised of dynamic and static models. The aim of the nonlinear static model is to investigate engine characteristics during its creation. The model includes, for instance, the altitude-velocity and fuel-scheduling characteristics. Such models are based on gas dynamics laws and typically include over 100 algebraic and transcendent equations [2-4]. The characteristics are partly defined in the form of empirical formulae, graphs and tables. The static operating line is then calculated by means of iterative numeric methods.

The detailed nonlinear dynamic model is similar to the static model, but its equations are written in dynamic form. These show the balance between characteristics of GTE elements in the form of continuous flows and energy conservation equations. In algebraic form, they define a static model; with differential notation defining a dynamic model. The number of determining parameters in this model depends on the engine scheme and is defined by the number of energy accumulators. The purpose of the detailed nonlinear dynamic model is to analyse engine transients. Although the above-mentioned model can be used for controller performance analysis, a large amount of calculation is involved and so generally simplified models are used. However, as processing power increases, it is becoming more feasible to use this model directly.

2.4 Dynamic Modelling for Control Systems Development

In this section, dynamic modelling techniques are introduced for controller design purposes including dynamic characteristic models (DCM), linear dynamic models (LDM) and real-time piecewise linear dynamic models (RPLDM). The starting point in any gas turbine engine controller development is the performance-based model. This model is a highly complex thermodynamic model of the engine. A full description of this type of model is given in Chapter 3. The detailed performance-based nonlinear model can be simplified to create a set of dynamic models. An analytical relationship exists between the model's dynamic and static parts; this relationship allows dynamic models of the controlled plant to be obtained quickly. As an example considering a single-shaft gas turbine engine, the detailed nonlinear static model is represented by the following system:

$$\begin{cases} \mathbf{f}_X(\mathbf{X}, \mathbf{U}, \mathbf{V}) = \mathbf{0} \\ \mathbf{Y} = \mathbf{f}_y(\mathbf{X}, \mathbf{U}, \mathbf{V}) \end{cases} \tag{2.1}$$

where $\mathbf{X} = [n, p^*_{\mathrm{g}}, T^*_{\mathrm{g}}, T^*_{\mathrm{n}}, p^*_{\mathrm{n}}]^T$ is the state vector;
$\mathbf{U} = [W_f, A_n]^T$ is the control vector;
$\mathbf{V} = [M, H, p_H, T_H]^T$ is the flight atmospheric conditions vector;
$\mathbf{Y} = [n_{\mathrm{red}}, p^*_c, \pi^*_c, \ldots]^T$ is the observed coordinates vector.

Taking the detailed dynamic model from the performance-based model, Equation (2.1) can be transformed into a system of differential equations with the same vectors **X**, **U**, **V** and **Y**:

$$\begin{cases} \dot{\mathbf{X}} = \mathbf{F}_X(\mathbf{X}, \mathbf{U}, \mathbf{V}) \\ \mathbf{Y} = \mathbf{F}_y(\mathbf{X}, \mathbf{U}, \mathbf{V}) \end{cases} \tag{2.2}$$

In steady-state conditions, all derivatives are equal to zero, and Equation (2.1) transforms into the system Equation (2.2). Consider a single-shaft engine static model and dynamic model. For this the compressor–combustor continuous flow equation is as follows:

$$W_g - W_a - W_f = \Delta W_a^{\mathrm{dyn}} = Z_1 \Rightarrow 0 \tag{2.3}$$

where Z_1 is a discrepancy or an equality violation factor. By performing a differentiation on the gas condition equation for the combustion chamber with:

$$\frac{pV}{T} = mR \tag{2.4}$$

the following equation is obtained:

$$\dot{p}_g^* = \frac{RT_g^*}{V_{\text{comb}}} \Delta W_a^{\text{dyn}} + \frac{p_g^*}{T_g^*} \dot{T}_g^* \tag{2.5}$$

Equation (2.5) provides a link between the pressure and temperature derivatives and the dynamic addition ΔW_{dyn}. Equations like (2.5) linking (2.1) and (2.2) are the basis for the methodology of dynamic modelling using a detailed static model and additional information about the engine construction. For instance, Equation (2.5) includes the combustion chamber volume V_{comb}.

The linear dynamic model can be obtained from the detailed nonlinear model using identification or linearisation, provided the model structure is determined *a priori* from thermodynamic model analysis. Usually, the linearity zone for such parameters as shaft speed, temperature and pressure is approximately 3-5%. In order to determine the linear dynamic model coefficients, a transient with small amplitude is used around the steady-state conditions. Linear models describe engine dynamic properties in the form of linear differential equations and transfer functions. These models are very visual and convenient for stability and control quality analysis. However, the accuracy of results decreases due to approximation.

2.5 Dynamic Characteristic

The dynamic characteristic (DC) is a graphical image of the nonlinear dynamic model [2]. In addition to the static line $\dot{n} = 0$, the DC of a single-shaft engine also includes a set of lines of constant accelerations $\dot{n}$ =const. In order to derive a DC from the static model, a series of operating points is calculated. The operating conditions are formed with varying discrepancies, which are then converted into corresponding accelerations as described above. A full explanation of the dynamic characteristic models is given in Chapter 3. When creating the DC, solutions are selected in which all the discrepancies are equal to zero except the discrepancy determining the mechanical energy balance between the turbine and compressor. The DC derivation from the detailed dynamic model is performed by means of setting up the control laws $\dot{n} = \text{const}$, $p_c = \text{const}$, $T_t = \text{const}$ and calculating the corresponding lines in the engine parameters plane with the axes: "shaft speed *versus* fuel flow."

The DC is widely used because of the visual comprehensibility of engine dynamic properties and because it retains the main shape of the engine nonlinearity [3]. However, the DC of engines with a lot of variable geometry is multi-dimensional. Significant assumptions and simplifications are introduced to represent the dynamic characteristic on the plane or in 3D space, which leads to the loss in their visual comprehensibility.

2.6 Adaptation of Controller Structure and Parameters at the Design Stage

The engine operation and flight conditions are usually known at the design stage. This enables the controller's properties to be altered within the aircraft flight envelope. The GTE characteristics for any flight conditions can be accurately predicted using a detailed nonlinear model. However, this would require a large amount of calculation. Therefore, the engine model is usually simplified to produce the DC combined with the formulae for the reduction to the sea-level conditions with linear interpolation [4].

The flight envelope is divided into three subareas, different in terms of gas-dynamic similarity (Figure 2.3). In area **I**, the engine is described by a single DC in reduced coordinates, and fairly accurate results are produced by the reduction formulae:

$$n_{\text{red}} = n_{\text{phys}} \sqrt{\frac{288.15}{T_{\text{in}}^{*}}}, \quad \dot{n}_{\text{red}} = \dot{n}_{\text{phys}} \frac{101325}{p_{\text{in}}^{*}}, \ etc. \tag{2.6}$$

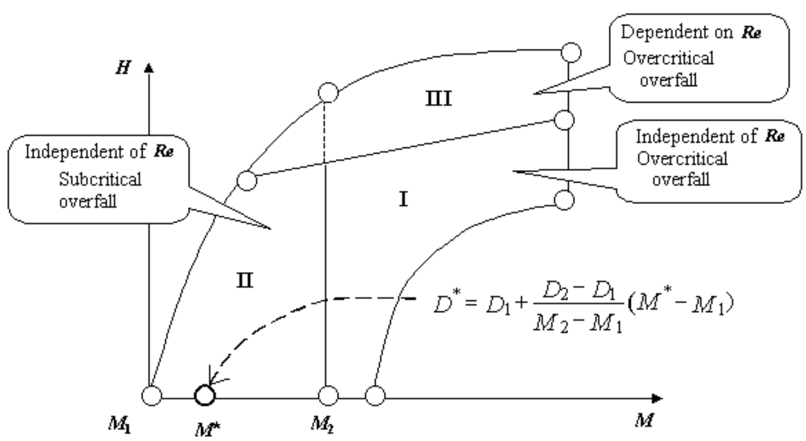

Figure 2.3. Adaptation of controller to flight conditions

In order to use the relationships indicated in Equation (2.6), the gas dynamic similarity conditions should be fulfilled. This includes the constancy of the engine geometry and the air properties such as humidity and *Re* number.

In other areas, these relationships are correct only at a qualitative level. In area II, the pressure overfall at the reactive nozzle becomes significant, and consequently, the DC is "stratified," and a separate model is created for each operating point. In area III, the effect of the *Re* number change is additionally taken into account.

In such a case, correcting coefficients are usually introduced into the thermodynamic model computations. In practice, the dynamic characteristics for particular flight conditions are determined by means of linear interpolation between the operating points.

A control program in reduced parameters:

$$n_{\text{red}} = f(\alpha_{\text{PLA}}) \tag{2.7}$$

is satisfactory if the engine is described by a single dynamic model. The program in Equation (2.7) sets up the demanded reduced shaft speed as a function of the angular position of PLA. Such control holds the engine within the gas dynamic stability region.

During transients, the $\dot{n}$ acceleration control program is used, where the shaft speed derivative is reduced using the engine inlet pressure. The various reduced parameter equations can also be used to control the transient operation. The temperature and pressure constraint programs are additionally introduced over the flight envelope to ensure gas dynamic stability. Thus, the controller acquires the ability to adapt in an open-loop scheme using *a priori* knowledge of the controlled plant properties. Note that the programs adapt to the flight conditions dependent upon p_{in}^* and T_{in}^*.

2.7 Linear Dynamic Models

This type of model is widely used in design and analysis of control systems. Linear dynamic modelling techniques are well established compared to nonlinear modelling. Control programs for steady-state conditions are usually designed and investigated using linear models.

Linear models are obtained from performance-based models via linearisation or identification methods. Also, they can be estimated from experimental data. Linear dynamic models are usually used in the form of state-space equations, transfer functions or Bode diagrams. These describe engine dynamics around a steady-state operating point. In Chapter 3, this modelling technique is discussed in more detail.

However, when exploring control programs for transient operation, linear models should incorporate nonlinear properties of the engine. The next section outlines a solution for this problem.

2.8 Real-time Piecewise Linear Dynamic Models

The real-time piecewise linear dynamic model (RPLDM) combines the engine’s nonlinearity and the LDM linearity. The source data for building the RPLDM are the nonlinear static lines and the LDM coefficients. The static characteristics are approximated by piecewise linear relationships determined at typically 7–10 operating points. The static line parameters between these points are then determined by means of interpolation. For the LDM coefficients, interpolation is used in the same manner. This is described in detail in Chapter 4. At the same time, it is assumed that the linear model parameters during transients are equal to the LDM coefficients for the closest static line point. The final calculated relationships Equation (2.8) establish the link between the **A**, **B**, **C** and **D** matrices of the LDM coefficients and the static line coordinates $\mathbf{X}_{st}$, $\mathbf{U}_{st}$ and $\mathbf{Y}_{st}$ by means of the operating parameter η :

$$\begin{aligned}\dot{\mathbf{X}}(t) &= \mathbf{A}(\eta)\left(\mathbf{X}(t)-\mathbf{X}_{st}(t)\right)+\mathbf{B}(\eta)\left(\mathbf{U}(t)-\mathbf{U}_{st}(t)\right)\\ \mathbf{Y}(t) &= \mathbf{C}(\eta)\left(\mathbf{X}(t)-\mathbf{X}_{st}(t)\right)+\mathbf{D}(\eta)\left(\mathbf{U}(t)-\mathbf{U}_{st}(t)\right)+\mathbf{Y}_{st}\end{aligned} \tag{2.8}$$

$$\eta = \sum_{i=1}^{n} z_i x_i(t) \tag{2.9}$$

RPLDM provides an acceptable level of accuracy and can be used in real time due to its simplicity as compared with a detailed nonlinear dynamic model. This enables control algorithms to be effectively tested on models and the controllers to be tested on electronic test beds. RPLDM models also have the potential to be used within on-board engine condition monitoring systems.

2.9 Control System Testing

Once a controller has reached this stage of design it is necessary to perform testing. There are several stages of testing which begins with a System Test Facility (STF) leading to an engine test bed and finally on the aircraft itself.

During initial testing it is common to use a STF. In a STF the control hardware and associated control laws can be exercised against a high fidelity model of the engine run in a computer. The control hardware may also include key actuation elements of the system for probing. This is commonly called hardware-in-the-loop or HIL.

In the second stage of testing the controller is exercised on the real engine on an engine test bed. This testing brings the operational environment of the controller closer to real-life operation. It is also common to operate an engine at an altitude test facility to explore the effects of the flight envelope.

The final stage of testing is on-wing testing where the engine is operated on a real aircraft and tests are performed over the flight envelope.

The different levels of testing become increasingly expensive as they approach real life. It is thus preferred to perform as many tests as possible at the STF stage. This is driving the development of more accurate identification and modelling techniques.

2.10 Concluding Remarks

This chapter has given a brief overview of the use of engine models at different stages in the controller development cycle. In Chapter 3, the derivation of off-line models is discussed in more detail. In Chapter 4, on-line dynamic models are discussed. This allows an appreciation of the relationships between the models and their derivation for specific control engineering needs.

The overview of mathematical models of gas turbine engines at life cycle stages reveals a certain contradiction between individual and general engine characteristics. The more accurate the mathematical models are, the closer they approach the particular plants, whereas their general properties are not taken into account. A general model for a whole class of engines is created in the process of the controller design. At the same time, characteristics of each particular engine are different from the general predicted model. A "draft" model of an engine can be more accurately defined by analysing characteristics of several engines. At the controller design stage, the general models enable the system characteristics to be improved. A controller created for general GTE is adapted individually for each particular engine. Every engine has its own individual properties and control characteristics. This is why, at the usage stage, the requirements qualitatively change, and every individual model provides useful information, especially if it is used for condition monitoring and fault diagnosis on the ground and in the air.

Mathematical models of gas turbine engines are extensively used in testing hardware, software, control laws and algorithms of the controller. This drives further investigation into identification and modelling of gas turbines to allow cost-effective testing based on computer simulation.