Preface

This book is intended for both undergraduate students and their staff as a guide to individual projects, group projects and case studies in mathematics. It covers all aspects of setting up projects and their assessment. The bulk of the text is devoted to giving worked examples of the various kinds of project. The author has benefited from a dual career first in an engineering department where he spent eight years which included looking after individual and group projects, often of a very practical nature. For the last twenty years, he has been within mathematics but still supervising different kinds of projects. This book is therefore written largely from experience. Mathematics here excludes statistics and operational research.

In the last few years there has been pressure on all undergraduate courses to include student centred learning. In mathematics this is difficult because most of the curriculum is devoted to learning mathematical skills. These skills are invariably assessed by examination as it is only in the controlled environment of an examination that the assessors can be sure who is being assessed. One does not assess skills such as spelling or piano grades by setting coursework or projects, hence many would ask how can you assess mathematics other than by a closed book examination? My reasoning is that with the increase in access the nature of many mathematics degrees has changed. If mathematics as a discipline is to keep pace with other undergraduate subjects it has to include modules that enable the student to develop in the broad sense. This means students must have the opportunity to work by themselves on a mathematical topic, or the opportunity to work with three or four other students towards a common goal. It is recognised that this must not be compulsory as there must be room for the clever student who has mathematical flair and may go on to a Fields Medal. This is the mathematics equivalent to the Nobel Prize. Incidentally there is a story that there is only no Nobel Prize in mathematics because

Alfred Nobel's wife eloped with a mathematician! As Nobel never married, the story cannot be true. However, perhaps he lost out in love to a mathematician. It is also said that Nobel had an intense personal dislike of the Swedish mathematician Gösta Mittag-Leffler who was attractive to the ladies and would have been the obvious candidate for the Nobel prize in Mathematics. Whatever the truth, it is a sad omission but as it happens many who have graduated in mathematics have gained a Nobel Prize–Paul Dirac (Physics 1933), Bertrand Russell (Literature 1950) and Richard Feynman (Physics 1965) to name but three.

These days mathematics must not be seen as an elitist subject only to be taken by specialists, but as a useful subject which many can benefit from studying. The usefulness of mathematics should be obvious to scientists, engineers and economists particularly as even the softer areas of these subjects become more quantitative, but its service as a basic discipline that aids the underlying thought processes needs emphasising too. Mathematics graduates have gone on to be politicians and lawyers as well as to succeed in the creative arts. The cry of "what use is Pure Mathematics" should be ignored; who in their right mind questions the use of music, literature or fine art? Mathematics has always been the most transferable of skills, but mathematicians have not traditionally been exposed to project work and group work as undergraduates. Traditionally, the mathematics undergraduate has had no forum in which they can discuss approaches to a problem with fellow students, or to voice their own opinions. This is no longer acceptable, and this text will help both students and lecturers see how projects and case studies can work well in mathematics.

In the text, there are passages that are verbatim extracts from student projects. These are indented and in a smaller font. In other places, the text may state that the following "was done in the project" or some such phrase. In these instances, the author has not copied the student's work but has used the same mathematical method or proof so that the reader understands how the student has approached a particular piece of mathematics. In all cases the author would like to thank all the students who have contributed projects and group work to this text, particularly those who he has supervised. Their anonymity has been preserved although they will easily recognise their own projects one hopes. Three individual projects have been included in their entirety in three separate appendices. Heartfelt thanks go to the three students concerned for permitting this, especially those for whom the project experience was less than ideal. The prime purpose of this text is to learn from the examples of others and to encourage students to try projects and case studies. One can often learn much from reading the works of other students, so these examples are very valuable. For the record, these projects have been faithfully reproduced as closely as possible from the originals. This means that the figures were scanned in and

therefore deliberately do not match the usual high standards of Springer-Verlag. On the other hand, the use of LaTeX has beautified the look of the mathematics, if not its content!

Thanks also go to the staff of the Mathematics and Statistics Department at the University of Plymouth. Many of the ideas, particularly those on assessment come from them. I thank those publishers who have permitted me to use figures from their books. Finally a big thank you to Noel Ford who drew all the cartoons to my sometimes less than well formed ideas.

Phil Dyke
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2
Assessment

2.1 Introduction

Assessment is the most important aspect of education to get right at all times, but especially when running a project or case study. These days students are not backward in telling you when things are going wrong, and if it involves their marks so much the worse. It is also true that now it seems more difficult to get students to do anything out of altruism than it used to be. It was once the case that students would expect "homework" to be set regularly, be marked and returned regularly. This was seen to be a standard part of any lecture course in mathematics. Nowadays, if homework is set too many students will not do it, even if this is accompanied by an apology and a Harrison Ford like lopsided grin. Despite it being helpful, a fact often duly recognised by the students themselves, the fact that it is not *essential* means that it does not get done. Or more accurately homework tends not to get done by those who would benefit most by doing it. Although educational theorists still talk about formative and summative assessment, in practice the distinction between the two is often fuzzy. After the first few weeks, everything students do usually needs to be connected to a mark that contributes to the final grade. Or, at the very least, be seen to be extremely useful towards gaining marks. In this chapter we tackle the all important question of how to assess individual projects, group projects and case studies, although it is the first two that take the lion's share of the chapter.

An important aspect is to make sure the student is committed quickly after

the start of the module. One way of ensuring this is to set an assessment quite near the beginning. Such an assessment need not be particularly summative, i.e. it need not contribute much toward the final grade, just enough to convince recalcitrant students to do something. It is largely formative and is there to help the student to know he is on the right lines. We call this assessment the "interim report".

2.2 Interim Reports

Interim reports are found in group projects and in individual projects. Generally they are absent from case studies. The group project, as the name implies, is a group of students working towards the production of a common piece of work. On the other hand the individual project is done by one person, with greater or lesser assistance from the supervisor. The role of the interim report is therefore subtly different in each case so let us discuss them separately.

To begin with individual project interim reports, these are primarily an indication that progress is being made with the project. Advice on how to write them is postponed until the next chapter. Here we shall concentrate solely on assessment. The supervisor should not be surprised by its contents, but it provides the student with an opportunity to review progress. It should also be assessed by another member of staff who can provide an independent view. The contents can be technical, outlining the mathematics done so far and signalling future work. Or it can be a general overview, describing what has been done. It is largely formative, but it is a good idea for it to attract some mark which is worth up to 10% of the total project mark. This mark should be the average given by the two assessors.

Group project interim reports are a little different. They are individual efforts, perhaps the only part of the group project that falls into this category (but see the next section on verbal presentations). Once more, guidance on how to write an interim group project report is found in Chapter 4, although it is worth looking at the guidance in Chapter 3 too as there are many similarities. The purpose of the interim report is still to assess progress so far, but it is also to help each student to identify his or her role within the group. A group project is based on the collective work of a group of students. In any particular group there may be dominance. One student may be physically dominant through appearance, a booming voice, a domineering attitude, etc. More likely one student could be intellectually superior, the boffin of the class perhaps. Another possibility is that the group consists of a "clique" plus an outsider. It is all too easy for this outsider to feel ostracised and leave the project for the rest to do.

In all of these cases, circumstances could lead to one or more members of the group not contributing sufficiently. The interim report forces every member of a group to home in on a sub-topic, do some work on it and write up what they have done. It also gives the module leader an indication of what each member of a group has done and will do, hence providing a valuable oversight of the entire project. Each interim report should still be second marked, although the final mark in this case could be agreed (over coffee perhaps) rather than a straight arithmetic mean. The mark given to the interim report here could be 15% rather than 10% to emphasise its importance. It is very important in both cases to give positive feedback to each student. At this early stage it is all too easy for students to become disheartened about the project or to go off at a tangent, or to get stuck. The feedback from the interim report is a useful vehicle to help the student formally. More seriously, if there is a severe problem later (for example health related), evidence from this stage can be very helpful.

2.3 Verbal Presentations

These days it is important for every student to get the opportunity to give a presentation. The era of the bright graduate in mathematics or engineering with a first class degree who can only mumble incoherently in interviews or meetings has passed. Being good on paper and in passing examinations is now only part of the story. All forms of project and case study can provide good vehicles for students to get used to presenting to others. Let us run through different ways this can be done and outline assessment procedures.

The most obvious, and the most daunting for the student, is to prepare a talk for the whole class. This is most often done in the context of an individual project where the student is given the opportunity either half-way through the year or at submission time to tell everyone about it. It has the advantage of being something that is new to the bulk of the audience (fellow students) and the staff present are normally sympathetic and do not ask too many awkward questions at the end. Nevertheless it is an ordeal for most students, particularly if they have not done anything like it before. The secret is to prepare well and run through the material a number of times before the event. Use of visual aids is encouraged. These used to be overhead projector transparencies written on or containing photocopied writing and equations, but increasingly now include PowerPoint presentations or computer projections and video clips. Here is a short list that students will find useful.

Figure 2.1 It is very important to give positive feedback to each student.

Figure 2.2 Staff are normally sympathetic and do not ask awkward questions.

In a good formal presentation the speaker should:

1. Await formal instructions from the Chairman or Announcer.
2. Thank the Chairman or Announcer.
3. Give a proper formal introduction to the talk in the form "I am .... from .... and my talk is on ....".
4. Always address the audience.
5. Keep the audience interested.
6. Give an outline of the talk at the start by going through a list of contents.
7. Present material in a logical, structured manner.
8. Use appropriate visual aids, usually overhead projector transparencies or PowerPoint.
9. Make sure the information on visual aids is clear and readable, and make sure they are not overcrowded.
10. Keep within the allotted time.
11. Conclude the talk properly (e.g. run through a list of conclusions).
12. Thank the Chairman and audience for their attention.
13. Respond clearly, concisely, correctly and politely to questions.

The presentation skills are of course important, but what must be right is the technical content. Students should remember that lecturers can see through the most glamorous presentation and soon detect any flaws or misunderstandings in the material of the project. The assessment of such project talks is done against two principal criteria, the mathematical content and technical level of the talk, and presentation skills. The weighting is either 50% each or 60%: 40% in favour of the content. There may be a heavier weighting in favour of content, but the presentation side should not be completely ignored. Again, these days verbal presentations are double marked to ensure fairness and quality. The verbal presentation is usually about 10% of the total individual project mark. Normally, students find the experience of standing and presenting very nerve wracking, and it is quite useful to be given some general guidelines on how performance relates to marks. The following list matches performance to grade:

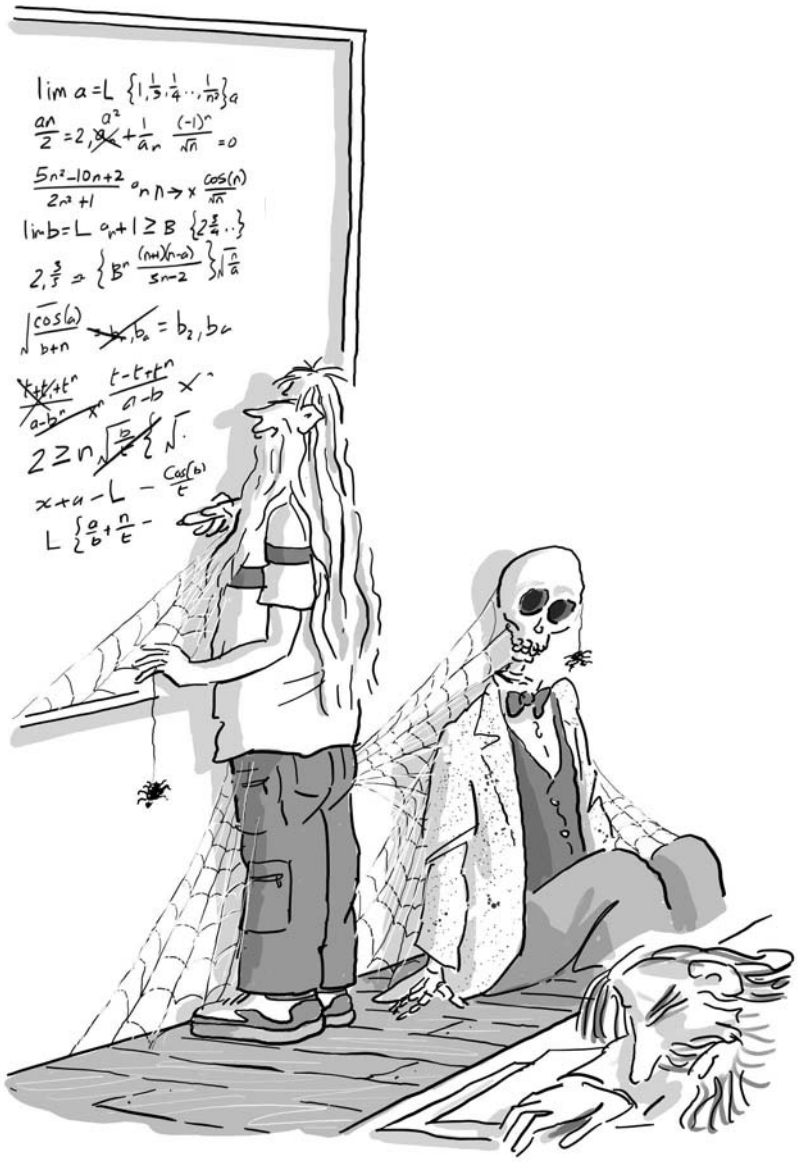

Figure 2.3 Students should keep within the allotted time.

Classification	Presentation
First	**Clear, insightful, good answers to probing questions.**
Upper Second	**Clear explanations, able to answer questions.**
Lower Second	**Some good explanations and answers, but sometimes unclear or unconvincing.**
Third	**Acceptable explanations of part of project, partial answers to some questions, but otherwise confused.**
Fail	**Completely muddled, or missed the point, or did not turn up!**

Students do not need convincing of the usefulness of giving a verbal presentation, both from the point of view of learning the material of their project and also in instilling confidence in situations such as the job interview. Many interviews these days include an element of presentation and students who have done this kind of thing before start with a great advantage.

In group projects, there is normally one group project presentation, but all members of the group must participate. This can be all group members taking a turn at the OHP, but it can be more adventurous. For example, a simulated interview, or one student taking on the role of Master of Ceremonies and introducing the rest in turn. This can present difficulties if the amount of time each member of the group is at the podium, so to speak, is very different. The assessor should take account only of the contribution in terms of technical content and presentation and this ought to be independent of the actual number of minutes it takes. The only time the student is penalised is if the presentation is far too long due to lack of discipline or organisation of material, or far too short due to ill-preparedness. Whatever format is used and however long each contribution takes, it is possible to give each participant an individual mark although inevitably there is less differentiation between content marks than between presentation marks.

There are other forms of verbal presentation that are well worth trying. One is the viva-voce, or viva for short. This is best suited to the individual project, but could be tried elsewhere. Normally the student is questioned for about fifteen minutes. The questioning is done by the assessor who may or may not be the project supervisor. In fact, an arrangement that works well is if the questions are led by an independent assessor with the project supervisor taking the role of the informed assessor, mostly listening but chipping in the odd

question. It is important that the supervisor is involved in assessing as a final year student often does not do justice to his or her knowledge in the stressful situation of a viva. The presence of the supervisor normally acts to calm the student, although of course one can always think of exceptions! This contrasts with the PhD viva in which the supervisor is definitely "prisoner's friend" and normally takes no role as assessor. Some postgraduate research students elect not to have their supervisor present: this has not been my experience for undergraduate student projects. It is important that there are two assessors for the project, and in the unlikely event that the student objects to one of these being his supervisor, two others have to be appointed. There is no point arguing with the student about this; however it must be made clear that there are severe disadvantages not allowing the supervisor to assess the project. For example the two people assessing the project may be unfamiliar with some of the circumstances of its production and not give credit at the correct weighting for work done. The assessment in this case is unlikely to be accurate. On the other hand to force a student to include an assessor/supervisor who, for whatever reason, is biased is to invite an appeal. Appeals are messy and best avoided. At the close of the viva, the student leaves the room and the two assessors agree a mark. This mark could be for the whole project and not just the student's performance at the viva. Indeed it is important that each student knows the criteria under which the viva is marked. Usually it is presentation skills and depth of knowledge shown. It sometimes happens that a student breaks down under vicious and over penetrative questioning. This is of course most unfortunate and may be the fault of an over zealous assessor who usually bears no malice but gets caught up in the subject matter of the project to such an extent that (s)he forgets that it is only an undergraduate student on the receiving end of questions and not an eminent professor. The supervisor should spot when this is happening and help the student, but if this does not happen there could once again be grounds for appeal. It is useful for a passive third assessor to be present in case of disputes. Finally of course telephone calls, knocks on the door and other external interference need to be eliminated as with any confidential meeting. I once conducted an oral examination with a raging storm outside which, with only partial success, we all tried to ignore!

Normally the viva is not marked separately and if it is, it is only given up to 10% of the marks. Oral presentations on the other hand are usually marked separately, again usually attracting 10% of the marks as a maximum.

Figure 2.4 The presence of the supervisor normally acts to calm the student.

2.4 Final Report

The actual writing of the report for the individual project is a subject tackled in the next chapter. Here we concentrate on its assessment.

Quite naturally, the final report is the ultimate goal and provides the main criterion by which to judge the success or otherwise of the entire venture. If more than one student is involved in producing the project report, then it is usually invidious to try and mark each contribution separately. Far better to give a single mark for the whole written report and use the other assessments to differentiate between students.

Some attention is now given to developing the criteria needed to arrive at a mark. To read a large piece of written work and then to decide "this is worth x%" is really not satisfactory and can leave academic staff open to student appeal. One way to be more helpful is to tell students how to get high marks in the written project. Here is a typical list:

Classification	Comment
First	**Elegant, excellent English, well structured, very few typographical errors.**
Upper Second	**Concise, good English, clear structure, not too many typographical errors.**
Lower Second	**Acceptable English, some structure, not very well proof-read, rambling prose.**
Third	**Some acceptable writing, some evidence of planning, many errors, muddled.**
Fail	**Illiterate and confused.**

It is emphasised that these are guidelines and that, for example, not all third class projects will be muddled. There will be some guidance also about the amount of time that should be spent on the project. Ten hours for every mark is consistent with 120 credit points every academic year. Since 1200 hours equates with 400 hours each term, and given a 10 week term, this implies a 40 hour week which is about right. More specific indications of attitude are given in the following list:

Classification	Typical commitment
First	**Enthusiastic, worked largely independently of supervisor, found own sources.**
Upper Second	**Found some extra references, diligent and well read.**
Lower Second	**Read the given texts thoroughly.**
Third	**Read the given texts, but was uncritical and sluggish.**
Fail	**Did not find or read anything relevant.**

So far nothing has been said about mathematical accuracy. A very well structured report, well written, thoroughly referenced but full of mathematical errors will not get good marks! Obviously it is also true that projects vary as to their mathematical sophistication. A student who is struggling with the technical aspects of mathematics is unlikely to do well in an individual project such as the hypergeometric functions project outlined in Section 3.7. However a project which is less mathematically demanding may well provide a good vehicle. The thorny question about levels of difficulty then arises. Is it permissible for a student to undertake a project with, in the extreme, no mathematical content? Perhaps the biography of a mathematician? The consensus seems to be that a certain level of mathematics is essential. For example, mathematical biographies, in addition to being scholarly need to contain a reasonably detailed account of the mathematical breakthroughs made by the subject of the biography. If the biography is of an ancient mathematician, say Archimedes or Apollonius then the report needs to contain a substantial amount of the mathematics of finding the area of a parabola using the method of indivisibles, or the geometry of spirals or whatever. An essay on the sociology of the time and some lamentation over all the lost Greek primary sources will not do. Then there are the projects that emerge from the general area termed Mathematics Education. Judging the quality of a project based around the impact of calculators on 'A' level mathematics against that on hypergeometric functions remains very difficult and the subject of some heated debates. The best advice to students is to do some advanced mathematics if at all possible or risk a poor project mark.

Copying has always been a problem to some extent, but in these days of the internet and floppy disc, it is so much easier to do and harder to detect. There are cases of completely plagiarised PhD theses that have gone undetected for years, and I am sure this is also true at undergraduate level. Most universities

have in place codes of conduct and severe disciplinary rules which prevent blatant plagiarism. However, the line between research and copying is a fine one especially at undergraduate level. Students must reveal all sources and include them in the reference list, even if this reference is a past project by an ex-student from a few years back. Building on the work of a previous undergraduate is commendable: copying from a previous project is unforgivable.

Although the emphasis above has been on assessing the written individual project report, much of it applies equally well to assessing any project report. The only additional aspect of a group report is assessing the integration of the different contributions. If this is well done, and an external examiner finds it hard to detect where one contribution ends and the next starts then credit should certainly be given. If it is entirely obvious that the report has been written by, say four individuals, then I am not convinced that any penalty should be exacted. On the other hand if each part is done in different type size and font, or there is discontinuity in page numbers (or no page numbers at all), or say two Figure 8's this detracts from the quality of the report and the mark must reflect this.

2.5 Moderating

Moderating is the name given to checking that all marks are fair and reasonable. In projects, this is done in several ways. In an individual project, it is a good idea for there to be a second marker for the project. Both attend any seminar or oral presentation involving the student, and both read and mark the project. The two assessors then agree on a mark, and this becomes the mark submitted to the examination board. In the event of disputes, some institutions involve a third assessor who acts as a referee. Each assessor writes a report on the project, and justifies the mark awarded. If the two grades are widely different, these reports should say why. A third assessor then brings judgement to bear and either agrees with one or other of the assessors, or strikes a happy medium. The third assessor's ruling should be final. If both assessors make similar comments, but award different marks, for example: "superb piece of work" 50% and "superb piece of work" 70% then the criteria given in the section above should be used to remind the first of these assessors that a "superb piece of work" equates to first class honours and ought to be awarded 70% rather than 50%. It is also common for one person, usually the final year tutor or equivalent to read all the individual project reports and check the comparative fairness of the marks. After this has been done, there is often a short(!) meeting at which minor adjustments can be made before marks are finally submitted

into the administrative system. Most academic staff are willing to concede that marking projects cannot be done to within an accuracy of three percent, and swapping the order of two projects between say 68% and 71% on the strength of the recommendation of the final year project coordinator's re-reading of both projects is normally not controversial.

Group projects are moderated by another member of staff overseeing all the assessment procedures. The moderator is involved in attending oral presentations and in reading the interim and final group reports. There is no involvement in peer assessment other than being aware (and commenting on if necessary) the procedure used.

2.6 Assessment of Case Studies

Some universities and colleges use case studies to mean group projects, in which case the above means of assessing group projects applies. Here a case study is defined as the presentation of an example of the use of mathematics to solve a specific problem, or a new application of mathematics. Normally it will span three to five hours of lectures. Students are thus attending lectures and tutorials in much the same way as in a standard module. At first sight therefore, there seems little reason to assess case studies in a manner different from a standard lecture course, that is, simply set an examination at the end and support this with some coursework, the weighting being perhaps 80% on the examination and 20% on the coursework. It is however possible to be more adventurous. Since a case study is an in-depth look at a particular mathematical technique, or some (to the student) new branch of mathematics it might be possible to set extended coursework with students working in pairs. Assessment can then involve some elements similar to the group project: verbal presentation and a common report. There should still be an examination as this is the fairest way of assessing the understanding of mathematics that has been given to the whole class. For case studies, an open book examination is particularly suitable, although in my personal experience there should be some restriction on the volume of material allowed into the examination room.

Figure 2.5 There should be some limit on the volume of material allowed into an open book examination.