In memory of my father

Preface

Symmetry has played an important role in art and architecture since early mankind. Symmetries occur in botany, biology, chemistry and physics. It is therefore not surprising that in various branches of mathematics symmetries and invariance principles have emerged and were put to good use as research tools. A prominent example is measure and integration theory where a rich theory has been developed for measures with remarkable symmetry properties such as Haar measures on locally compact groups, invariant measures on homogeneous spaces, or particular measures on $\mathbb{R}^n$ with special symmetries. In connection with invariant statistics research has focussed on specific situations in which measures that do not have 'full' symmetry occur.

This book also deals with measures having symmetry properties. However, our assumptions on the symmetry properties and on the general framework in which we are working are rather mild. This has two somewhat contradictory consequences. On the one hand, the weakness of the hypotheses we impose and the ensuing generality cause the mathematics with which we have to cope to be deep and hard. For instance, certain techniques which have been very efficient in differential geometry are not available to us. On the other hand, the generality of our approach allows us to apply it to an amazingly wide range of problems. This new scope of applications is the positive side of the generality of our attack.

The main results of this book belong to measure and integration theory and thus to pure mathematics. However, in the second part of the book many concrete applications are discussed in great detail. We consider integration problems, stochastic simulations and statistics. The examples span from computational geometry, applied mathematics, computer aided graphical design to coding theory. This book therefore does not only address pure and applied mathematicians, but also computer scientists and engineers. As a consequence, I have attempted to make the discourse accessible to readers from a great variety of different backgrounds and to keep the prerequisites to a minimum. Only in Chapter 2 where the core theorems are derived did it appear impossible to adhere strictly to this goal. However, I have arranged the presentation in such a fashion that a reader who wishes to skip this chapter completely, can read the remaining parts of the book without loss of under-

standing. As far as they are relevant for the applications, the main results of Chapter 2 are summarized at the beginning of Chapter 4.

This book arose from my postdoctoral thesis (Habilitationsschrift, [71]). I would like to thank Karl Heinrich Hofmann, Jürgen Lehn and Gunter Ritter again for acting as referees and for their valuable advice, Siegfried Graf for providing me with an interesting reference, and Hans Tzschach for the encouragement he has given me to even consider the writing of this thesis. Relative to the postdoctoral thesis, in the present book I have added some new results and a number of illustrative examples. Of course there are editorial adjustments, and the text has been made more 'self-contained'; this should enhance its readability without forcing the reader to consult too many references. I would like to thank Karl Heinrich Hofmann for his advice and support. Finally, I want to thank my wife Susanne for her patience with me when I was writing my postdoctoral thesis and this book.

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