Preface

Many partial differential equations arising in practice are parameter-dependent problems and are of singularly perturbed type for small values of this parameter. These include various plate and shell models for small thickness in solid mechanics, the convection-diffusion equation, the Oseen equation, and Navier-Stokes equations in fluid flow problems where the fluid is assumed to have small viscosity, and finally equations arising in semi-conductor device modelling. Analysis of such equations by numerical methods such as the finite element method is an important task in today's computational practice. A significant design aspect of numerical methods for such parameter-dependent problems is robustness, that is, that the performance of the numerical method is independent of, or at least fairly insensitive to, the parameter. Numerous methods have been proposed and analyzed both theoretically and computationally for a variety of singularly perturbed problems—we merely refer at this point to the three recent monographs [97, 99, 108] and their extensive bibliographies.
Most numerical methods employed in the study of singularly perturbed problems are low order methods. In contrast, the present work is devoted to a complete analysis of a high order finite element method, the *hp*-version of the Finite Element Method (FEM), for a class of singularly perturbed problems on curvilinear polygons. To the knowledge of the author, this work represents the first *robust exponential convergence* result for a class of singularly perturbed problems under realistic assumptions on the input data, that is, piecewise analyticity of the coefficients of the differential equation and the geometry of the domain.
This work is at the intersection of several active research areas that have their own distinct approaches and techniques: numerical methods for singular perturbation problems, high order numerical methods for elliptic problems in non-smooth domains, regularity theory for singularly perturbed problems in terms of asymptotic expansions, and regularity theory for elliptic problems in curvilinear-polygons. Although, naturally, the present work draws on techniques employed in all of these fields, new tools and regularity results for the solutions had to be developed for a rigorous robust exponential convergence proof.

This book comprises research undertaken during my years at ETH Zürich. I take this opportunity to thank Prof. Dr. C. Schwab for many stimulating discussions on the topics of this book and for his support and encouragement over the years.

Leipzig, June 2002 *J.M. Melenk*