

Optimization: The Competitive Edge

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Abstract

The purpose here is twofold. First, we will briefly review the state of the art. Examples will show that optimization is matured to the point where we can make daily use of this technology to reduce design times and improve product quality.

The second purpose is to identify the benefits of optimization and demonstrate that this is the one technology that is useful across a broad spectrum of design activities. Given today's emphasis on conservation of natural resources and reduced energy consumption, the motivation for using optimization is compelling. Indeed, this is a tool that will provide its users the competitive edge.

1 Basic Concepts

Numerical optimization solves the general problem: Find the set of design variables, X , that will :

$$\text{Minimize } F(X) \quad (1)$$

Subject to:

$$g_j(X) \leq 0 \quad j = 1, M \quad (2)$$

$$X_i^L \leq X_i \leq X_i^U \quad i = 1, n \quad (3)$$

The function, $F(X)$, is referred to as the objective or merit function and is dependent on the values of the design variables, X , which themselves include member dimensions or shape variables of a structure or radius of a pipe bend in computational fluid dynamics as examples. The limits on the design variables, given in Equation 3, are referred to as side constraints and are used simply to limit the region of search for the optimum. For example, it would not make sense to allow the thickness of a structural element to take on a negative value. Thus, the lower bounds are set to a reasonable minimum value. If we wish to maximize $F(X)$, for example, maximize fuel economy, we simply minimize the negative of $F(X)$.

The $g_j(X)$ are referred to as constraints, and they provide bounds on various response quantities. A common constraint is the limits imposed on stresses at various points within a structure. Then if $\bar{\sigma}$ is the upper bound allowed on stress, the constraint function would be written, in normalized form, as

$$\frac{\sigma_{ijk}}{\bar{\sigma}} - 1 \leq 0 \quad (4)$$

i = element,

j = stress component,

k = load condition

Additionally, we could include equality constraints of the form

$$h_k(X) = 0 \quad k = 1, L \quad (5)$$

Normally, equality constraints can be included in the original problem definition as two equal and opposite inequality constraints.

This problem statement provides a remarkably general design approach. However, though the methods for solving optimization problems are well established, industrial applications have lagged far behind the technology. This is largely due to an almost complete lack of education at the academic level as well as a general resistance to change in the engineering community.

However, we are seeing a dramatic increase in optimization applications today and can demonstrate a multitude of successes. Many of these are in the automotive industry and examples given here will necessarily focus on those. This is not to say that optimization has not been used in elsewhere, such as in aerospace. Indeed most of the R&D in engineering optimization has been developed with funding from NASA and the Air Force. Also, most aerospace companies will claim to use optimization. Yet published results are uncommon, partly due to proprietary issues and partly due to the very limited use of this technology.

2 History

Numerical methods for solving the optimization problem have been under development for over fifty years. However, until the 1980s optimization was mainly a research topic, with rare applications to real design tasks. From the 1980s, optimization has been added to several commercial finite element programs. Beginning in the late 1980s, general purpose commercial optimization software has become available, with graphics based software coming in the 1990s.

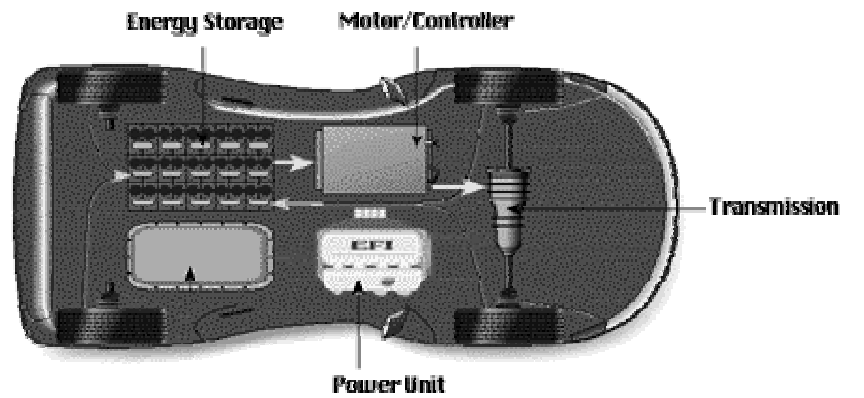
Today, robust, commercial software is available to solve a wide variety of design problems. In the case of structural optimization, the optimizer is incorporated directly into the finite element analysis program. For more general applications, where we do not have the refined methods available for structural optimization, the analysis is coupled externally to the optimizer.

3 General Optimisation

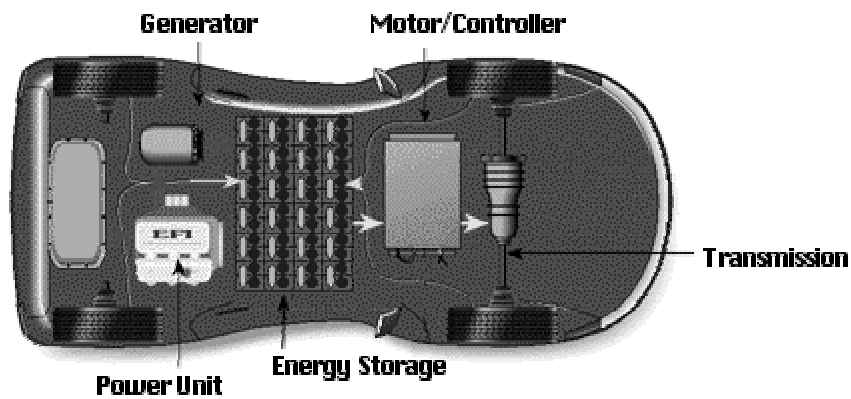
The basic concept here is to couple the optimization software to the analysis software of choice, such as nonlinear structural analysis, computational fluid dynamics, or thermal analysis, as examples. Using the graphical interface, the user identifies the inputs to the analysis program which are to be treated as design variables. Also, the user identifies quantities in the analysis output which may be the objective function or which may be constrained. The user then simply executes the combined program. Options normally available include gradient or non-gradient based optimization, response surface approximations (2,3) and design of experiments. Also, distributed or parallel computing can dramatically reduce computational times, even when the analysis is very complex and time consuming (4).

Using these methods, problems in the range of 10 to 15 variables can be routinely solved. Using gradient based methods, problems in the range of thousands of variables and millions of constraints are possible, limited primarily by computer resources. For such large problems, it is almost essential that gradient information is calculated analytically for computational efficiency. Reference 1 discusses numerous applications of this type of optimization, including airfoil, heat exchanger, conceptual aircraft and ship design as examples.

An example of general purpose optimization is shown in Figure 1, where the control strategy for a hybrid vehicle is designed. Two types of control strategy are used as shown in the figure.



Parallel Vehicles



Series Vehicles

Figure 1: Alternative Hybrid Vehicles

This is a multi-objective optimization task where we wish to maximize fuel economy, minimize hydrocarbon emissions and minimize nitrous oxide emissions of the parallel control strategy vehicle.

There were six design variables;

1. Battery pack's high state of charge

2. Battery pack's low state of charge
3. Electrical Launch Speed (vehicle speed below which vehicle operates as a zero emissions vehicle)
4. Charge torque (torque loading on the engine to recharge the battery pack when the engine is on)
5. Off torque fraction (fraction of torque capability of the engine for a given speed at which the engine may shut off)
6. Minimum torque fraction (fraction of the torque capability of the engine for a given speed at which the motor may act as a generator)

Constraints included limits on acceleration and grade climbing ability. The analysis software was the ADVISOR program from the National Renewable Energy Lab in Golden Colorado and this was coupled with the VisualDOC (5) optimization software to perform the optimization.

The optimization process increased fuel economy by 6.5%, reduced hydrocarbon emissions by 3.6% and reduced nitrous oxide emissions by 11.5%, though none of the design variables changed by more than 10% from their initial values.

Next, consider the design of a heat sink for electronic applications. Here VisualDOC was coupled with the FLUX2D thermal analysis program from Cedrat Corporation (6). The initial design is shown on the left side of Figure 2. Heat is generated by the thyristor and dissipated by the heat sink. The objective is to minimize the material of the heat sink and the design variables are the thickness of the base and height and width of the fins. Constraints include heat dissipated to the air, heat dissipated between the heat sink and the supporting chassis and the maximum temperature allowed in the thyristor. The initial design was chosen to have an unreasonably thick base to test the optimization. The optimum design is shown on the right side of Figure 2 and is very similar to heat sinks commonly found in electronic devices. This demonstrates the ease with which a commercial analysis program can be coupled with optimization.

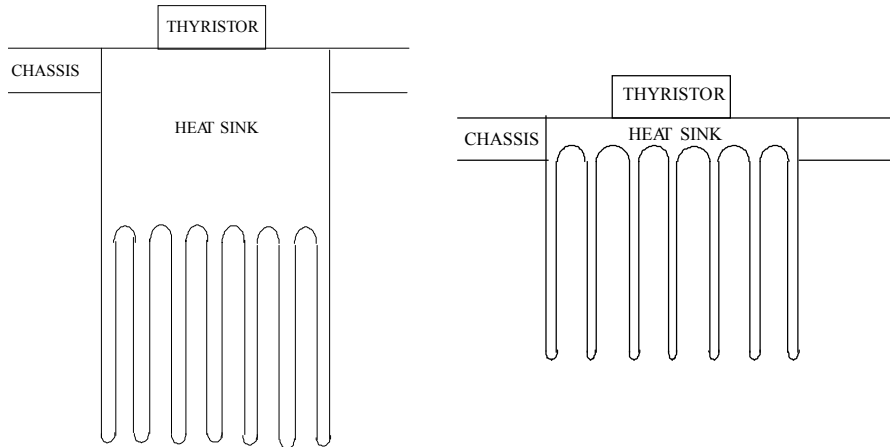


Figure 2: Heat Sink Geometry Optimization

As a final example to demonstrate coupling existing commercial analysis software with optimization, the Fluent CFD code (7) was coupled with VisualDOC. As a simple demonstration, the lift to drag ratio of an airfoil was maximized. The initial design was a NACA 0012 airfoil and the design condition was at very low speed (20 m/s). The design variables were the camber, position of maximum camber, maximum thickness and angle of attack. Response surface approximations were used to optimize the airfoil and the lift/drag ratio was increased by 160%. The optimization required fifteen Fluent analyses. The optimum airfoil is shown in Figure 3.

These examples serve to demonstrate that we have well established technology and software to couple a wide range of commercial analysis software with optimization. Coupling of the analysis programs used here typically required less than one week, most of which was spent by the optimization engineer to become familiar with the analysis software, with only a small portion of the time spent on the actual coupling of the programs.

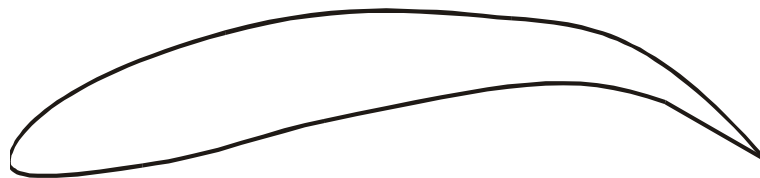


Figure 3: Optimum Airfoil

4 Structural Optimisation

Structural optimization methods are particularly well developed. Today, structural optimization, based on linear finite element analysis, can be routinely performed for member sizing, shape and topology optimization. Almost any calculated response can be treated as the objective function or can be constrained. Most of the time, mass is treated as the objective function to be minimized, though it is also common to maximize frequencies (stiffness), minimize the maximum stress, etc. Constraints typically include limits on stresses, strains, frequencies, dynamic response, thermal response, buckling loads and aeroelastic response. A typical structural optimization problem may consist of perhaps 10 to 500 design variables with 1,000,000 constraints, although larger problems are solvable. Indeed, in this author's experience, a mass minimization problem with over 100,000 design variables has been solved subject to frequency constraints, where the finite element model included over 800,000 degrees of freedom.

The key to today's efficiency in structural optimization is approximation techniques (8,9). Here, the original problem is approximated in terms of intermediate variables and intermediate responses. These approximations are gradient based and gradients are efficiently calculated as part of the finite element analyses (10,11). The approximate problem is then solved, a new finite element analyses is performed, and the process is repeated to convergence. These approximations go far beyond simple linearization and are of such high quality that the design variables can typically be changed by up to 50% before a new approximation is needed. The result is that, for member sizing and shape optimization we require only about ten detailed finite element analyses and for topology optimization about twenty detailed analyses. This is a key issue because finite element models of the order of one million degrees of freedom are becoming commonplace and a single analyses can be quite expensive. Thus, with this efficiency, we can achieve an optimum design for a cost well below the cost of just achieving an acceptable design in the past.

Figures 4-6 provide an indication of the type of design tasks that are routinely solved using modern structural optimization.

Figure 4 shows topology optimization of a truck front cross-member. The structure was first modeled by filling the available design space with solid elements. Topology optimization was applied by letting the density of each element be a design variable and maximizing the stiffness of the structure. This example included over 10,900 design variables. Those elements whose density was reduced to zero were removed from the structure to provide the optimum topology shown in the right half of the figure. Optimization required 20 detailed finite element analyses to achieve this result. This example is several years old and today smoothing techniques are used to generate a more smooth topology. After this first step, shape optimization can be applied to further refine the structure, including stress and other constraints.

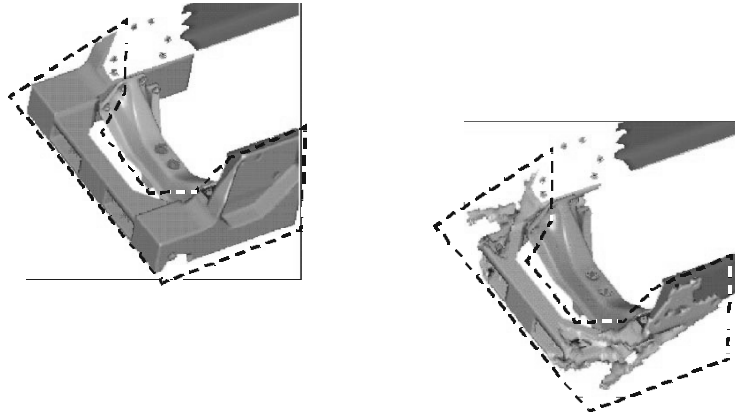


Figure 4: Topology Optimization

Figure 5 shows a heavy truck where the front suspension mount is to be designed. The objective was to minimize mass, subject to the requirement that the maximum stress not exceed the maximum stress in the existing design. As shown in Figure 6, the mass was reduced by 30% with no reduction in strength. The optimum design was achieved using ten detailed finite element analyses.

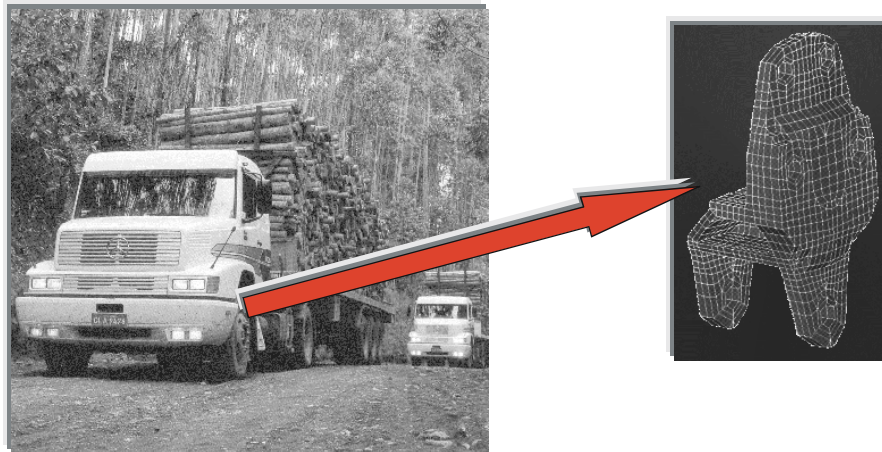


Figure 5: Truck Suspension Support

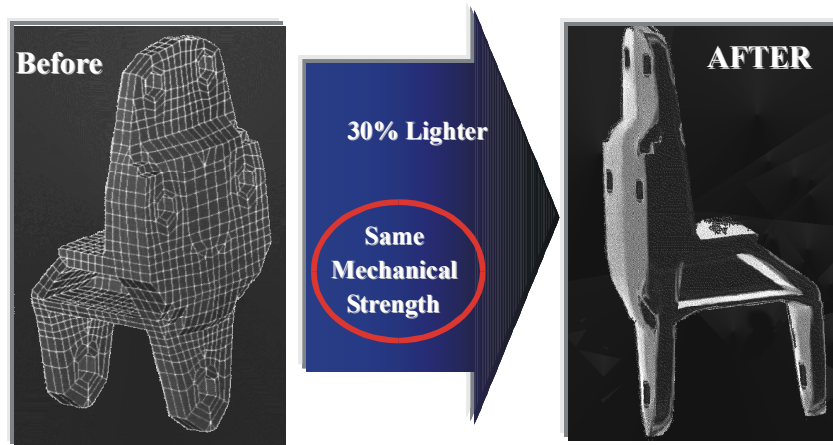


Figure 6: Truck Suspension Support Results

These problems were solved using the GENESIS structural optimization software (12). While these examples are typical of structural optimization, they represent only a small fraction of design tasks being solved today. Also, much larger problems are feasible where thousands of design variables and millions of constraints can be included. Recently, a mass minimization automotive body design problem was solved subject to increasing the fundamental frequency by 10%. The finite element model was 800,000 degrees of freedom and 105,000 member sizing variables were considered. An optimum design was achieved using 14 detailed finite element analyses.

5 Economic Motivation

The examples presented here, as well as a multitude of proprietary tasks that have been solved, require much less time and effort than that needed to achieve even near optimum designs using traditional “cut and try” methods. Today it is undeniable that optimization produces better designs in less time than we can produce by other methods. Also, the exceptional generality of these methods gives us a design tool for almost all aspects of engineering design, or other applications where computer based analysis is available. Indeed, we do not even need a computer program.

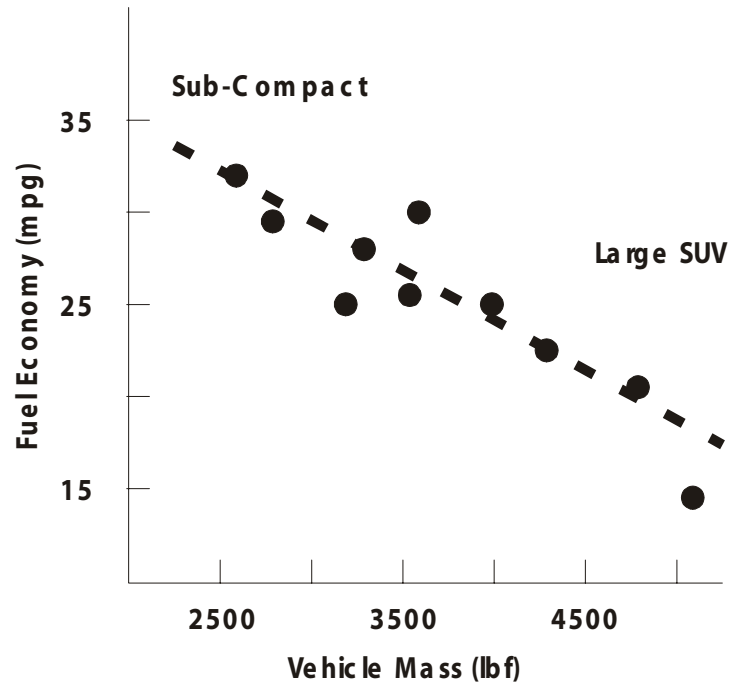


Figure 7: Fuel Economy

We can use experimental results, together with response surface approximations to improve our results (13).

Consider now the economic and natural resource benefits of using optimization. Again, from the automotive industry, Figure 7 shows the relationship between highway fuel economy and mass of typical passenger vehicles.

From the figure, it is clear that reducing mass by 50% will increase economy by about 100%. Of course, the larger vehicles carry more passengers or offer other features that justify their size. However, regardless of vehicle class, if we can reduce mass alone, we can clearly improve economy.

What are the benefits of a one percent reduction in gasoline use in the U.S.? Currently, we use about seven million barrels of gasoline (385,000,000 gallons) daily. In other words, we consume about one gallon of gasoline for each American every day. If we can reduce fuel consumption by only one percent, the annual cost savings (at \$1.50/ gallon) is over two billion dollars!

From Figure 7, we see that a one percent in mass reduction alone improves economy by about two percent. Virtually every published result, where optimization is applied to an existing design to reduce mass without reducing strength, shows that we can save about five percent by using optimization. Of course this is not possible in such areas as sheet metal hoods and roofs, but it is possible for floor panels, internal door panels, suspension parts, etc. Therefore, it is entirely reasonable to argue that we can reduce vehicle mass by a meager one half

of one percent using optimization, thereby gaining a one percent efficiency improvement without any new materials, power plants or other new technology.

Now consider applying optimization to aircraft design. This has been done for many years at the conceptual design level, but rarely at the detailed design level. Yet, if we can reduce mass of a transport aircraft by only 200 lbf, we can add a paying passenger. On a large commercial aircraft, with over 100,000 pounds of structure, it is easy to predict that optimization can add a passenger or two. Furthermore, if we use optimization to simultaneously design the structure and aerodynamic shapes and perhaps power plants, the benefits of optimization become even more compelling. Finally, considerable mass savings are possible in various passenger accommodations such as seats. In other words, we should not just focus on primary structure or aerodynamic shapes, but apply the technology to all aspects of the design.

The key idea is that optimization provides the tools to create higher quality designs in less time. Also, it can be used improve safety, such as crash worthiness (14). Overall, optimization is an ideal tool for conservation of natural resources, whether applied to existing technology or to technologies (such as fuel cells) of the future.

6 Summary

Although there are always advancements possible, numerical optimization is now a mature technology that can be applied to almost all aspects of design. The question is then, “Why is optimization not more widely used?” The answer to this question has eluded this author for nearly 30 years. Part of the answer is that it is seldom taught to undergraduates as a design tool, so few engineers really understand its power. Part of the answer is that, too often, optimization experts at companies demand that engineers come to them to do optimization, rather than spreading the technology throughout the company. Perhaps most of the answer is that we are comfortable doing things as we always have. When presented with these tools, the response is too often “Can’t you see that we’re too busy and too broke to consider something that will save us time and money?”

This is not to say that optimization is not being used today. Indeed, applications are growing at an accelerated rate. It is just that the technology is only used to a fraction of its potential. Whatever the reason for the present status, it is incumbent on the engineering community to make better use of this and other technologies that will improve designs and reduce the consumption of scarce resources.

Although the most sophisticated and interesting applications of optimization are proprietary, several examples have been offered to indicate the technology and the benefits obtainable from its use. While the examples presented here for structural optimization used GENESIS and for general applications used VisualDOC, there are numerous additional commercial software programs available today. Also, for computational fluid dynamics and many other analysis tasks, the user has a wide range of commercial analysis tools to choose from.

To those who say optimization is too complicated or not ready for routine use in design, we can say, categorically, that they are wrong. Commercial software products available for optimization require very little training and provide substantial design capabilities. Those who embrace this technology will indeed have a clear competitive edge. More importantly, this technology can go far to conserve natural resources, without reducing our quality of life.

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