Preface

Matrix analysis is a research field of basic interest and has applications in scientific computing, control and systems theory, operations research, mathematical physics, statistics, economics and engineering disciplines. Sometimes it is also needed in other areas of pure mathematics.

A lot of theorems in matrix analysis appear in the form of inequalities. Given any complex-valued function defined on matrices, there are inequalities for it. We may say that matrix inequalities reflect the quantitative aspect of matrix analysis. Thus this book covers such topics as norms, singular values, eigenvalues, the permanent function, and the Löwner partial order.

The main purpose of this monograph is to report on recent developments in the field of matrix inequalities, with emphasis on useful techniques and ingenious ideas. Most of the results and new proofs presented here were obtained in the past eight years. Some results proved earlier are also collected as they are both important and interesting.

Among other results this book contains the affirmative solutions of eight conjectures. Many theorems unify previous inequalities; several are the culmination of work by many people. Besides frequent use of operator-theoretic methods, the reader will also see the power of classical analysis and algebraic arguments, as well as combinatorial considerations.

There are two very nice books on the subject published in the last decade. One is *Topics in Matrix Analysis* by R. A. Horn and C. R. Johnson, Cambridge University Press, 1991; the other is *Matrix Analysis* by R. Bhatia, GTM 169, Springer, 1997. Except a few preliminary results, there is no overlap between this book and the two mentioned above.

At the end of every section I give notes and references to indicate the history of the results and further readings.

This book should be a useful reference for research workers. The prerequisites are linear algebra, real and complex analysis, and some familiarity with Bhatia's and Horn-Johnson's books. It is self-contained in the sense that detailed proofs of all the main theorems and important technical lemmas are given. Thus the book can be read by graduate students and advanced undergraduates. I hope this book will provide them with one more opportunity to appreciate the elegance of mathematics and enjoy the fun of understanding certain phenomena.

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I wish to express my gratitude to my son Sailun whose unique character is the source of my happiness.

Sendai, December 2001 *Xingzhi Zhan*