1. Introduction

1.1 What is a Stochastic System?

A "stochastic system" is understood here as a *dynamic* system that has some kind of uncertainty. The type of uncertainty will be specified in a precise mathematical sense when dealing with methods of analysis and design. At this point, it is sufficient to say that the uncertainty will include disturbances acting on the system, sensor errors and other measurement errors, as well as partly unknown dynamics of the system. The uncertainties will be modelled in a probabilistic way using random variables and stochastic processes as important tools.

Theories of stochastic systems are very useful in many areas of systems science and information technology, such as controller design, filtering techniques, signal processing and communications. They give systematic techniques on how to model and handle random phenomena in dynamic systems.

Some typical illustrations of the usefulness of stochastic systems are given later in this chapter. They show that the concepts of stochastic dynamic systems can be useful for forecasting (Example 1.1), control under uncertainty (Example 1.2) and the design of filters (Example 1.3).

This book is aimed as an introduction to the properties of stochastic dynamic systems in discrete time. There are several reasons why the emphasis is on discrete-time systems only. One is that, today, processing equipment for filtering and control is very often based on digital hardware, so data are available only in discrete time. Another reason is that discrete-time stochastic processes are much easier to handle than their continuous-time counterparts, which have certain mathematical subtleties that are far from trivial to handle in a stringent way. Nevertheless, continuous-time processes will occasionally be discussed, especially as far as sampling is concerned.

Most of the material centres around the treatment of linear systems using variance criteria as measurements of performance. This is no doubt very useful in many areas of application. The combination of linear dynamics and quadratic performance criteria also leads to neat mathematical analysis. One should, however, remember that aspects other than low variance may sometimes be of importance. There can also be strong nonlinear effects to consider. Such aspects are only discussed briefly in the book, and the mathematics then

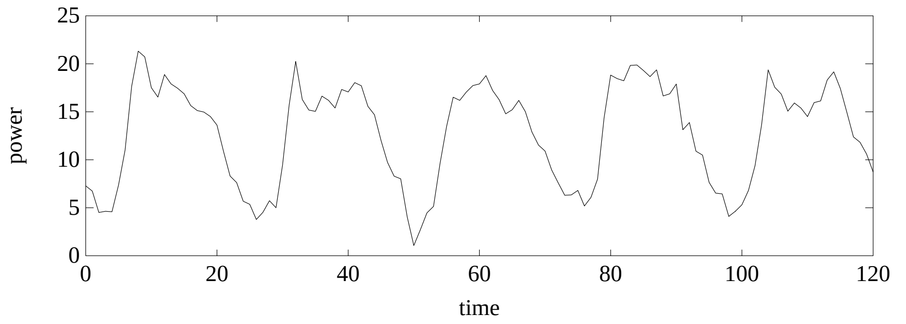

Fig. 1.1. Electric power consumption for a period of 120 h

no longer show the neat character of the linear quadratic case. Both input–output models and state space formalisms will be used extensively in the book. In the linear case, there are always close links between these two ways of treating dynamics, and it is fruitful to see how any concept appears in both types of model.

For illustration of the theories of stochastic systems that can be used, a few examples are in order.

Example 1.1 The consumption of electrical energy in an area varies considerably over time. A typical pattern is shown in Figure 1.1.

The energy consumption shows a regular variation through the day and decays to low values at night. There is also a random effect that adds to the regular effect. This random effect has several causes: effect of weather, special needs in industry, popular TV programs, *etc.* In order to generate the amount of power that is needed for every time instant, it is important to be able to forecast the demand a few hours ahead. The regular component of the consumption may be known, but there is a need to describe (*i.e.* model) the random contribution, and use that description to find good forecasts or *predictions* of its future value using currently available measurements. □

Example 1.2 In the processing industry, there are many examples of production of paper, pulp, concrete, chemicals, *etc.*, where variations in raw material, temperature and several other effects produce random variations in the final product. For several reasons, the producer may want to reduce such variations. One reason could be the quality requirements of the customers. Another could be the need for more efficient saving of energy and raw material. A third could be that smaller variations allow a more economical setpoint. This is illustrated in Figure 1.2, which shows how a reduced variation can allow the setpoint to be chosen closer to a critical level.

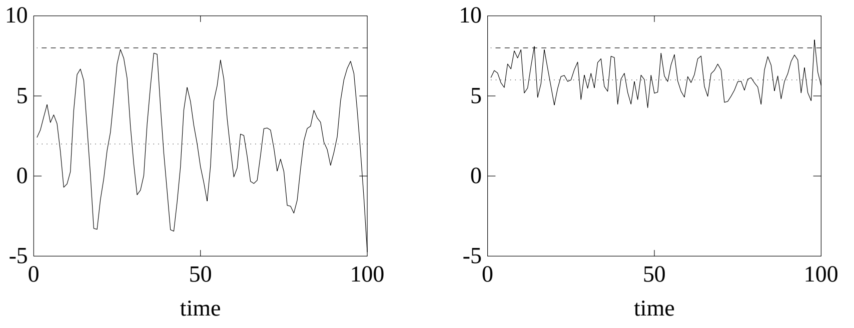

Fig. 1.2. Output variations (*solid lines*) around setpoints (*dotted lines*); critical values that presumably should not be passed (*dashed lines*); crude regulator (*left*) and a well-tuned regulator (*right*)

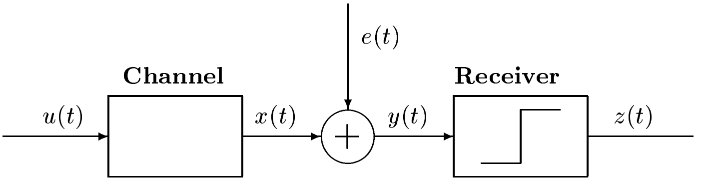

Fig. 1.3. Block diagram for a simple radio communication

To achieve efficient control of the process, it is often necessary to have a stochastic model of how the output is influenced both by the inputs (the control variables) and by disturbances. Such a model can then be the basis for the design of regulators, which seek to minimize the influence of the disturbances. □

Example 1.3 As yet another illustration, consider mobile radio communication, which in a very simplified form can be described as follows. The message to be transmitted is digitized. In this example it is represented as a binary signal, $u(t) = \pm 1$; see Figure 1.3.

The channel refers to the "system" or "filter" that describes how the signal is distorted before it arrives at the receiver. A typical reason for such distortions is that the signal propagates along several paths to the receiver. Signals that arrive after reflection travel a longer distance than direct signals and introduce a delay. There is often also noise, for example sensor noise in the receiver, $e(t)$, that adds to the signal, $x(t)$, so that the actual measurement is $y(t)$. A simple approach to reconstructing the transmitted signal is to take the

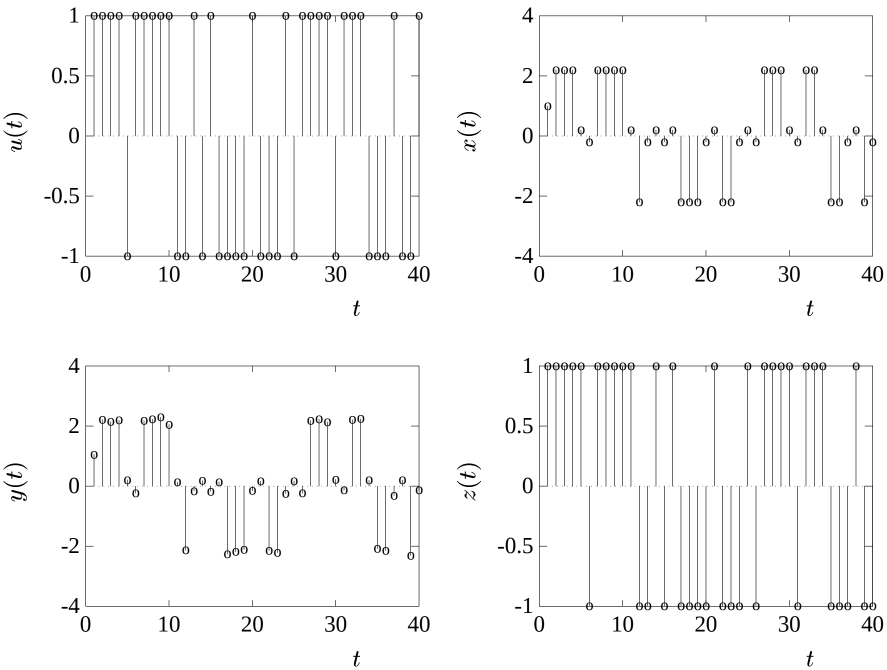

Fig. 1.4. Examples of signals for a digital radio communication

sign of $y(t)$ to form a binary signal $z(t)$. It should resemble the transmitted signal $x(t)$ for a good communication system. The procedure for determining $z(t)$ from the measurements $y(t)$ is called *equalizing*. The transmission causes a distortion of the transmitted signal, which is called *intersymbol interference*. A good equalizer will include a dynamic filter operating on $y(t)$ and not only the sign operator. To design such a filter, it is important to have a good description (*i.e.* a model) of the channel, and the statistical properties of the transmitted signal $x(t)$ and the disturbance $e(t)$. The "best" equalizer is a compromise between different objectives. Should there be no noise and the channel model be invertible, it is, of course, optimal to filter $y(t)$ by the inverse of the channel model. However, the inverse is often not stable, which makes the design more complicated. Another difficulty is how to take appropriate consideration of the noise. In the extreme case, when only the effect of the noise is considered, a filter giving zero as output would be ideal. In the general case, the filter must be a compromise between damping the noise and trying to "invert" the channel by a stable filter.

Figure 1.4 illustrates, by simulation for a simple case, what the signal $u(t)$, $x(t)$, $y(t)$ and $z(t)$ may look like.

This example can also illustrate the concept of *smoothing*. In order to reconstruct the transmitted signal $x(t)$ as efficiently as possible, it seems pertinent to allow the output $z(t)$ to depend not only on $y(s)$, $s \leq t$, but also on future data, $y(s)$, $s \leq t + \tau$. Such a principle would introduce a delay in the received message, so that ideally $z(t) = u(t - \tau)$. However, such a (small) delay can often be accepted, especially if it improves the quality of the outcome. □

In various communication systems, such as radar, sonar and radio communications, it is convenient to describe the signals as being complex-valued. For example, in radar, the amplitude of the echo (response) is a measure of the effective size of the target, and the phase (due to Doppler shift in the carrier frequency) is a measure of the target's radial velocity towards the radar. In many parts of the book, complex-valued signals and processes are treated in order to make the treatment as general as possible. In other parts, though, the more traditional approach of considering only real-valued signals is employed.

The need for complex-valued signal models can be heuristically motivated in various ways.

- The signals are often of the narrow-band type, meaning that they have their energy concentrated in a small frequency region. The signals can therefore be (approximately) characterized as sinewaves. Interesting information is contained in the amplitude and the phase. To model amplitudes, phases and how they are affected by linear filtering, it is convenient to introduce complex-valued modelling of the signal.
- A radio communication signal contains a low-frequency message that is modulated using a carrier signal of high frequency. The transmitted signal then has a frequency content that is varied slightly around the carrier frequency. Distortion affects this frequency content. After demodulation, when retrieving the low-frequency message, it turns out that the frequency content is not symmetric. This can be viewed as a sign that a complex-valued description of the signal is needed.

Not only may the signal be complex-valued, but the dynamic system itself may also be complex-valued. Section 3.A gives a brief account of complex-valued models of narrow-band signals and the properties of linear dynamic complex-valued systems.

Bibliography

There is a huge literature on stochastic dynamic systems. For some alternative books on estimation and control, see, for example:

Anderson, B.D.O., Moore, J.B., 1979. *Optimal Filtering.* Prentice Hall, Englewood Cliffs, NJ.

Åström, K.J., 1970. *Introduction to Stochastic Control.* Academic Press, New York.

Borrie, J.A., 1992. *Stochastic Systems for Engineers.* Prentice Hall International, Hemel Hempstead, UK.

Brown, R.G., 1983. *Introduction to Random Signal Analysis and Kalman Filtering.* John Wiley & Sons, New York.

Grimble, M.J., Johnson, M.A., 1988. *Optimal Control and Stochastic Estimation*, vol. 2., John Wiley & Sons, Chichester.

Jazwinski, A.H., 1970. *Stochastic Processes and Filtering Theory.* Academic Press, New York.

Kailath, T., Sayed, A.H., Hassibi, B., 2000. *Linear Estimation.* Prentice Hall, Upper Saddle River, NJ.

Lewis, F.L., 1986. *Optimal Estimation.* John Wiley & Sons, New York.

Maybeck, P.S., 1979–1982. *Stochastic Models, Estimation and Control*, vols 1–3. Academic Press, New York.

Needless to say, there are also many books dedicated to stochastic processes in general.

For some collections of historical key papers on estimation of stochastic systems, see:

Kailath, T. (Ed.), 1977. *Linear Least-Squares Estimation.* Dowden, Hutchinson and Ross, Inc., Stroudsburg, PA.

Sorenson, H. (Ed.), 1985. *Kalman Filtering: Theory and Application.* IEEE Press, New York.

This book deals with analysis and design for given stochastic models. To build (or estimate) dynamic models from experimental data is called system identification. For some general texts on that subject, see:

Ljung, L., 1999. *Identification – Theory for the User*, second ed. Prentice Hall, Upper Saddle River, NJ.

Söderström, T., Stoica, P., 1989. *System Identification.* Prentice Hall International, Hemel Hempstead, UK.